Center of Mass

We will need to determine the center of mass of objects for upcoming problems. The center of mass is a point in an object where the mass seems to be concentrated.

Types of Motion - Large Objects

The motion of large objects can be divided into two types, ***translational*** and ***rotational***.

translational motion - the motion of an object from one point to another

rotational motion - the motion of an object about one point (pivot point or fulcrum)

Figure 10.15 The wrench is rotating around the mark on the wrench while the mark is moving in a straight line.

Physics - McGraw-Hill Ryerson, Page 491

Torque (MHR - Page 490)

Torque occurs when a force is applied to an object and that force causes the object to rotate.

"Seesaw" Demonstration

1. Balance the rod on the coffe can.

2. a) What happens if a 20 g mass is placed at the center of the seesaw (the pivot point)?

   Even though a force has been applied to the rod, the rod does not rotate.

   b) What happens if a 20g mass is placed to the right of the pivot point?

   The rod rotates, therefore torque has been produced.

   c) What was different about the two trials?

   A force must be applied at some distance from the pivot point.

3. Place a 20 g mass on one side of the pivot point. Where would another 20 g mass have to be placed in order to keep the rod balanced?

   The second mass must be placed at the same distance from the pivot point as the first, but on the other side of the pivot point.

4. Place a 50 g mass on one side of the pivot point. Where would a 100 g mass have to be placed in order for the masses to be in a state of equilibrium?

   The 100 g has to be placed at half the distance from the pivot point as the 50 g mass.

☆

Torque can be defined as:

$$\boxed{\cancel{\vec{\tau} = r\vec{F}\sin\theta}}$$

perpendicular

τ * -> torque (Nm) $\tau = r F_{\perp}$ ← ⊥ component of Force

* this symbol represents the Greek letter *tau*

r -> distance from pivot point to the application of the force (m)

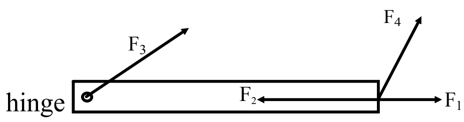
F -> force applied (N)

θ -> angle between r and F when they start at the same point (degrees)

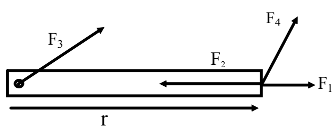
Torque is a ***vector***. The direction of torque is based on the direction in which the force would cause the object to rotate if it were acting alone.

CW: clockwise (-)
CCW: counter-clockwise (+)

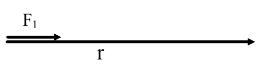
The diagram below shows four forces acting on a door. Which forces will cause the door to rotate?



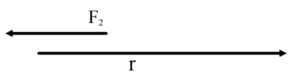
Only the component of F_4 perpendicular to r produces torque.



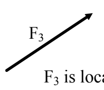
We can verify our previous answers by examining the equation.



F_1: $\theta = 0^o$
$\sin 0^o = 0$
$\tau = 0$ Nm

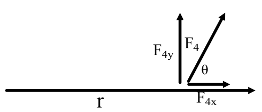


F_2: $\theta = 180^o$
$\sin 180^o = 0$
$\tau = 0$ Nm



F_3 is located at the pivot point.

F_3: $r = 0$ m
$\tau = 0$ Nm

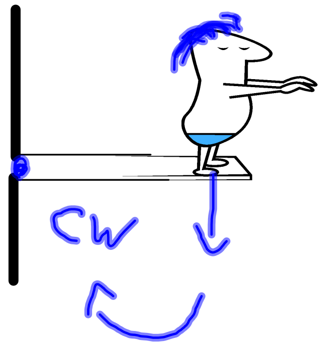


F_4: $r \neq 0$ m and $\sin\theta \neq 0$
F_{4x} will cause the door to rotate!

Label the Pivot Point

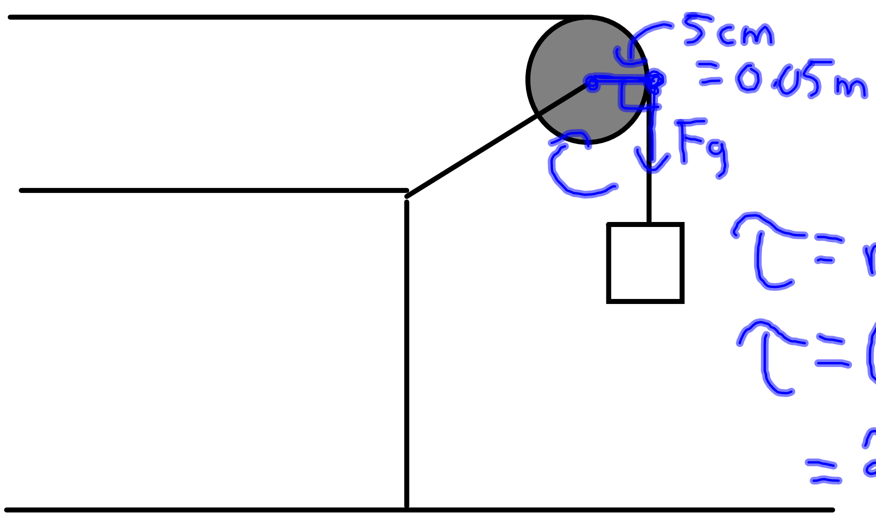
Example: A 490 N man stands at the end of a diving board at a distance of 1.5 m from the point at which it is attached to the tower. What is the torque the man exerts on the board?
(735 Nm, CW or -735 Nm)

$$\tau = rF\sin\theta$$
$$= (1.5\,m)(490)\sin 90^\circ$$



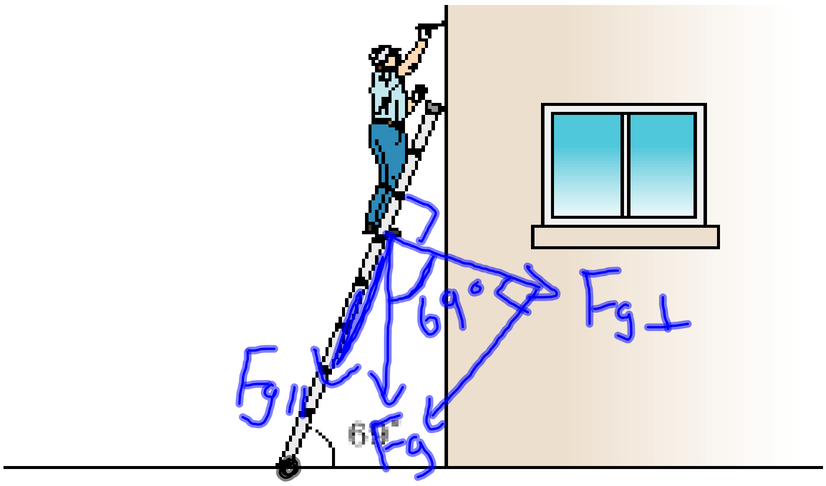
$$\tau = -735\ Nm$$

Example: A 5.0 kg mass is attached as shown to a pulley of radius 5.0 cm. What torque is produced by the mass?
(2.5 Nm, CW or -2.5 Nm)



$$\tau = rF\sin\theta$$
$$\tau = (0.05)(5.kg)9.81$$
$$= 2.5\ Nm\ CW$$
$$= -2.5\ Nm$$

Example: A 64 kg painter is standing three fourths of the distance up a ladder that is 3.0 m long. If the ladder makes an angle of 69° with the ground, what torque does the painter's weight exert on the ladder? (5.1×10^2 Nm, CW)



$$\tau = rF_\perp$$

$$= \underbrace{(0.75)(3.0)}_{\text{dist up ramp}}\underbrace{(64kg)(9.81)\cos 69}_{F_{g\perp}\text{ to ladder}}$$

$$\tau = 506 \text{ Nm CW}$$

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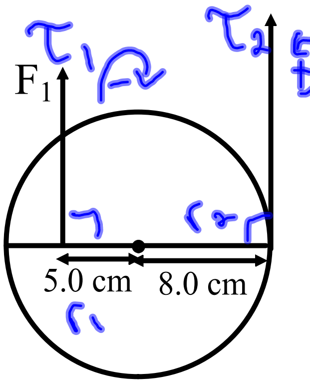
Net Torque

Just as net force sometimes plays a part in a problem, so does net torque. Net torque is the vector sum of all torques.

$$\tau_{net} = \sum \text{Torques}$$

Example: Two forces act on the cylinder as shown in the diagram below. If $F_1 = 10$ N and $F_2 = 15$ N, what is the net torque on the cylinder? (0.70 Nm, CCW)

$-$ $+$



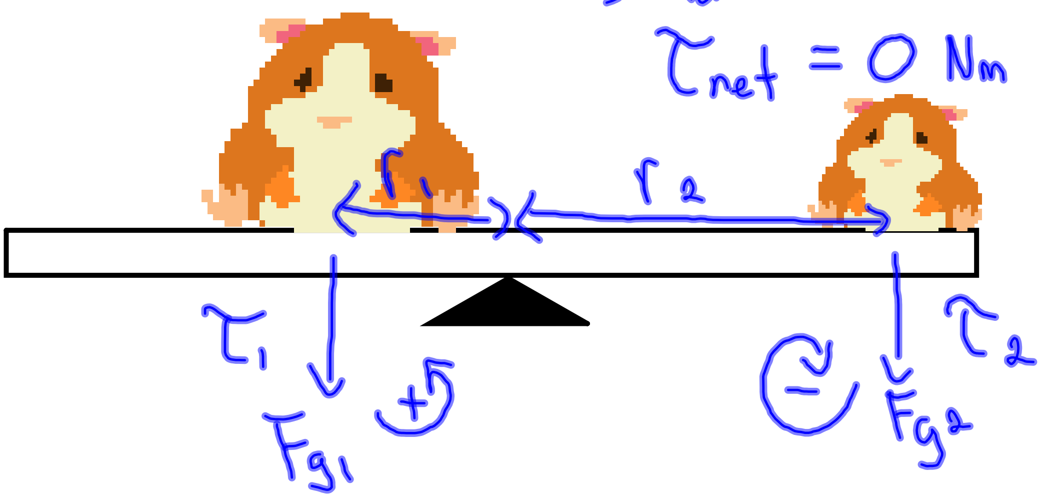
$$\tau_{net} = \tau_1 + \tau_2$$
$$= r_1F_1 + r_2F_2$$
$$= -(0.05)(10) + (0.08)(15)$$
$$= -0.5\text{ Nm} + 1.2\text{ Nm}$$
$$\boxed{= 0.7\text{ Nm}}$$

•

Example: A massless board serves as a seesaw for two giant hamsters as shown below. One hamster has a mass of 30 kg and sits 2.5 m from the pivot point. At what distance from the pivot point must a 25 kg hamster place himself to balance the seesaw? (3.0 m)

Balanced so

$\tau_{net} = 0 \text{ Nm}$



$$\tau_{net} = \tau_1 + \tau_2$$

$$0 = (2.5)(30\text{ kg})(9.81) - r_2(25)(9.81)$$

$$0 = 735.75 - 245 r_2$$

$$\frac{-735.75}{-245.25} = r_2$$

$$\boxed{3.0\text{m} = r_2}$$

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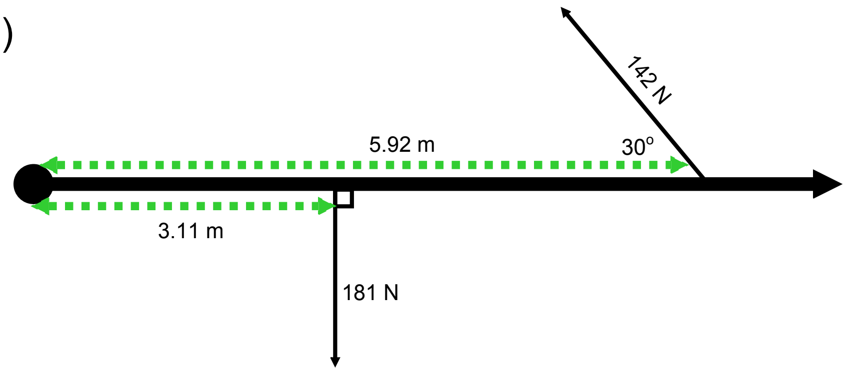
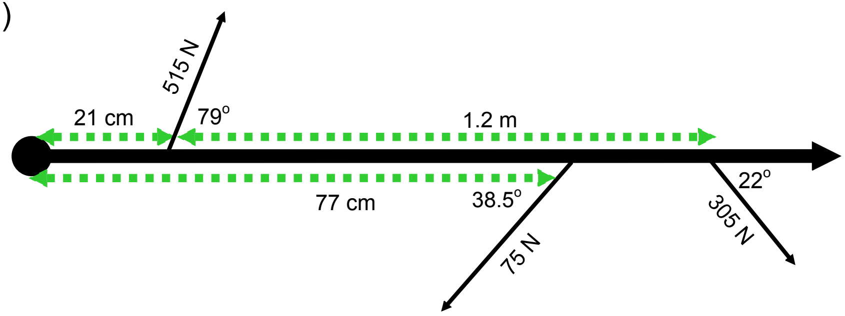
Net Torque Practice

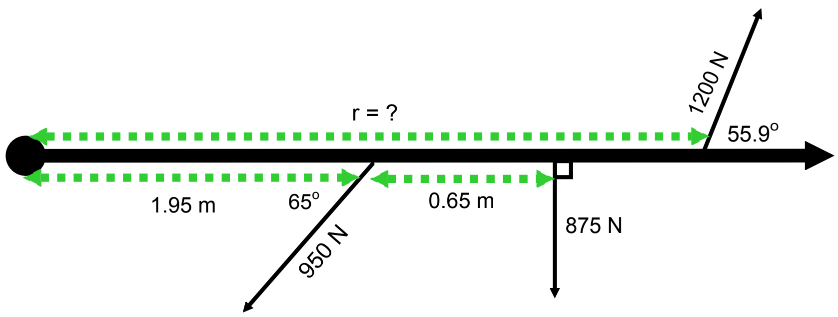
#1)



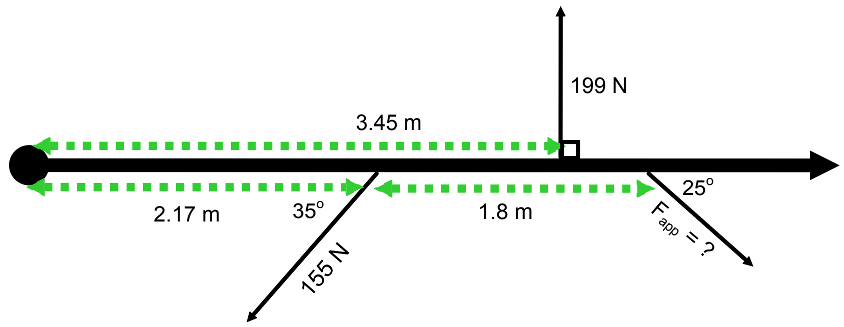
#2)



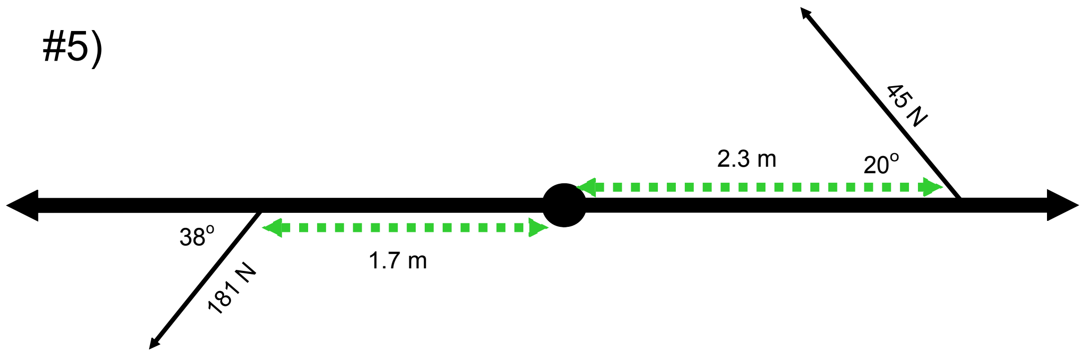
#3) τ_{net} = 0 Nm



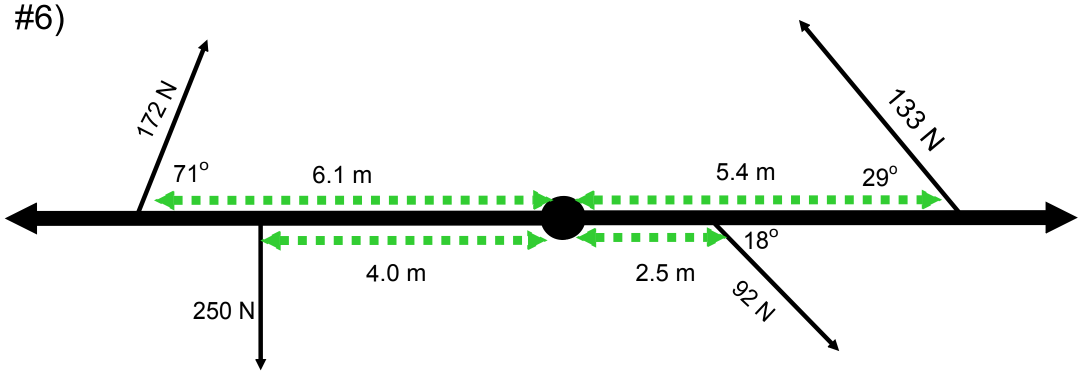
#4) τ_{net} = 0 Nm



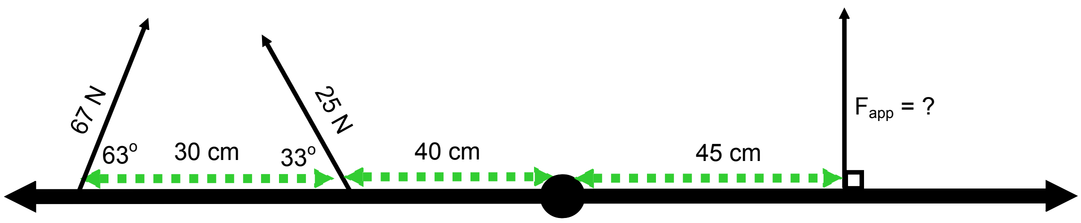
#5)



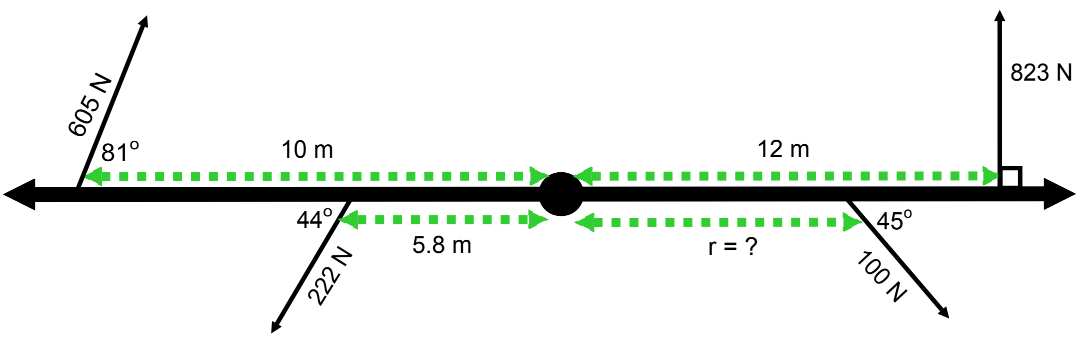
#6)



#7) τ_{net} = 0 Nm



#8) τ_{net} = 0 Nm



Static Equilibrium

An object is in static equilibrium if:

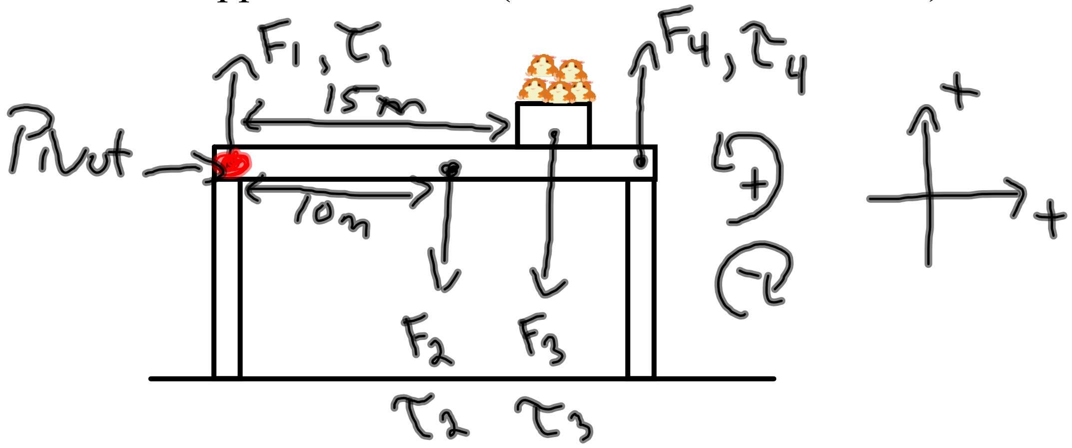
1. $\mathbf{v} = 0$ m/s
2. $\mathbf{F_{net}} = 0$ N
3. $\boldsymbol{\tau_{net}} = 0$ Nm

Steps for Solving Static Equilibrium Problems

1. Draw a diagram.
2. Label all forces.
3. Choose a pivot point. It is helpful to place the pivot point where an unknown force exists.
4 Label distances from the pivot point to the forces. (r values)
5. Choose a coordinate system.
6. Resolve a force into its perpendicular components if the force doesn't fit into the chosen coordinate system.
7. Write F_{netx} and F_{nety} equations.
8. Write a τ_{net} equation.
9. Solve the equation(s) for the unknown.

* If a solid object has mass, treat the object as if all its mass were concentrated at a point - the center of mass.

Example: A uniform 1500 kg beam, 20.0 m long, supports a 15000 kg box of hamsters 5.0 m from the right support column. Calculate the magnitude of the forces on the beam exerted by each of the vertical support columns. (1.2×10^5 N, 4.2×10^4 N)



$$F_{net\,y} = F_1 + F_2 + F_3 + F_4$$

$$0 = F_1 - (1500)(9.81) - (15000)(9.81) + F_4$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

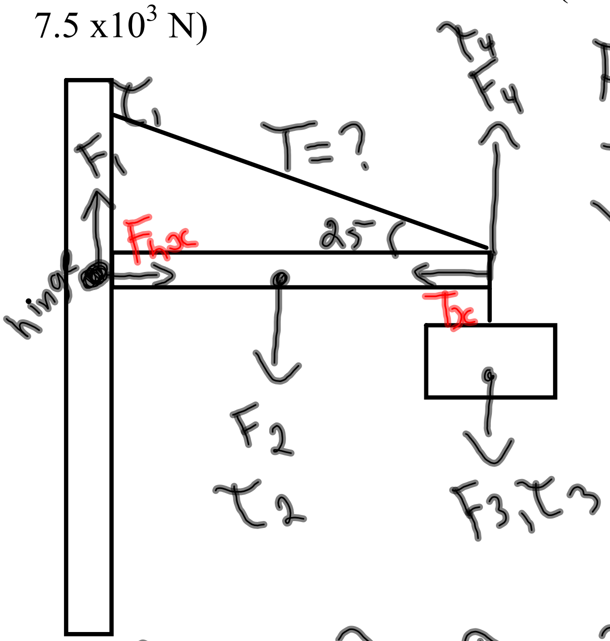
$$0 = r_1F_1 + r_2F_2 + r_3F_3 + r_4F_4$$

$$0 = 0 - (10)F_2 - (15)F_3 + 20F_4$$

$$0 = -147000 - 2{,}207{,}000 + 20F_4$$

$$F_4 = 120000\text{ N}$$

Example: A uniform beam of mass 50.0 kg and length 3.00 m is attached to a wall with a hinge. The beam supports a sign of mass 300 kg which is suspended from its end. The beam is also supported by a wire that makes an angle of 25° with the beam. Determine the components of the force that the hinge exerts and the tension in the wire. (6.8×10^3 N, 2.5×10^2 N, 7.5×10^3 N)



Find Tension, T

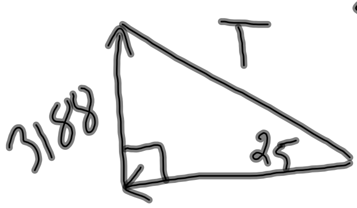
$\tau_{net} = 0\ Nm$

$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$

$0 = 0 - (1.5)(50)(9.81) - (3)(300)(9.81) + (3)F_4$

$0 = -735 - 8829 + 3F_4$

$3188\,N = F_4$



$\sin 25 = \frac{3188}{T}$

$F_4 = T \sin 25$

$T = \frac{3188}{\sin 25}$

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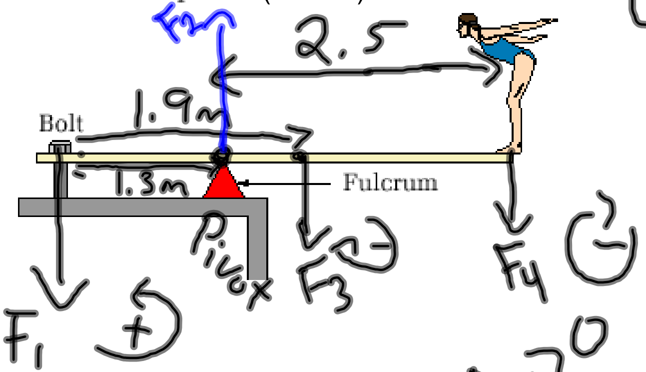
$\boxed{T = 7500\,N}$

PRACTICE PROBLEMS

31. An Olympic diver with a mass of 54 kg stands at the end of a uniform, 3.8 m diving board with a mass of 25 kg. The fulcrum supporting the diving board is 1.3 m from the bolted end. What force must the bolt at the end exert on the diving board to hold the board in place? (1100 N)



$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = r_1 F_1 + \underbrace{\cancel{r_2 F_2}}_{\to 0} + r_3 F_3 + r_4 F_4$$

$$0 = +(1.3)F_1 - (0.6)(25)(9.81) - 2.5(54)(9.81)$$

$$0 = 1.3F_1 - 147\ \text{Nm} - 1324\ \text{Nm}$$

$$F_1 = 1132\ \text{Nm}$$

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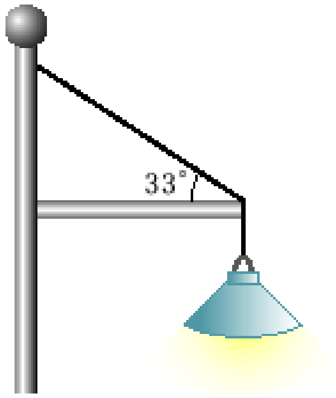
33. A 24 kg light fixture is hanging from a uniform, 3.5 kg horizontal beam that is 1.6 m long. A supporting cable makes an angle of 33° with the beam.



(a) Find the tension in the cable.

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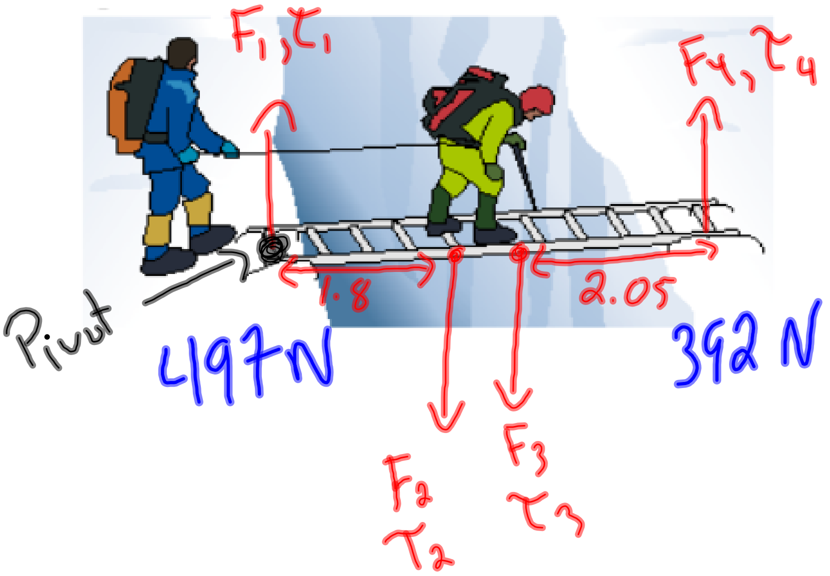
27. Mountain climbers have placed a 3.6 kg uniform ladder across an icy crevasse. The edges of the crevasse are 4.1 m apart. The first climber starts to cross the crevasse on the ladder and reaches a point 1.8 m from the edge. The mass of the climber and her gear is 87 kg. With what force is the ice on each side of the crevasse pushing up on the ladder?



$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = 0 - (1.8)(87)(9.81) - (2.05)(3.6)(9.8) + (4.1)F_4$$

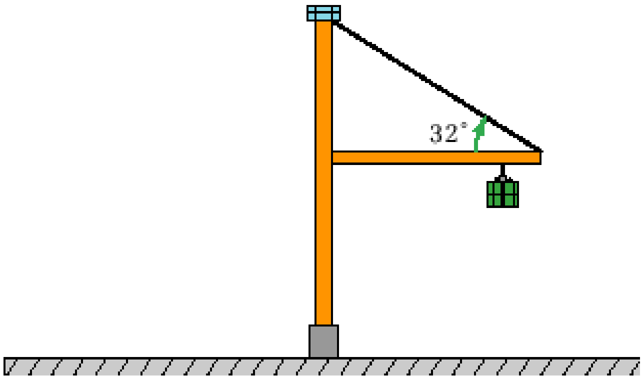
$$\boxed{F_4 = 392\text{ N}}$$

$$F_{net} = F_1 + F_2 + F_3 + F_4$$

$$0 = F_1 - (87)(9.81) - (3.6)(9.81) + 392$$

$$\boxed{F = 497\text{N}}$$

28. A crane with a movable pulley system on a horizontal arm is moving a large container. The 355 kg container is hanging from a cable that is 6.15 m out on the 7.50 m arm. The arm has a mass of 345 kg. A cable that is attached to its end makes an angle 32.0° with the horizontal arm.



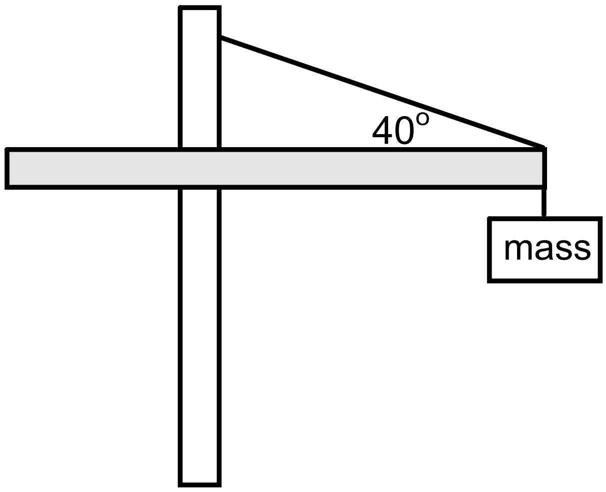
(a) What is the tension in the cable supporting the arm? (8582 N)

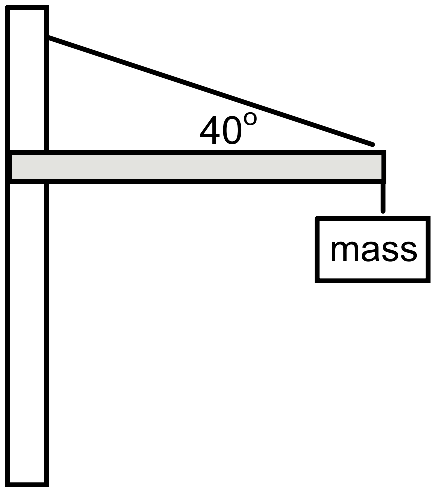
A construction crane is designed such that part of the boom acts as a counterweight. The boom is constructed of uniform material with a linear density of 25 kg/m. The left side of the crane is 10 m long and the right side is 15 m.

a) If the mass at the right end is 300 kg what is the tension in the cable? (T = 6200 N)

b) What is the tension in the cable if there was no left side of the boom? (T = 7400 N)

c) Suppose each cable can support a tension of 12000 N. What is the maximum mass that each crane can support? (Left: 680 kg; Right: 600 kg)





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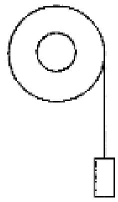
Physics 122

Handout – Torque

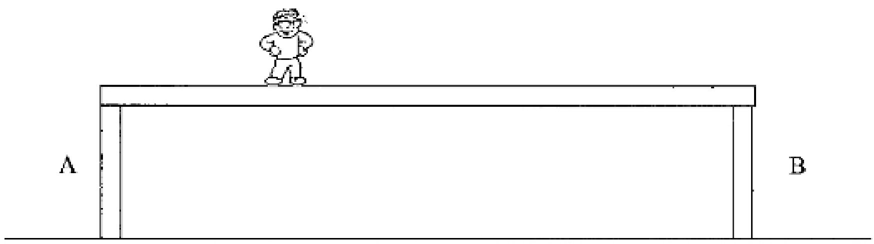
1. Consider a light string wound around a frictionless and massless wheel. The free end of the string is attached to a 1.2 kg mass that is allowed to fall freely. The wheel has a radius of 0.25 m. What torque is produced? (-2.94 Nm)



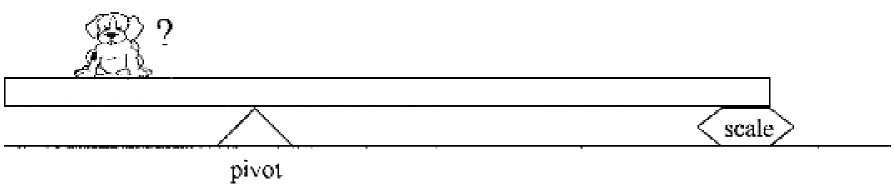
2. The magnitude of the maximum torque exerted by a person riding a bike when all his weight is put on the pedal is 92 Nm. What is the mass of the person if the pedals rotate in a circle of radius 17 cm? (55 kg)

3. A small boy of mass 30 kg is at one end of a seesaw of total length 3.0 m. Where must a girl of mass 21 kg sit in order that the seesaw be in equilibrium? (2.1 m from the pivot on the opposite as the boy)

4. Bob is standing on a bridge. The bridge itself weighs 10 000 N. The span between pillars A and B is 80 m. Bob is 20 m from the center of the bridge. Bob's mass is 100 kg. Assuming the bridge is in equilibrium, find the magnitude of the force exerted by pillar B on the bridge. (5.2×10^3 N)

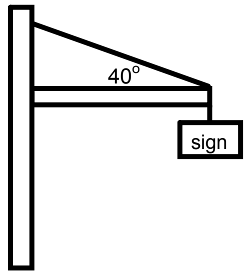


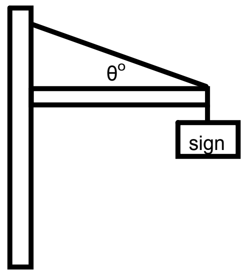
5. A 60 kg uniform board 2.4 m long is supported by a pivot 80 cm from the left end of the board and by a scale at the right end. Where should a 40 kg puppy be placed if the scale is to read 100 N? (61 cm from the left end of the board)



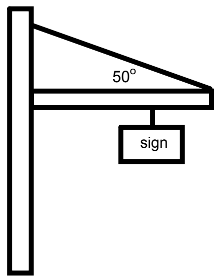
More Torque!

1. Determine the tension in the wire and the components of the force from the hinge. The beam has a mass of 170 kg, the sign has a mass of 75 kg, and the beam is 6.0 m long. (T = 2442 N, F_{hy} = 834 N, F_{hx} = 1870 N)



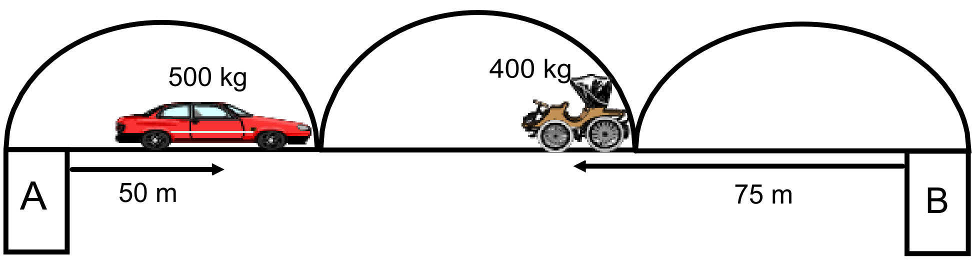


2. The cable in the diagram to the right will break if the tension reaches 1500 N. What is the smallest angle that can be made with the horizontal? The beam is 15 m long and has a weight of 1050 N. The sign has a weight of 500 N. (θ = 43°)



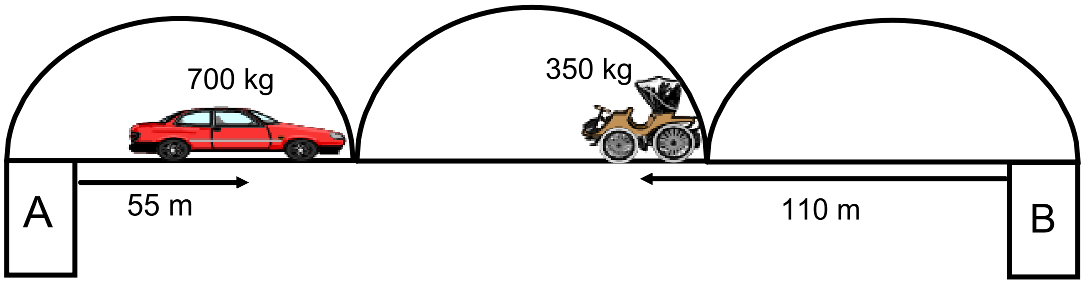
3. If the cable will break under a stress of 2300 N, what is the largest mass that can be hung from the beam? The beam is 150 kg and 8.0 m long. The cable makes an angle of 50° with the beam and the sign is 5.5 m from the left end of the beam. (152 kg)

4. The Morrissey bridge will collapse if column A must support more than 50000 N of weight. The bridge spans 225 m and has a mass of 8500 kg. Will the bridge collapse under the circumstances depicted in the diagram? (No, F_A = 46815 N)

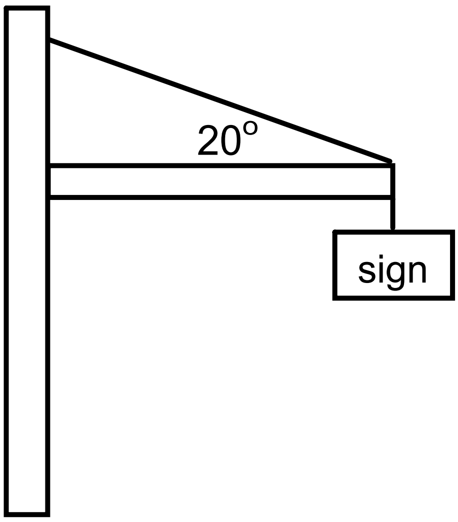


Torque Quiz - Solve on your own paper

1. Find the force each column supports. The bridge spans 300 m and has a mass of 12 500 kg.



2. Determine the tension in the wire. The beam has a mass of 225 kg, the sign has a mass of 1055 kg, and the beam is 15 m long.



3. Find the net torque.