The Thin Lens Equation

- We must assume that all of the lenses we discuss are thin lenses; otherwise we will have to take into account lateral displacement of the light rays through the lens.

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f}$$

- This is called the lens equation.
- d_o = distance to the object from optical centre
- d_i = distance to the image from optical centre
- f = focal length of the lens

Sign Convention

1. All distances are measured from the optical centre of the lens.
2. Distances of real objects and images are positive.
3. Distances of virtual images are negative.
4. Object heights (h_o) and image heights (h_i) are positive when measured upward and negative when measured downward from the principle axis.

- To take into account the sign convention, the magnification equation becomes:

$$M=\frac{h_i}{h_o}=-\frac{d_i}{d_o}$$

$M=\frac{h_i}{h_o}$ $M=-\frac{d_i}{d_o}$

- A negative magnification means that the image is inverted from the original orientation.

* A negative focal length means the lens is diverging. and vice-versa.

Sample Problems

1. An object 8.0 cm high is 18 cm from a converging lens having a focal length of 10.0 cm.

a. How far is the image from the optical centre of the lens?

h_o d_o f

$d_i = ?$

$h_o = 8.0\text{cm}$

$d_o = 18\text{cm}$

$f = 10\text{cm}$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{d_i} = \frac{1}{10} - \frac{1}{18}$$

$$\frac{1}{18} + \frac{1}{d_i} = \frac{1}{10}$$

$$\frac{1}{d_i} = 0.0444\ldots$$

$$\Rightarrow d_i = (0.0444)^{-1}$$

$$\boxed{d_i = 22.5}$$

b. How tall is the image?

$d_i = ?$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} \Rightarrow \frac{h_i}{8} = -\frac{22.5}{18}$$

upside down

$$h_i = \frac{-22.5 \times 8}{18} = \boxed{-10\text{cm}}$$

2. A diverging lens has a focal length of –4.0 cm. If an object is placed 8.0 cm from the lens, how from the optical centre is the image?

$d_o = 8\text{cm}$

$d_i = ?$

$f = -4.0\text{ cm}$

$** \; 0 \leq d_i \leq -4$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{8} + \frac{1}{d_i} = \frac{1}{-4} \Rightarrow \frac{1}{8} + \frac{1}{d_i} = \frac{-1}{4}$$

$$\frac{1}{d_i} = \frac{-1}{4} - \frac{1}{8} = -0.375$$

$$d_i = (-0.375)^{-1}$$

$$\boxed{= -2.7\text{cm}}$$