
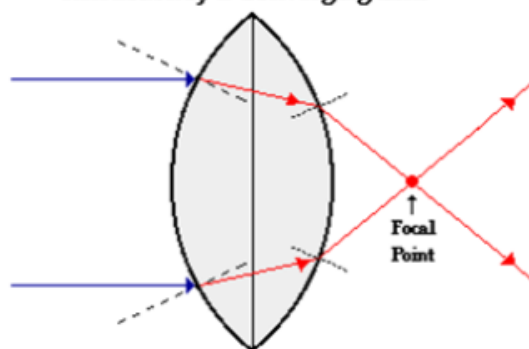


Lenses

- ⇒ Lenses are divided into two types:
 - Converging Lenses – Focuses light rays to a point.
 - Diverging Lenses – Spreads light rays apart.
- ⇒ Refraction in Lenses
 - Whether a lens focuses or spreads light rays depends on the shape of the lens → where the angle of incidence is relative to the normal.

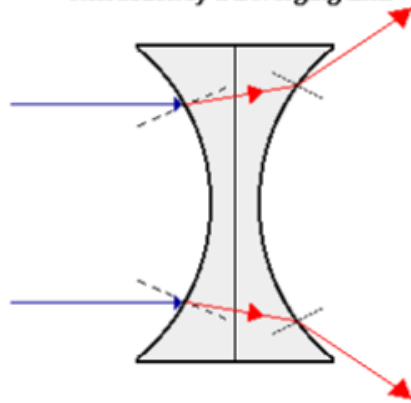
Refraction by a Converging Lens

Online Demo 



Incident rays which travel parallel to the principal axis will refract through the lens and converge to a point.

Refraction by a Diverging Lens



Incident rays traveling parallel to the principal axis will refract through the lens and diverge, never intersecting.

- ⇒ A lens is usually a circular piece of glass with uniformly curved surfaces that refract light passing through.
- ⇒ A series of rays will be refracted by different amounts depending on where they hit the surface.
- ⇒ An image is formed where the light rays meet.

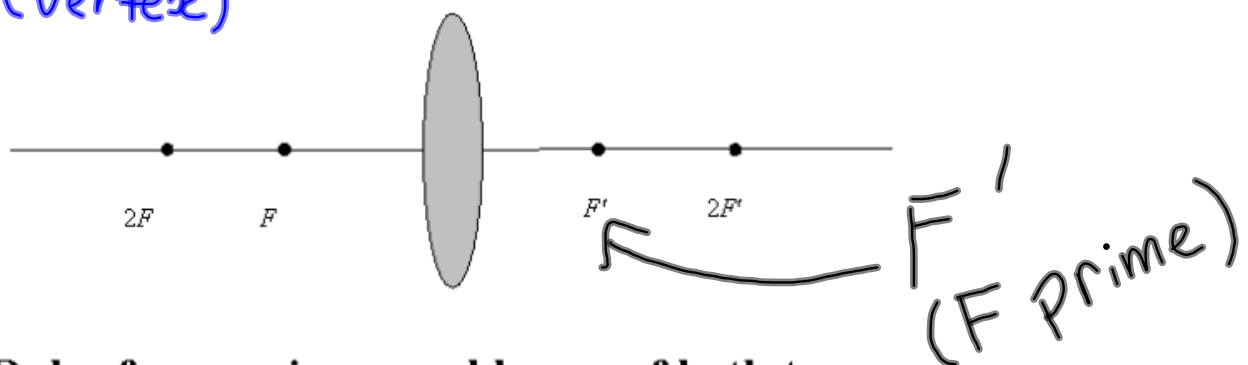
Terminology

Principle axis: A line that goes through the centre of the lens.

Principle focus (F or F'): The focal point on the principle axes.

Optical centre: The centre of gravity of the lens.

(Vertex)



Rules for rays in curved lenses of both types:

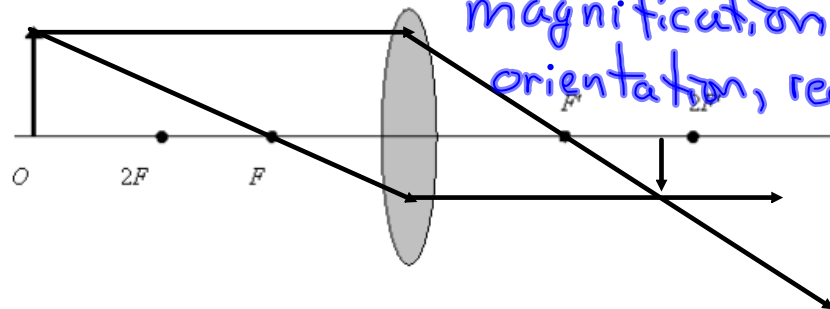
1. A ray that is parallel to the principle axis is refracted so that it passes through the principle focus (F or F').

2. A ray that passes through (or appears to pass through) the principle focus (F or F') is refracted parallel to the principle axis.
3. A ray that passes through the optical centre goes straight through, without bending.

4 Characteristics of an image:

magnification, location, orientation, real or virtual

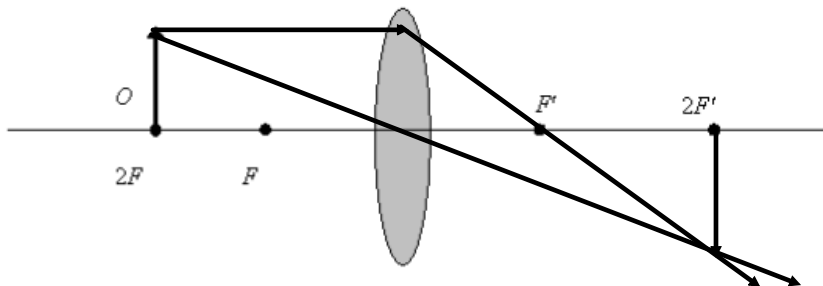
1. Object beyond $2F$:



⇒ Characteristics

○ Smaller, between F' and $2F'$, inverted, real.

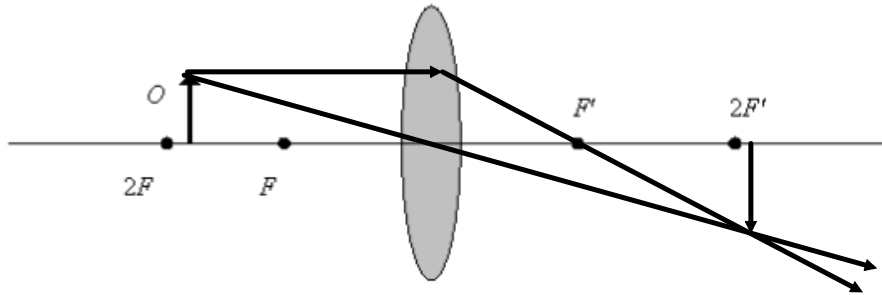
2. Object at $2F$:



⇒ Characteristics

Same size, inverted, @ $2F'$, real

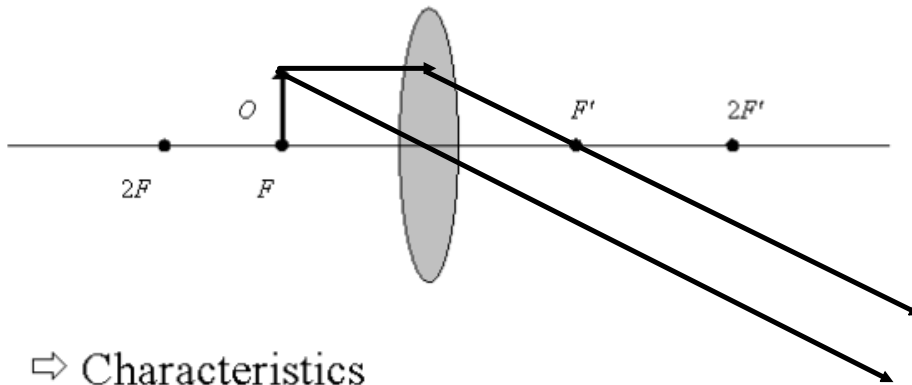
3. Object between F and $2F$



⇒ Characteristics

- Enlarged, inverted, beyond $2F'$
real.

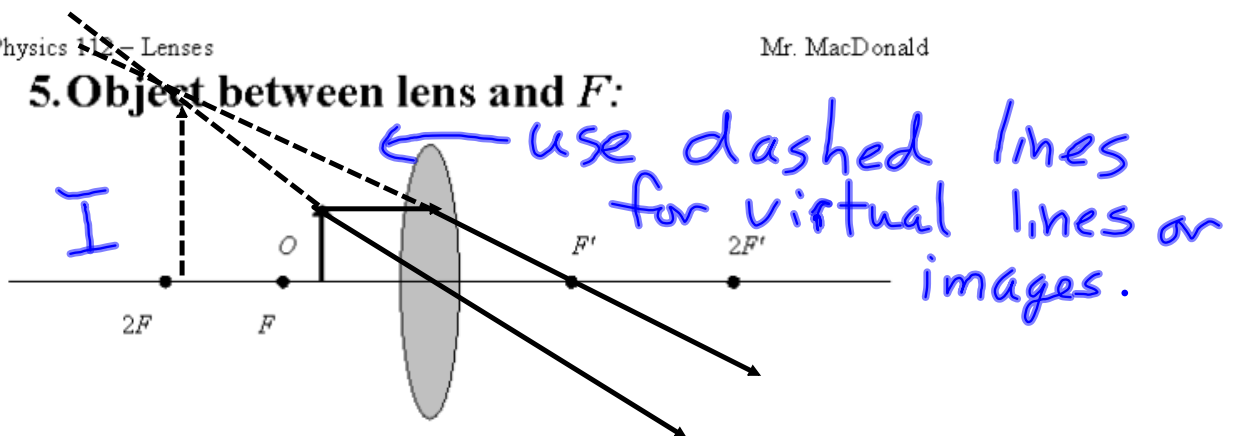
4. Object at F :



⇒ Characteristics

No image formed

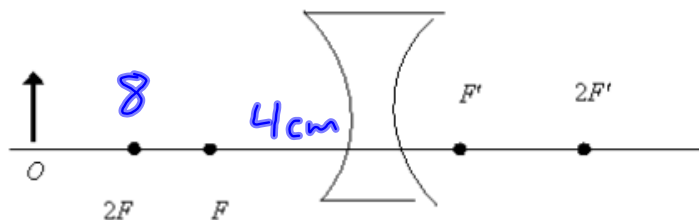
5. Object between lens and F :



⇒ Image is:

- enlarged, Virtual, behind object, upright

⇒ In a diverging lens, parallel rays are refracted so that they radiate out from a virtual focus (like with diverging mirrors).



⇒ For all positions of the object, the image is virtual, erect, smaller, and is always located between the principle focus and the optical centre.

The Thin Lens Equation

⇒ We must assume that all of the lenses we discuss are thin lenses; otherwise we will have to take into account lateral displacement of the light rays through the lens.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

⇒ This is called the lens equation.

⇒ d_o = distance to the object from optical centre

⇒ d_i = distance to the image from optical centre

⇒ f = focal length of the lens

Sign Convention

1. All distances are measured from the optical centre of the lens.
2. Distances of real objects and images are positive.
3. Distances of virtual images are negative.
4. Object heights (h_o) and image heights (h_i) are positive when measured upward and negative when measured downward from the principle axis.

⇒ To take into account the sign convention, the magnification equation becomes:

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

⇒ A negative magnification means that the image is inverted from the original orientation.

* A negative focal length means the lens is diverging. and vice-versa.

Sample Problems

1. An object h_o 8.0 cm high is d_o 18 cm from a converging lens having a focal length of 10.0 cm f .

a. How far is the image from the optical centre of the lens? d_i

$$\begin{array}{lcl}
 h_o = 8 & \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} & \rightarrow \frac{1}{d_i} = \frac{1}{10} - \frac{1}{18} \quad \boxed{x^{-1}} \\
 d_o = 18 & & \frac{1}{d_i} = 0.0444... \\
 f = 10 & \frac{1}{18} + \frac{1}{d_i} = \frac{1}{10} & d_i = \frac{1}{0.0444...} = \boxed{22.5 \text{ cm}} \\
 d_i = ? & &
 \end{array}$$

b. How tall is the image?

$$\begin{array}{lcl}
 h_i = ? & \frac{h_i}{h_o} = -\frac{d_i}{d_o} & \rightarrow \boxed{h_i = -10 \text{ cm}} \\
 d_i = 22.5 & & \\
 d_o = 18 & \frac{h_i}{8} = -\frac{22.5}{18} & \\
 h_o = 8 & & f
 \end{array}$$

2. A diverging lens has a focal length of -4.0 cm. If an object is placed d_o 8.0 cm from the lens, how from the optical centre is the image? d_i

$$\begin{array}{lcl}
 d_i = ? & \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} & \\
 d_o = 8 & & \\
 f = -4 \text{ cm} & \frac{1}{8} + \frac{1}{d_i} = \frac{1}{-4} & \\
 & \frac{1}{d_i} = -\frac{1}{4} - \frac{1}{8} = -\frac{3}{8} & \\
 & d_i = -\frac{8}{3} = \boxed{-2.7 \text{ cm}} & \text{Virtual}
 \end{array}$$

④

$$h_o = 10 \text{ cm}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$d_o = 60 \text{ cm}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$f = -20 \text{ cm}$$

$$d_i = ?$$

$$h_i = ?$$

$$\frac{1}{60} + \frac{1}{d_i} = \frac{-1}{20}$$

$$\frac{1}{d_i} = \frac{-1}{20} - \frac{1}{60}$$

$$\frac{1}{d_i} = -0.06667$$

$$d_i = \frac{1}{-0.06667}$$

$$x^{-1}$$

$$\text{or} \frac{1}{x}$$

$$d_i = -15 \text{ cm}$$

$$\frac{h_i}{10 \text{ cm}} = \frac{-(-15)}{60}$$

$$h_i = \frac{+15}{60} \times 10 = 2.5 \text{ cm}$$

#6 $d_{\text{sun}} (d_o) = 1.5 \times 10^{14} \text{ cm}$

Lenses Quiz

1. A converging lens in a slide projector has a focal length of 15 cm. The screen is 52 cm away. How far from the lens should a 3.5 cm high slide be placed to have a focused image? What will be the size of the image?
2. A converging lens is used to magnify letters in a newspaper. The focal length is 22 cm and the letters are normally 0.4 cm high. What is the size of the image if the letters are located 5.25 cm from the lens?

Examples

A convex lens has a focal length of 12 cm. How far from the lens should an object be to produce a real image 4 times larger than the object?

$d_o = ?$

$$f = 12 \text{ cm} \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad \rightarrow \quad -4 = -\frac{d_i}{d_o} \quad \text{* Solve for } d_i$$

$$M = -4 \quad M = -\frac{d_i}{d_o} \quad \rightarrow \quad -4d_o = -d_i$$

$\underline{4d_o = d_i} \quad \text{* Sub into lens equation}$

\uparrow
real

$$4 \times \frac{1}{d_o} + \frac{1}{4d_o} = \frac{1}{12} \quad \rightarrow \quad \frac{5}{4d_o} = \frac{1}{12}$$

$$\frac{4}{4d_o} + \frac{1}{4d_o} = \frac{1}{12} \quad \rightarrow \quad 60 = 4d_o$$

$15 \text{ cm} = d_o$

A convex lens has a focal length of 8.0 cm and is used as a magnifying glass. How far from the lens should an object be to produce a virtual image 3 times larger than the object?

$$f = 8 \quad +3 = -\frac{d_i}{d_o} \quad \frac{1}{d_o} + \left(\frac{1}{-3d_o} \right) = \frac{1}{8}$$

$$M = +3$$

$$-3d_o = d_i$$

$$\frac{1}{d_o} - \frac{1}{3d_o} = \frac{1}{8}$$

$$\frac{3}{3d_o} - \frac{1}{3d_o} = \frac{1}{8}$$

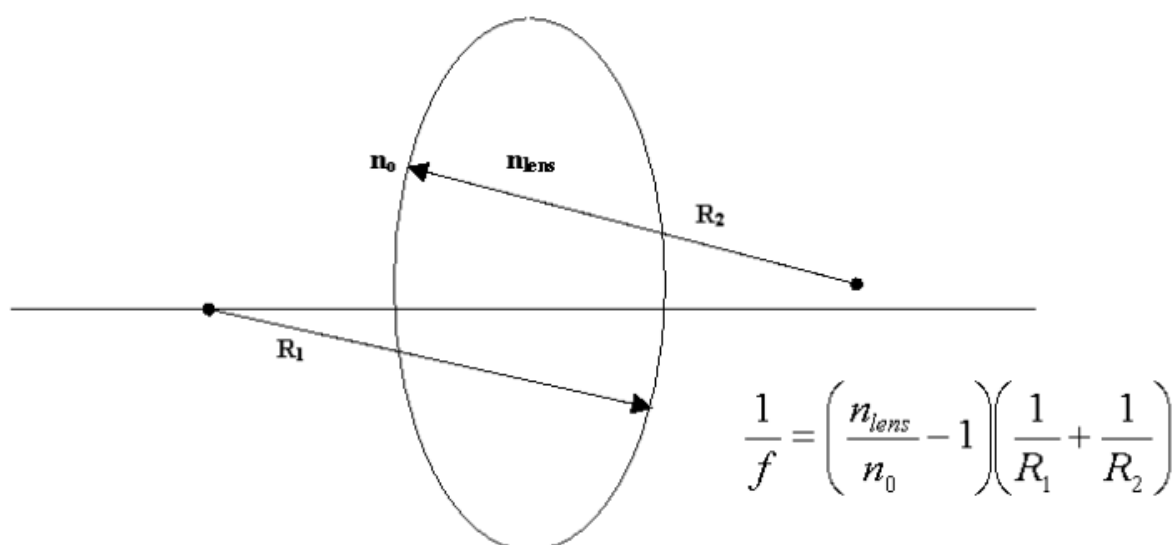
$$\frac{2}{3d_o} = \frac{1}{8}$$

$$16 = 3d_o$$

$5.3 \text{ cm} = d_o$

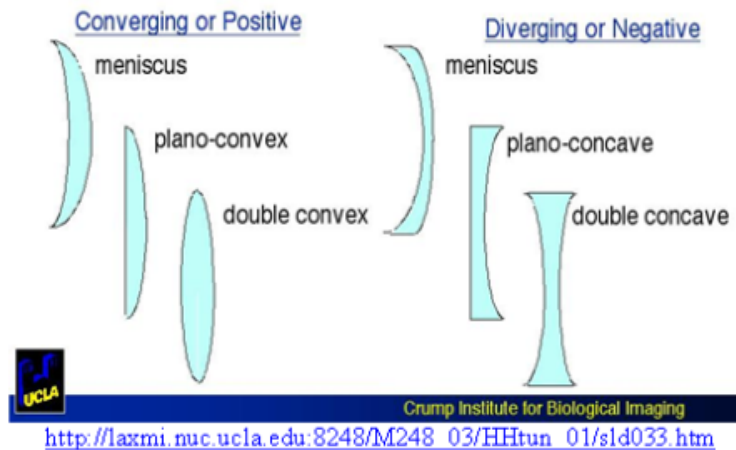
Lens Makers Formula

- Lenses always consist of two sides, but the lens does not have to be symmetric. One side could be diverging and the other converging etc.
- Knowing the curvature for the lenses allows for a formula relating the focal length and index of refraction:



- n_0 is the index of refraction of the surrounding medium.
- Depending on the shape of the lens, as light approaches the lens from the outside, the radius is measured as either positive or negative.
 - A positive radius measurement is used if the side is convex.
 - A negative radius measurement is used if the side is concave.
 - When solving for focal length:
 - If $f > 0$, then the lens is converging.
 - If $f < 0$ then the lens is diverging.

Types of Lenses



Examples

1. What is the focal length of a Plexiglas plano-convex lens that has a radius of 15.7 cm?

$$\frac{1}{f} = \left(\frac{1.51}{1} - 1 \right) \left(\frac{+1}{15.7} + \frac{1}{\infty} \right)$$

$R = \infty$

$$\frac{1}{f} = (0.51) \left(\frac{1}{15.7} \right)$$

2. What is the measure of each radius of a double concave ruby lens that has a focal length of -25 cm?

3. In air a flint glass converging meniscus lens has a focal length of 14 cm. What will be the focal length of the lens if it is submerged in water?