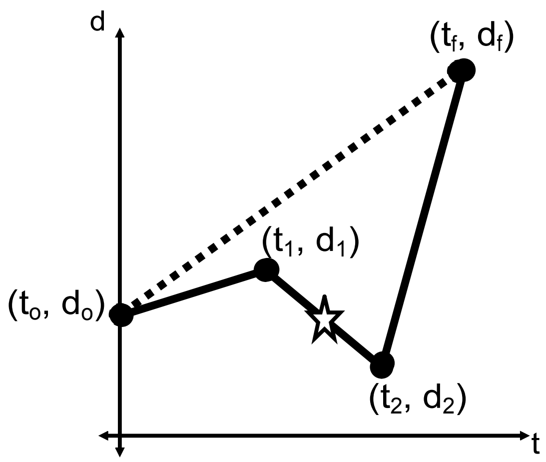
1D Motion: Mathematical Analysis

We want to be able to solve problems involving the following:

- displacement, $\vec{d}$
- time, t
- velocity, $\vec{v}$
- acceleration, $\vec{a}$

You may remember some of the equations of motion from grade 10. We will derive the equations to use based on our graphical study of motion in grade 11.

Instantaneous Velocity = Change in position (displacement) per unit time. It is the slope of the d-t graph at a point (t,d).



$$v_{☆} = slope = \frac{d_2 - d_1}{t_2 - t_1} = \frac{\Delta d}{\Delta t}$$

Remember that if the displacement works out to be negative that means the object has traveled in the direction that our frame of reference has labeled to be negative.

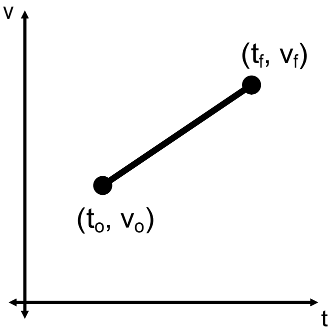
If you want average velocity the concept is the same but you have to use the initial and final points:

$$v_{avg} = \frac{d_f - d_o}{t_f - t_o}$$

$$\Delta d = v_{avg} \Delta t$$

For many of our problems we are only concerned with average velocity.

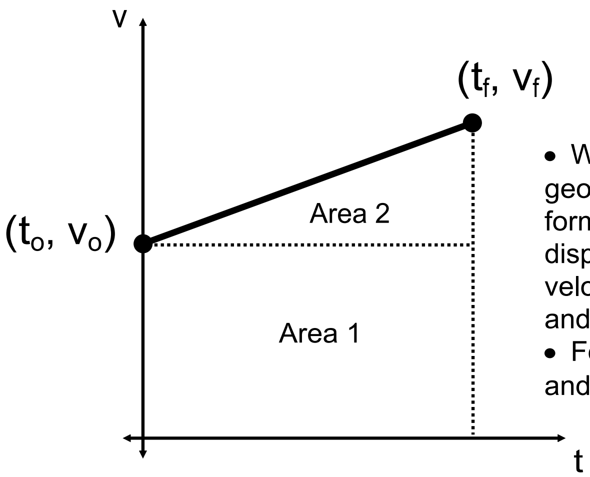
Acceleration works very similarly to velocity. The instantaneous velocity is the slope of the v-t graph at a point (t, v) and the average acceleration is found using the initial and final information. We will limit ourselves to objects undergoing a constant acceleration.



$$a = \frac{v_f - v_o}{t_f - t_o} = \frac{\Delta v}{\Delta t}$$

Read problems carefully as quite often the initial velocity and time is zero, but not always.

Displacement for an object undergoing constant acceleration equals the area between the v-t graph and the time axis



- We can use the geometry to derive a formula that relates displacement to initial velocity, acceleration, and time.
- For simplicity $t_o = 0$ and t_f is written as t.

Displacement = Area 1 + Area 2

$$d = v_o t + \frac{1}{2}(v_f - v_o)t$$

This is a perfectly valid formula but many times we are given information about acceleration. We can incorporate acceleration by using a little math trick:

$$d = v_o t + \frac{1}{2}(v_f - v_o)t \times \frac{t}{t}$$

$$d = v_o t + \frac{1}{2}\left(\frac{v_f - v_o}{t}\right)t^2$$

remember that: $a = \frac{v_f - v_o}{t_f - t_o} = \frac{\Delta v}{\Delta t}$

$$d = v_o t + \frac{1}{2}at^2$$

Motion Equations

$$v_{avg} = \frac{d_f - d_o}{t_f - t_o}$$

$$a = \frac{v_f - v_o}{t_f - t_o} = \frac{\Delta v}{\Delta t}$$

$$\Delta d = v_{avg}\Delta t$$

$$d = v_o t + \frac{1}{2}at^2$$

Remember that everything is a vector (sign of the variable depends on its direction) except time and that the change in time can never be negative!

Motion Examples

A car accelerates from zero to 35 m/s in 7.3 seconds.
a) What is the average acceleration?
b) What distance was covered during the acceleration?

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a) What is the stopping distance required for a car initially traveling 100 km/h that skids to a stop in 4.3 s? (59 m)
b) Assuming the same acceleration as in (a) what distance is needed if the car is traveling 120 km/h? (86 m)

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Standing near the edge of a cliff a baseball is launched straight up with a velocity of 15 m/s. The ball is in the air for a total of 4.5 s before it hits the ground at the bottom of the cliff. Find the height of the cliff (magnitude of the acceleration of gravity, g = -9.8 m/s^2).

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