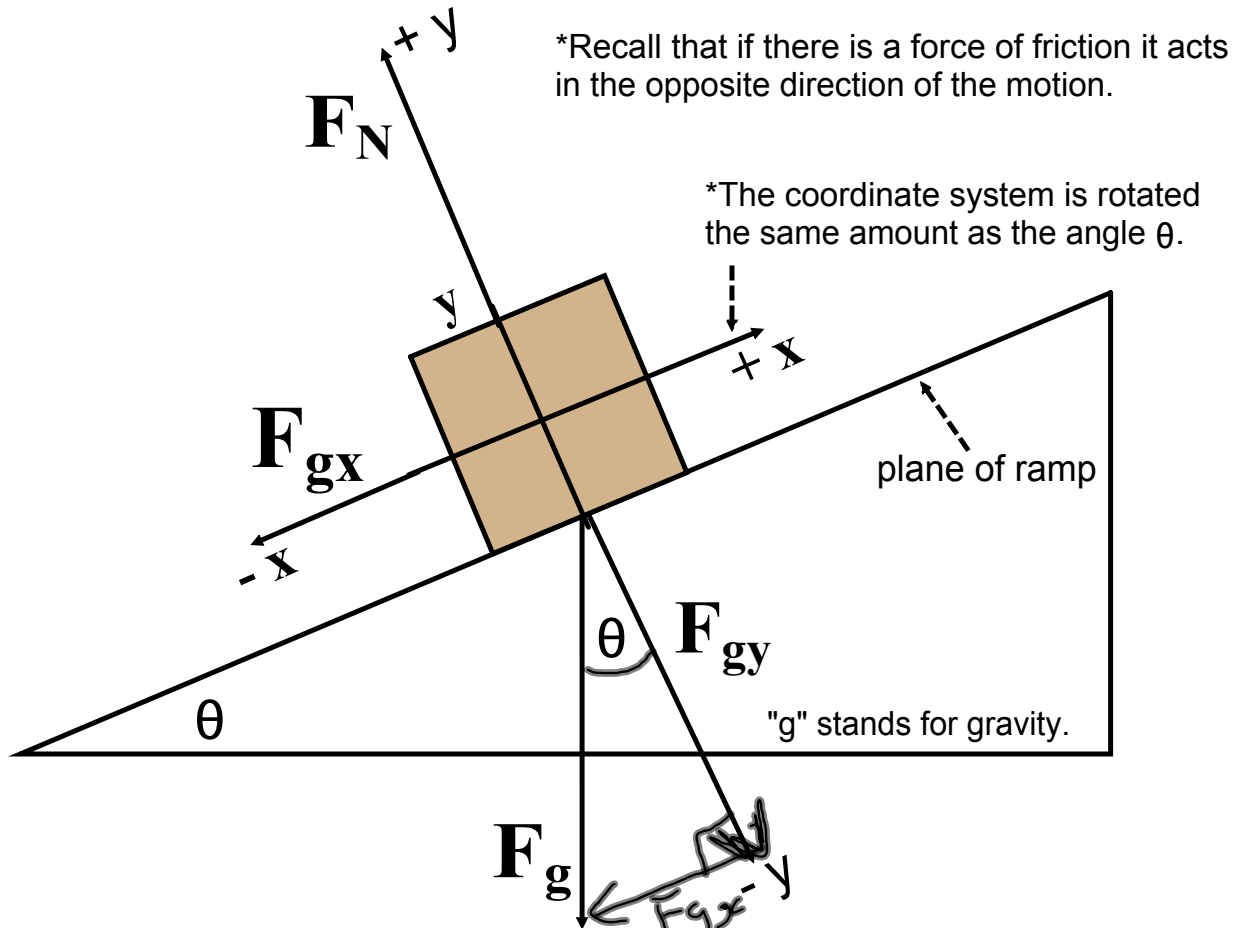


## Type III - Inclined Planes, Hills, Ramps

(printed copy for students)



$F_{gy}$  and  $F_g$  are separated by  $\theta$  because of two similar triangles.

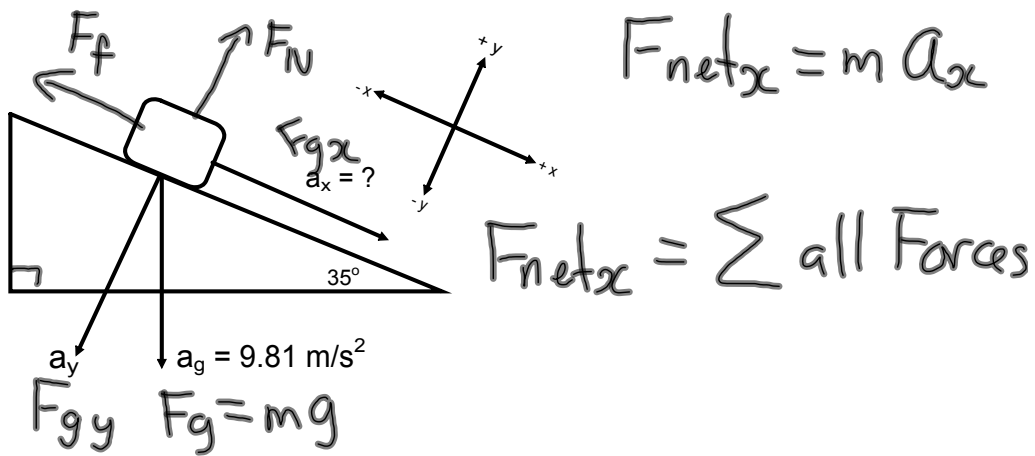
$$F_{gx} = F_g \sin \theta \quad \longleftarrow \text{component parallel to the plane.}$$

$$F_{gy} = F_g \cos \theta \quad \longleftarrow \text{component perpendicular to the plane.}$$

**NOTE!** The *sin* and *cos* have switched places. This will only happen when dealing with objects on a ramp.

**NOTE FURTHER!** Every  $F$  in the above diagram can be replaced with an  $a$  for acceleration.

1. A 2300 kg block is sliding down a hill inclined at an angle of  $35^\circ$ . The coefficient of kinetic friction is 0.13. What is the acceleration of the ~~car?~~ <sup>block</sup>



$$F_{netx} = F_f + F_{gx}$$

$$F_f = \mu F_N$$

$$= 0.13 (mg) \cos 35^\circ$$

$$= 0.13 (2300)(9.81) (\cos 35^\circ)$$

$$= 2400 \text{ N}$$

$$F_{gx} = mg \sin \theta$$

$$= (2300)(9.81) \sin 35^\circ$$

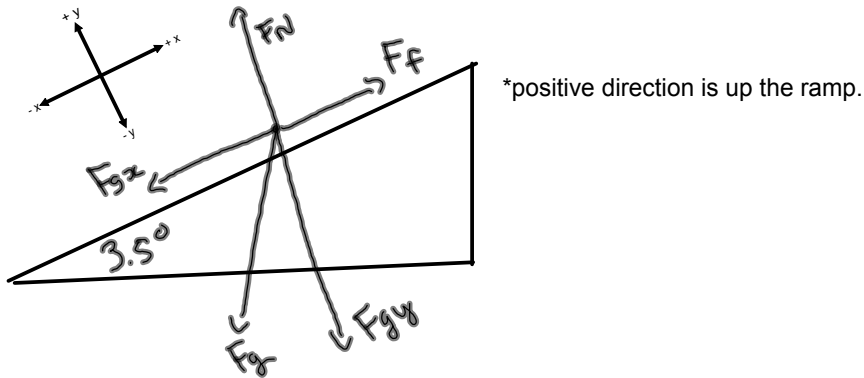
$$= 12900 \text{ N}$$

$$\begin{aligned}
 \star \quad F_{netx} &= -2400 + 12900 \\
 &= 10500 \text{ N}
 \end{aligned}$$

$$a_x = \frac{F_{netx}}{m} = \frac{10500}{2300} = \boxed{4.58 \text{ m/s}^2}$$

## Examples

2. A skier coasts down a  $3.5^\circ$  slope at a constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope. (0.061)



$$\mu = \frac{F_f}{F_N} \xrightarrow{\text{force} = \text{mass} \times \text{acceleration}} \mu = \frac{m \vec{a}_{fx}}{m \vec{a}_{Ny}}$$

$a_f$  = acceleration from frictional force.

$a_N$  = acceleration from normal force.

$$\vec{a}_{nety} = \vec{a}_{gy} + \vec{a}_{Ny} \quad \text{"y" direction}$$

negative vertical direction

constant speed

$$0 = -(9.81) \cos 3.5^\circ + \vec{a}_{Ny}$$

$$\underline{9.79 \text{ m/s}^2 = \vec{a}_{Ny}}$$

$$\vec{a}_{netx} = \vec{a}_{gx} + \vec{a}_{fx} \quad \text{"x" direction}$$

negative horizontal direction

$$0 = -9.81 \sin 3.5^\circ + \vec{a}_{fx}$$

acts up the ramp.  
so is positive

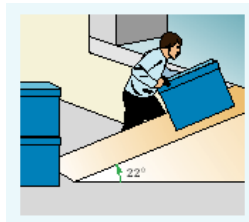
$$\underline{0.599 \text{ m/s}^2 = \vec{a}_{fx}}$$

$$\mu = \frac{\vec{a}_{fx}}{\vec{a}_{Ny}}$$

$$\mu = \frac{0.599}{9.79} = 0.061$$

## Sample Problems - Inclined Planes Handout

1. A trunk weighing 562 N is resting on a plane inclined at  $30.0^\circ$  from the horizontal. Find the components of the trunk's weight parallel and perpendicular to the plane.
2. A 562 N trunk is placed on a frictionless plane inclined at  $30.0^\circ$  from the horizontal. Find the magnitude and direction of the trunk's acceleration.
3. A worker places a large plastic waste container with a mass of 84 kg on the ramp of a loading dock. The ramp makes an angle of  $22^\circ$  with the horizontal. The worker turns to pick up another container before pushing the first one up the ramp. If the coefficient of static friction is 0.47, will the crate slide down the ramp?



4. A 1975 kg car is rolling down a hill inclined at an angle of  $15^\circ$ . What is the acceleration of the car? Neglect friction.
5. A skier coasts down a  $3.5^\circ$  slope at a constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope.
6. You slide a 325 N trunk up a  $20.0^\circ$  inclined plane with a constant velocity by exerting a force of 211 N parallel to the inclined plane.
  - a) What is the sum of your applied force, friction and the parallel component of the trunk's weight? Justify your answer.
  - b) What is the magnitude and direction of the force of friction?
  - c) What is the coefficient of friction?

## Answers to sample Problems.

1.  $F_{gx} = 281 \text{ N}$     $F_{gy} = 487 \text{ N}$

2.  $a_{gx} = 4.905 \text{ m/s}^2$  down the ramp.

3. No,  $F_{sf} > F_{gx}$   
 $F_{sf} = 359 \text{ N}$ ,  $F_{gx} = 309 \text{ N}$

4.  $a_x = 2.53 \text{ m/s}^2$

6. a)  $F_{net,x} = 0$

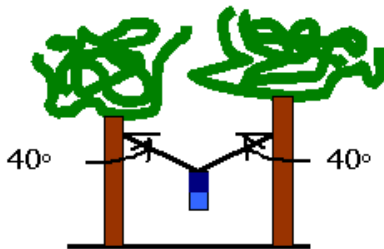
b)  $|F_f| = 100 \text{ N}$  down the ramp.

c)  $\mu = 0.32$

## Handout

Physics 122/121  
Handout: Problems I, II and III

1. On a camping trip you stretch a rope between two trees and hang your backpack from the middle of it to keep it safe from bears. The mass of your backpack is 36.0 kg and each half of the rope makes an angle of  $40.0^\circ$  with the horizontal.
  - a) Find the amount of weight supported by each half of the rope.
  - b) Find the magnitude of the tension in each rope.



2. A 2.5 kg brick is pulled at a constant speed across a table by a cord that makes an angle of  $20^\circ$  with the horizontal. There is 7.0 N of force in the cord.
  - a) Calculate the force of friction between the brick and the table.
  - b) Calculate the normal force.
3. Joey moves a 26 kg wagon at a constant speed by pushing on the handle that makes an angle, theta, with the horizontal. Joey exerts a force of 54 N on the handle and the force of friction on the wagon is 34 N.
  - a) Calculate the angle the handle of the wagon makes with the horizontal.
  - b) What is the magnitude of the normal force acting on the wagon?
4. A 10 N block is held motionless on a frictionless inclined plane which makes an angle of  $30^\circ$  with the horizontal. What force would be needed to hold the block in position?
5. An object weighing 600 N is pulled up a frictionless incline at a constant speed using a rope. If the incline makes an angle of  $42.0^\circ$  with the horizontal, what is the magnitude of the force that is applied to the rope?
6. A 10 kg object, starting from rest, slides down a frictionless incline with a constant acceleration of  $2.0 \text{ m/s}^2$ . What angle does the incline make with the horizontal?
7. An object with a mass of 7.2 kg is allowed to slide from rest down an inclined plane. The plane makes an angle of  $30^\circ$  with the horizontal and is 65 m long. The coefficient of friction between the plane and the object is 0.45. What is the velocity of the object at the bottom of the plane?
8. A piano is accelerating down a ramp that is inclined at an angle of  $38.5^\circ$  above the horizontal. The acceleration is  $4.62 \text{ m/s}^2$ . What is the coefficient of friction between the piano and the ramp?

## Answers

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- Each half of the rope supports half of the weight of the backpack, 176 N.
  - The tension in each rope is 274 N.
- The force of friction is 6.6 N, in a direction opposite to the motion of the brick.
  - The magnitude of the normal force is 22 N.
- The handle makes an angle of  $51^\circ$  with the horizontal.
  - The normal force is  $3.0 \times 10^3$  N, up.
- A 5.0 N force exerted up the incline would be needed.
- It is 401 N.
- The incline makes an angle of  $12^\circ$ .
- The velocity of the object is  $-12$  m/s.
- The coefficient of friction is 0.19.

## ***Practice***

A 120 N weight is pushed to obtain constant speed along the ground. The applied force is 80 N at an angle of  $40^\circ$  to the horizontal. What is the coefficient of static friction? (0.35)

A 25 kg box is placed on an incline  $33^\circ$  to the horizontal. The coefficient of kinetic friction is 0.38. Find the acceleration of the box. ( $2.2 \text{ m/s}^2$  down the ramp)

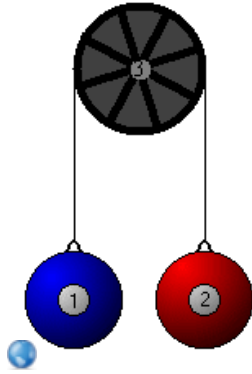


## Multiple Masses and Finding Net Force

Chapter 10.2 of text:

Read Pg 478 - 489

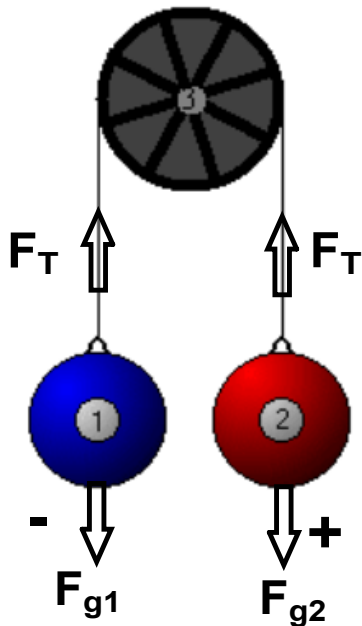
Problems Pg 485 #s 19 - 23, Pg 488 #s 24 - 28



This is an example of a system where there are multiple masses, the Atwood machine.

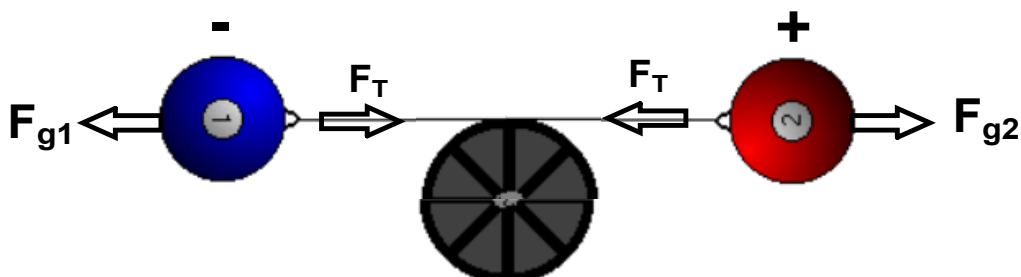
We will apply the concept of forces to determine the resulting acceleration.

### Define the Direction of Forces

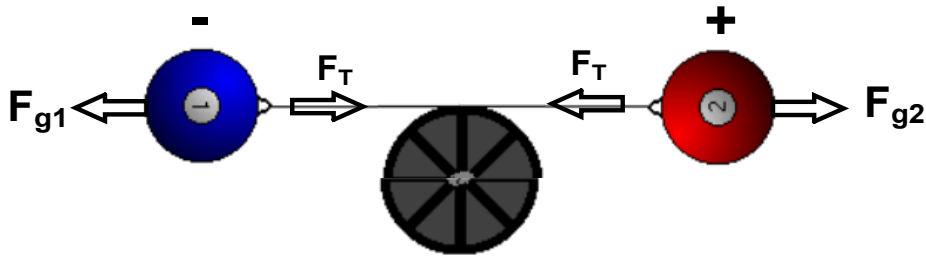


Our problems will not include friction and the pulley will be massless.

You may find it easier to picture, or draw, the system horizontally.



## Acceleration of the Masses: Newton's 2nd Law



$$F_{net} = ma$$

$$F_{net} = \sum \text{Forces}$$

$$m = \sum \text{masses that accelerate}$$

$$F_{g1} + F_T + F_T + F_{g2} = (m_1 + m_2)a$$

$$-m_1g + F_T + (-F_T) + m_2g = (m_1 + m_2)a$$

$$m_2g - m_1g = (m_1 + m_2)a$$

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$\star \vec{a} = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

To Find Tension:

$$F_{g1} + F_T = m_1a$$

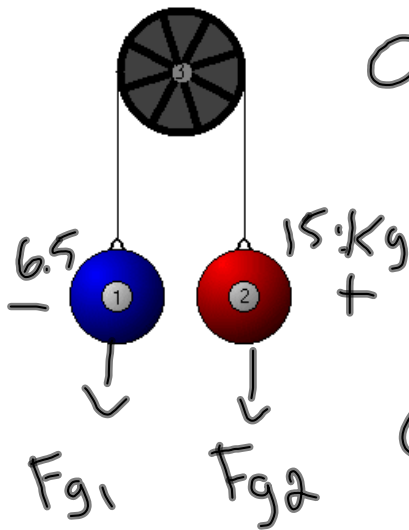
or

$$F_{g2} + F_T = m_2a$$

★ How would the formula for acceleration change if positive was to the left?

## Atwood Machine Examples

What is the acceleration of an Atwood machine with masses of 6.5 kg and 15 kg on opposite sides of the pulley? What is the magnitude of the force of tension in the rope?



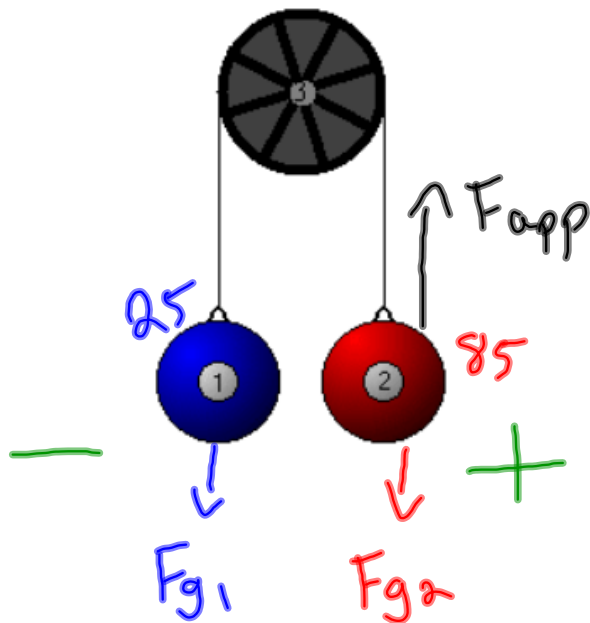
$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$a = \frac{(15 - 6.5)9.81}{15 + 6.5}$$

$$a = + (0.395)(9.81)$$

$$a = +3.88 \text{ m/s}^2$$

A counter weight of 25 kg is used to help a person of mass 85 kg to do chin ups. What is the force applied by the person if he accelerates at  $1.2 \text{ m/s}^2$ ?



$$F_{net} = m a$$

$$\sum F = (\sum m) a$$

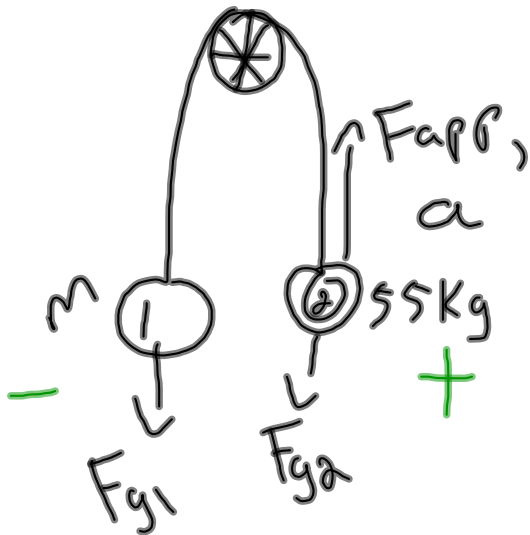
$$F_{g1} + F_{g2} + F_{app} = (m_1 + m_2) a$$

$$-(25)(9.81) + (85)(9.81) + F_{app} = (25 + 85)(-1.2)$$

$$588.6 + F_{app} = -132$$

$$F_{app} = -721 \text{ N}$$

A counterbalance is set up to help someone lift an object. The largest mass a person can lift is 33 kg. What must be the minimum mass of the counter weight for a 55 kg object to be lifted with an acceleration of  $1.5 \text{ m/s}^2$ ? (mass = 36 kg)



$$F_{g1} + F_{g2} + F_{app} = (m_1 + m_2) a$$

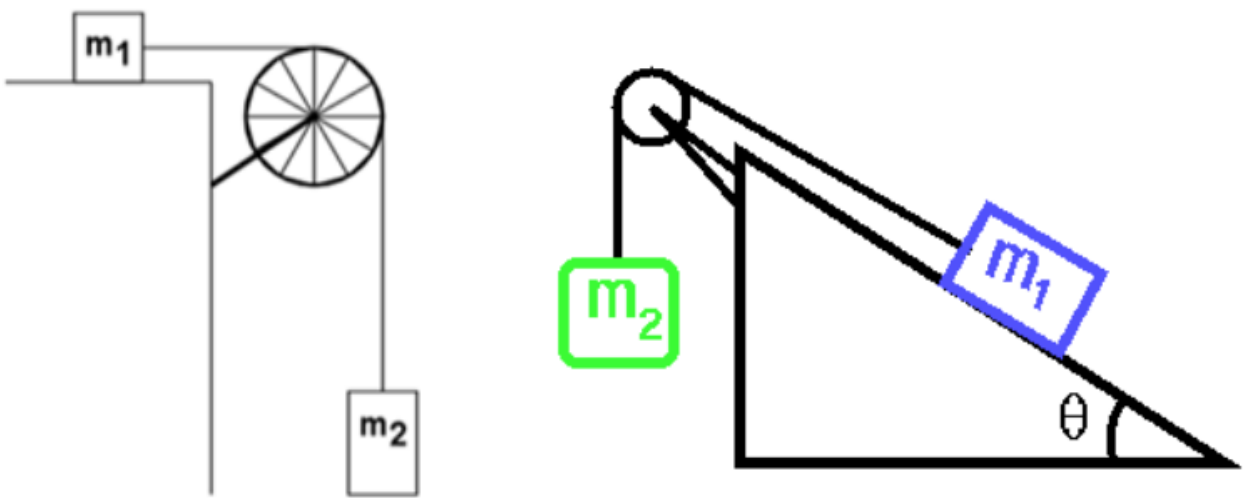
$$-9.81m + (55)(9.81) - (33)(9.81) = (m_1 + 55)(-1.5)$$

$$-9.81m + 539.55 - 323.73 = -1.5m - 82.5$$

$$298.32 = 8.31m$$

$$36 \text{ kg} = m$$

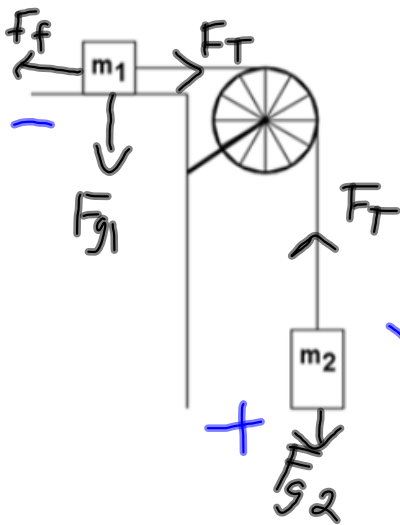
## *Objects Connected at an Angle*



We approach this problem like the others:

- Picture the motion in one dimension.
- Apply Newton's second law.

In the diagram below  $m_1 = 425 \text{ g}$ ,  $m_2 = 735 \text{ g}$ , the coefficient of kinetic friction is 0.34, and there is not friction from the pulley and string. What is the acceleration of the masses?



$$\Sigma F = \Sigma m \cdot a$$

Equal + opposite

$$+ F_{g2} + F_f = (m_1 + m_2)a$$

$$F_{g2} + F_f = (m_1 + m_2)a$$

$$F_f = \mu F_N \quad \leftarrow F_N = F_{g1} \text{ in magnitude}$$

$$= (0.34)(m_1 g)$$

$$= (0.34)(0.425)(9.81) = 1.412 \text{ N}$$

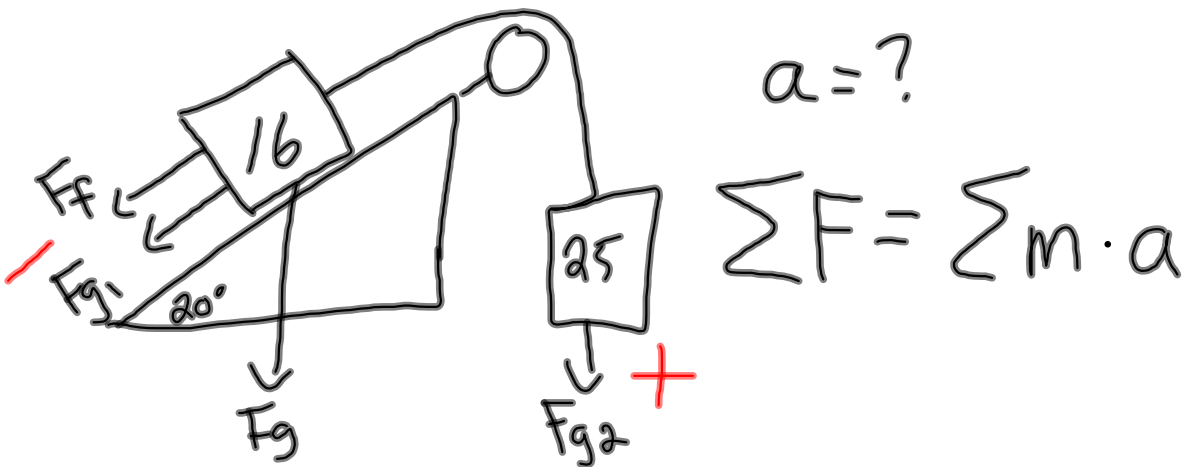
$$m_2 g - 1.412 = (0.425 + 0.735)a$$

$$(0.735)(9.81) - 1.412 = (1.16)a$$

$$\cdot 7.21 - 1.412 = 1.16a$$

$$5.0 \text{ m/s}^2 = a$$

A counterweight is used to slide an object up an inclined plane of  $20^\circ$ . The counterweight has a mass of 25 kg and is suspended with a massless string and a frictionless pulley. The coefficient of friction on the plane is 0.19. What is the acceleration of a 16 kg object?



$$F_{g1} + F_f + F_{g2} = (m_1 + m_2)a$$

$$-(16)(9.81)\sin 20^\circ - F_f + (25)(9.81) = (16 + 25)a$$

$$-53.7 - F_f + 245 = 41a$$

$$F_f = \mu F_N$$

$$= (0.19)(16)(9.81)\cos 20^\circ = 28 \text{ N}$$

$$\cdot -53.7 - 28 + 245 = 41a$$

$$3.98 \text{ m/s}^2 = a$$



A counterweight is used to slide an object up an inclined plane that makes a  $40^\circ$  angle with the horizontal. The counterweight has a mass of 35 kg and is suspended with a massless string and a frictionless pulley. The coefficient of kinetic friction on the plane is 0.23.

a) For the acceleration of the object not to exceed  $0.42 \text{ m/s}^2$  up the ramp, what must be the mass of the object? (39 kg)

b) What mass would result in an acceleration of  $0.21 \text{ m/s}^2$  down the ramp? (80 kg)