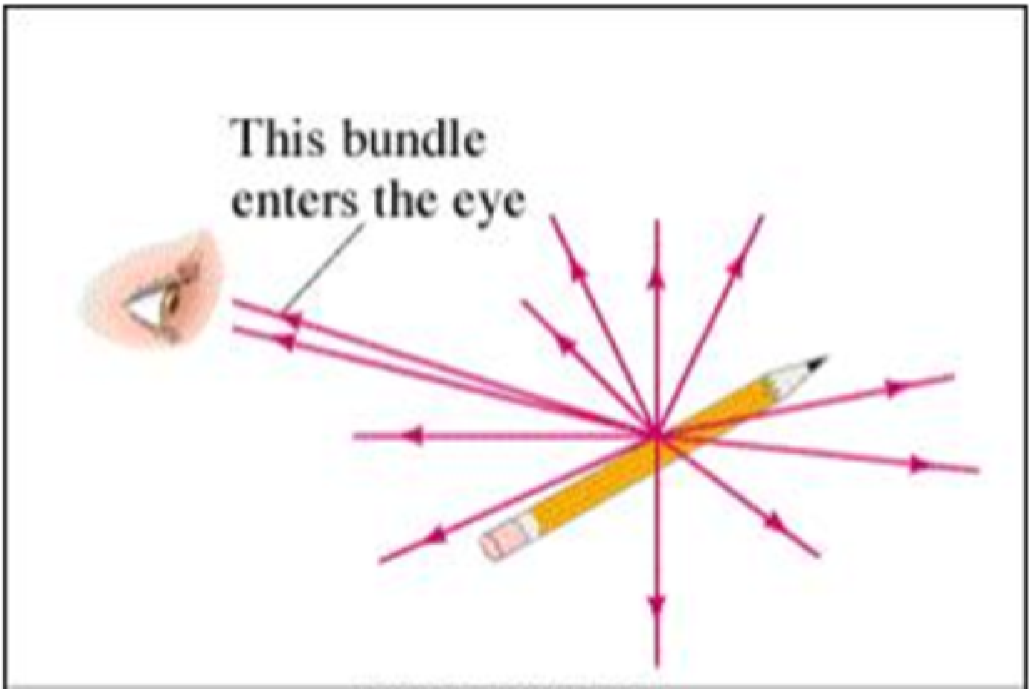
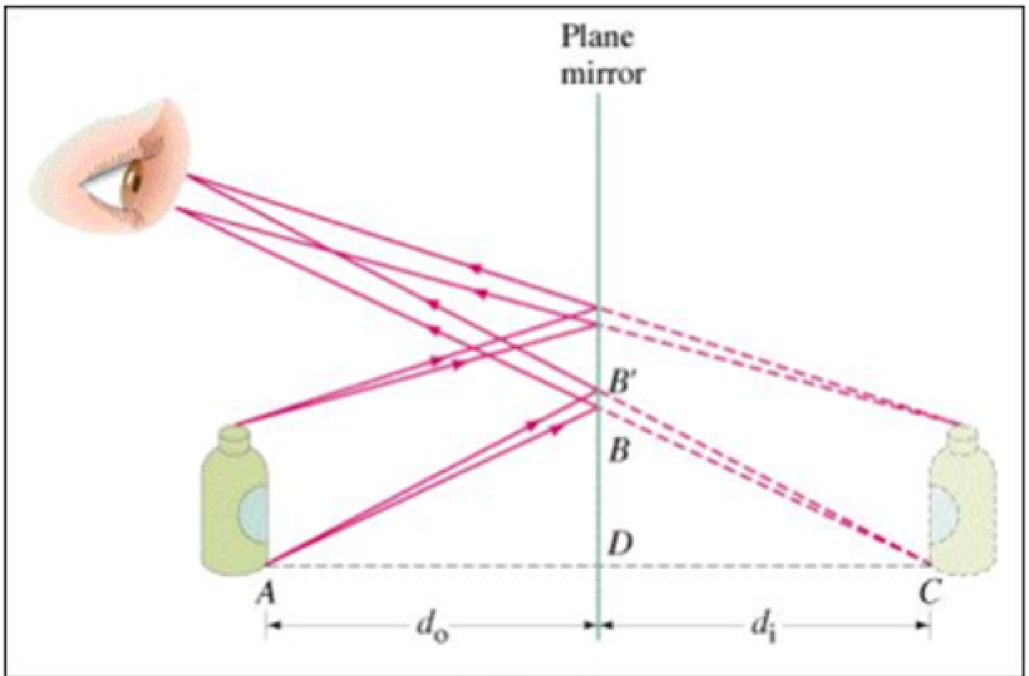


FG23_001.PCT



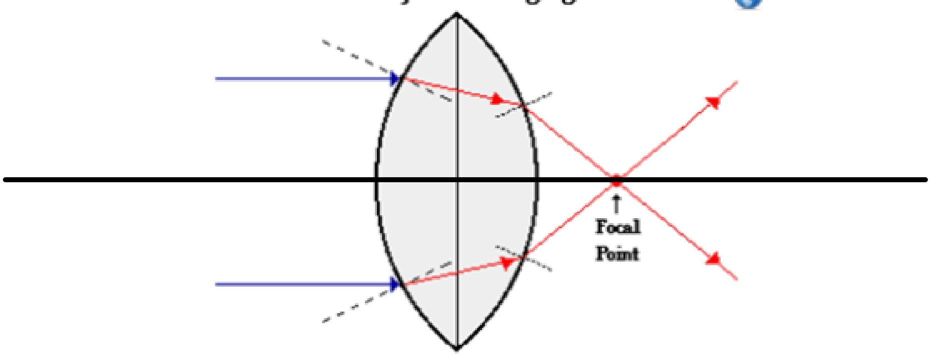
FG23_006.PCT

Lenses

- ⇨ Lenses are divided into two types:
  - Converging Lenses – Focuses light rays to a point.
  - Diverging Lenses – Spreads light rays apart.
- ⇨ Refraction in Lenses
  - Whether a lens focuses or spreads light rays depends on the shape of the lens → where the angle of incidence is relative to the normal.

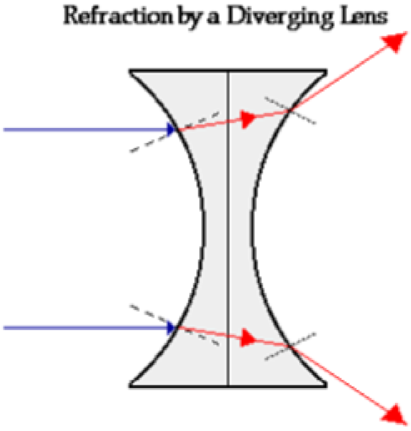
Online Demo

Refraction by a Converging Lens



Incident rays which travel parallel to the principal axis will refract through the lens and converge to a point.

Refraction by a Diverging Lens

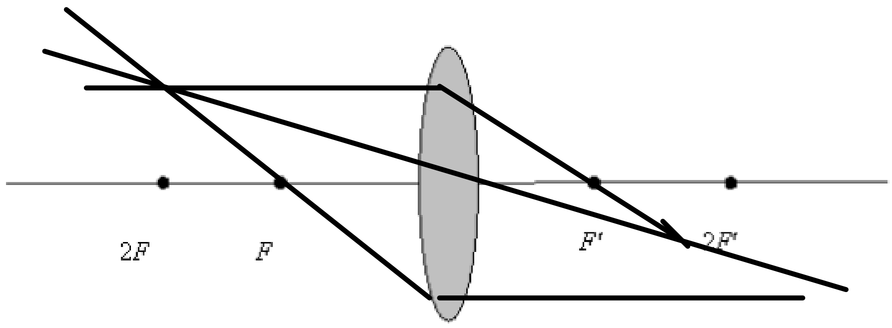


Incident rays traveling parallel to the principal axis will refract through the lens and diverge, never intersecting.

- ⇨ A lens is usually a circular piece of glass with uniformly curved surfaces that refract light passing through.
- ⇨ A series of rays will be refracted by different amounts depending on where they hit the surface.
- ⇨ An image is formed where the light rays meet.

Terminology

Principle axis: A line that goes through the centre of the lens.
Principle focus (F or F`): The focal point on the principle axes.
Optical centre: The centre of gravity of the lens.

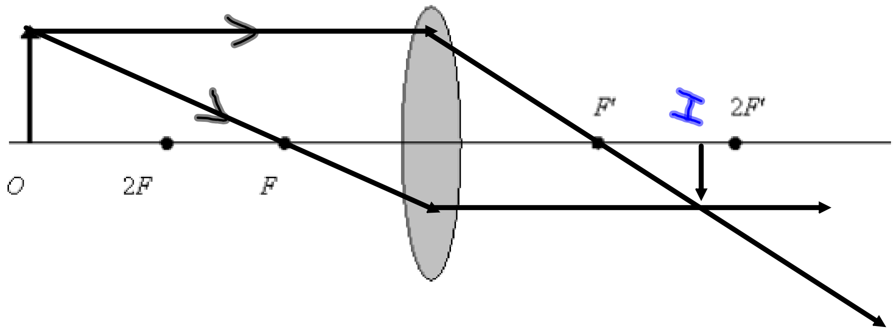


Rules for rays in curved lenses of both types:

1. A ray that is parallel to the principle axis is refracted so that it passes through the principle focus (F or F`).

2. A ray that passes through (or appears to pass through) the principle focus (F or F') is refracted parallel to the principle axis.
3. A ray that passes through the optical centre goes straight through, without bending.

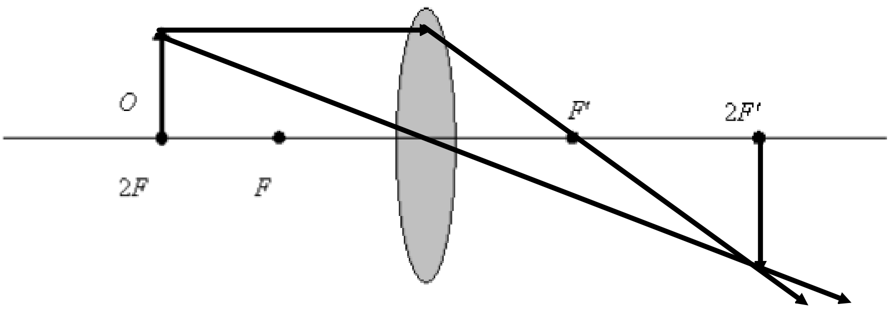
1. Object beyond *2F*:



⇨ Characteristics (of the image)

- Between F' and 2F', smaller, inverted, real

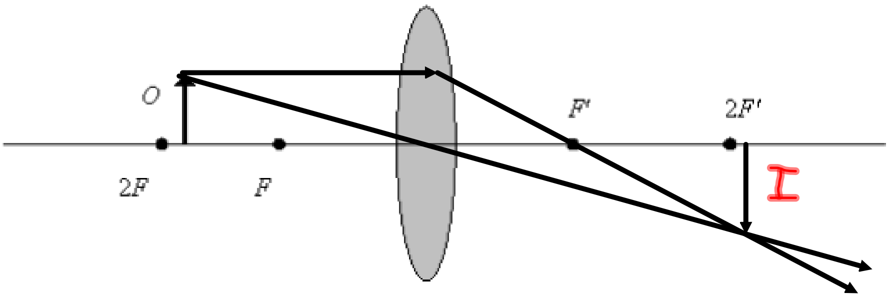
2. Object at 2*F*:



⇨ Characteristics

@ 2F', real, inverted, same size

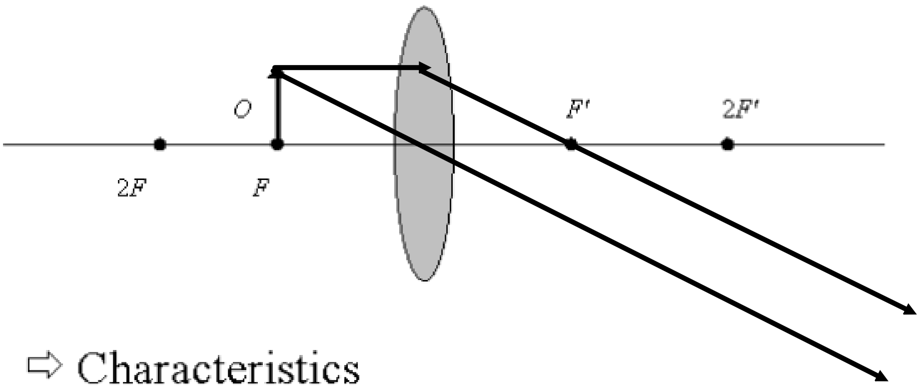
3. Object between *F* and 2*F*



⇨ Characteristics

- Beyond 2F'; real, inverted, enlarged

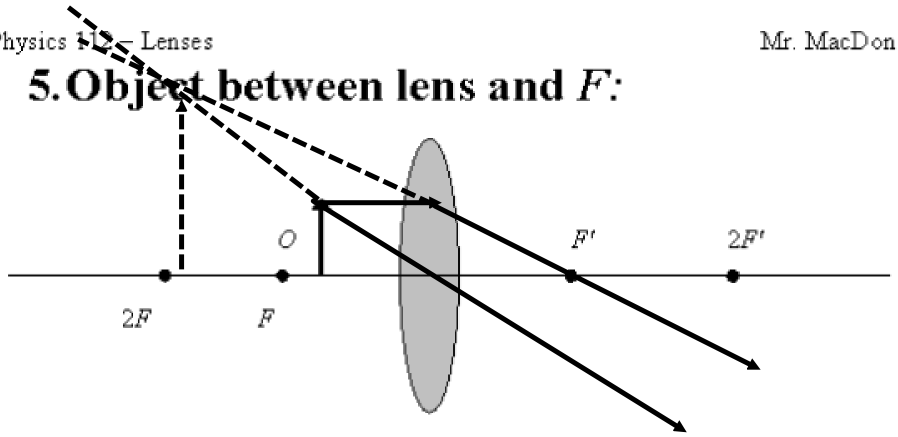
4. Object at *F*:



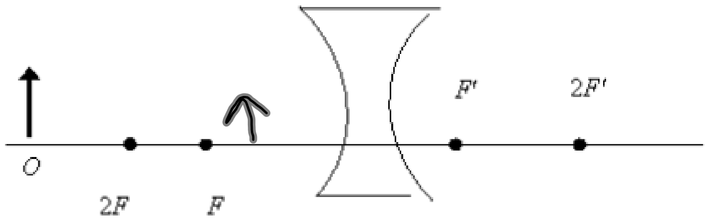
⇨ Characteristics

No image formed

5. Object between lens and F:



- Image is:
  - Virtual, same side of lens as object, upright, enlarged.
- In a diverging lens, parallel rays are refracted so that they radiate out from a virtual focus (like with diverging mirrors).



- For all positions of the object, the image is virtual, erect, smaller, and is always located between the principle focus and the optical centre.

The Thin Lens Equation

⇨ We must assume that all of the lenses we discuss are thin lenses; otherwise we will have to take into account lateral displacement of the light rays through the lens.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

⇨ This is called the lens equation.

⇨ d_o = distance to the object from optical centre

⇨ d_i = distance to the image from optical centre

⇨ f = focal length of the lens

Sign Convention

1. All distances are measured from the optical centre of the lens.
2. Distances of real objects and images are positive.
3. Distances of virtual images are negative.
4. Object heights (h_o) and image heights (h_i) are positive when measured upward and negative when measured downward from the principle axis.

⇨ To take into account the sign convention, the magnification equation becomes: $M = \frac{h_i}{h_o}$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$M = \frac{-d_i}{d_o}$

⇨ A negative magnification means that the image is inverted from the original orientation.

Sample Problems

1. An object 8.0 cm high is 18 cm from a converging lens having a focal length of 10.0 cm.
   a. How far is the image from the optical centre of the lens?

$d_i = ?$
$h_o = 8cm$
$d_o = 18cm$
$f = 10cm$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{18} + \frac{1}{d_i} = \frac{1}{10}$$

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{18}$$

$$\frac{1}{d_i} = 0.10 - 0.0556$$

$$\frac{1}{d_i} = 0.0444$$

$$d_i = (0.0444)^{-1}$$

$$d_i = 22.5cm$$

$a^b/_c$ x^{-1} $1/x$

   b. How tall is the image?

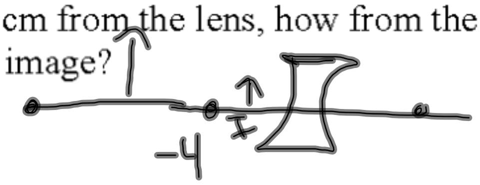
$h_i = ?$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{8} = -\frac{(22.5)}{18}$$

$$h_i = -10cm$$

2. A diverging lens has a focal length of –4.0 cm. If an object is placed 8.0 cm from the lens, how from the optical centre is the image?



$d_i = ?$
$d_o = 8cm$
$f = -4cm$

$$-4 \le d_i \le 0$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{8} + \frac{1}{d_i} = \frac{-1}{4}$$

$$\frac{1}{d_i} = \frac{-1}{4} - \frac{1}{8}$$

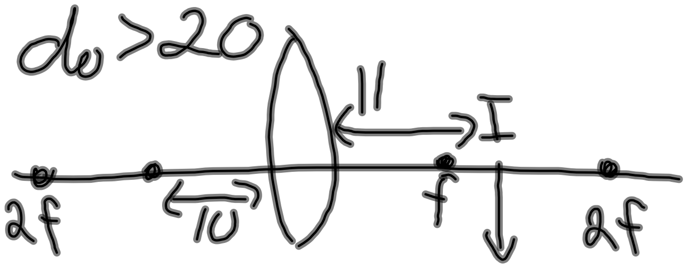
$$\frac{1}{d_i} = -0.375$$

$$d_i = (-0.375)^{-1} = -2.7cm$$

2) $d_o = ?$

$d_i = 11\,cm$

$f = 10\,cm$



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \longrightarrow \frac{1}{d_o} + \frac{1}{11} = \frac{1}{10}$$

$$\frac{1}{d_o} = \frac{1}{10} - \frac{1}{11}$$

$$\frac{1}{d_o} = 0.0091$$

$$\boxed{d_o = 110\,cm}$$

#1-13

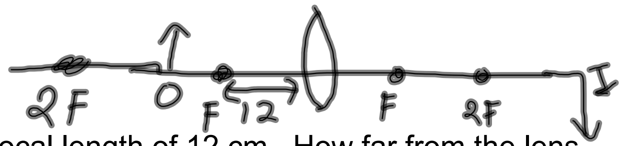
(8 if you want a challange!)

Lenses Quiz

1. A converging lens in a slide projector has a focal length of 15 cm. The screen is 52 cm from the lens. How far from the lens should a 3.5 cm high slide be placed to have a focused image? What will be the size of the image?

2. A converging lens is used to magnify letters in a newspaper. The focal length is 22 cm and the letters are normally 0.4 cm high. What is the size of the image if the letters are located 5.25 cm from the lens?

Examples



A convex lens has a focal length of 12 cm. How far from the lens should an object be to produce a real image 4 times larger than the object?

$12 \le d_o \le 24 \rightarrow -4 = \frac{-d_i}{d_o}$

$d_o = ?$

$f = 12$ $\quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

$M = -4$ (real, inverted image) $\quad M = \frac{-d_i}{d_o}$ (* solve for d_i)

$4d_o = d_i$ * sub into lens eq.

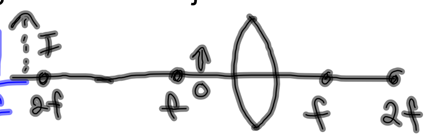
$\frac{1 \times 4}{d_o \times 4} + \frac{1}{4d_o} = \frac{1}{12}$

$\frac{4}{4d_o} + \frac{1}{4d_o} = \frac{1}{12} \rightarrow \frac{5}{4d_o} = \frac{1}{12}$

$60 = 4d_o$

$15\text{cm} = d_o$

A convex lens has a focal length of 8.0 cm and is used as a magnifying glass. How far from the lens should an object be to produce a virtual image 3 times larger than the object?



$d_o = ?$ $\quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

$f = 8$ $\quad 0 \le d_o \le 8$

$M = +3$ (virtual) $\quad M = -\frac{d_i}{d_o}$

$3 = \frac{-d_i}{d_o} \rightarrow -3d_o = d_i$ * sub into lens eq.

$\frac{1}{d_o} + \frac{-1}{3d_o} = \frac{1}{8} \rightarrow \frac{1}{d_o} - \frac{1}{3d_o} = \frac{1}{8}$

$\frac{3}{3d_o} - \frac{1}{3d_o} = \frac{1}{8}$ $\quad \frac{2}{3d_o} = \frac{1}{8}$

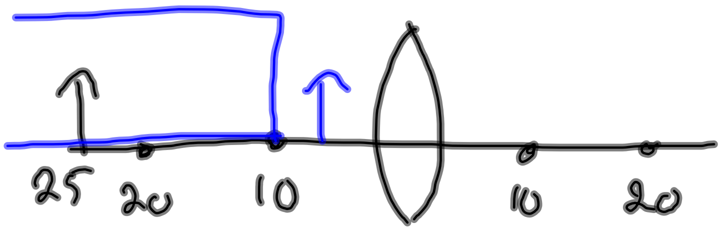
$16 = 3d_o$

$5.3\text{cm} = d_o$

#15-16 of sheet

An object is 25 cm from a converging lens that has a focal length of 10 cm. Which one of the following is true for the location of the image?

a) 0 < di < 10
b) 10 < di < 20
c) di > 20
d) di = ∞

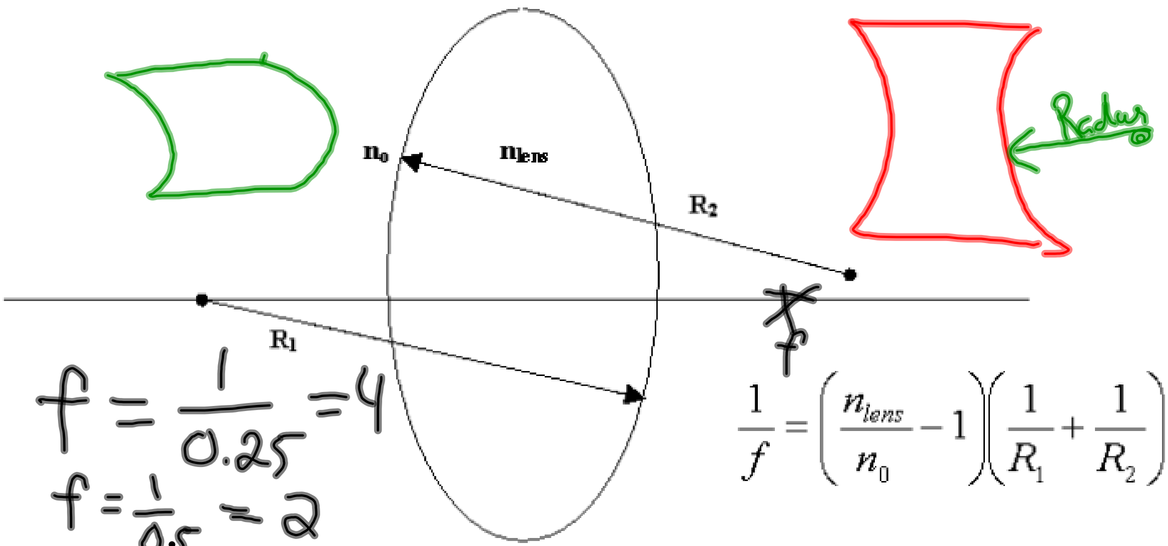


An object is 12 cm from a converging lens that has a focal length of 15 cm. Which one of the following is true for the location of the image?

a) 0 < di < 15
b) 15 < di < 30
c) di < -15
d) -15 < di < 0

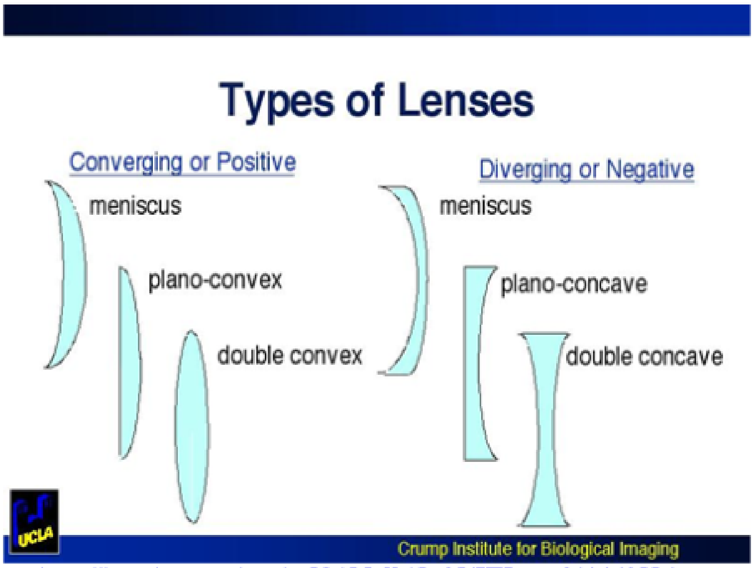
Lens Makers Formula

- Lenses always consist of two sides, but the lens does not have to be symmetric. One side could be diverging and the other converging etc.
- Knowing the curvature for the lenses allows for a formula relating the focal length and index of refraction:



$$\frac{1}{f} = \left(\frac{n_{lens}}{n_0} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

- n_o is the index of refraction of the surrounding medium.
- Depending on the shape of the lens, as light approaches the lens from the outside, the radius is measured as either positive or negative.
  - A positive radius measurement is used if the side is convex.
  - A negative radius measurement is used if the side is concave.
  - When solving for focal length:
    - If $f > 0$, then the lens is converging.
    - If $f < 0$ then the lens is diverging.



http://laxmi.nuc.ucla.edu:8248/M248_03/HHtun_01/sld033.htm

Examples

1. What is the focal length of a Plexiglas plano-convex lens that has a radius of 15.7 cm?

$f = ?$

$n_{lens} = 1.51$

$n_o = 1.0003$

$R_1 = \infty$

$R_2 = +15.7$

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_o} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{1.51}{1.0003} - 1\right)\left(\frac{1}{\infty} + \frac{1}{15.7}\right)$$

($\frac{1}{\infty} \to 0$)

$$\frac{1}{f} = (0.51)(0.0637)$$

$$\frac{1}{f} = 0.0325$$

x^{-1}

$1/x$

$$\frac{1}{0.0325}$$

$$\boxed{f = 30.8\,\text{cm}}$$

A crown glass double-convex lens is placed from air into water. How will the focal length change in water compared to that in air?
a) Increase
b) decrease
c) will not change

#17 - 24, 26, 29, 32-34

A double-convex lens with n = 1.2 is place from air into water. Which one of the following best explains what will happen?
a) it will continue to converge light
b) it will now diverge light
c) focal length will be zero
d) focal length will be infinity (undefined)

If the index of refraction of a lens equals that of the medium in which it is placed, what will be the focal length?
a) Zero
b) Undefined

2. What is the measure of each radius of a double concave ruby lens that has a focal length of –25 cm?

3. In air a flint glass converging meniscus lens has a focal length of 14 cm. What will be the focal length of the lens if it is submerged in water?