

Universal Gravitation (Page 574 MHR)

Chapter 12

Theories of Planetary Motion

1. Tycho Brahe (1546-1601)

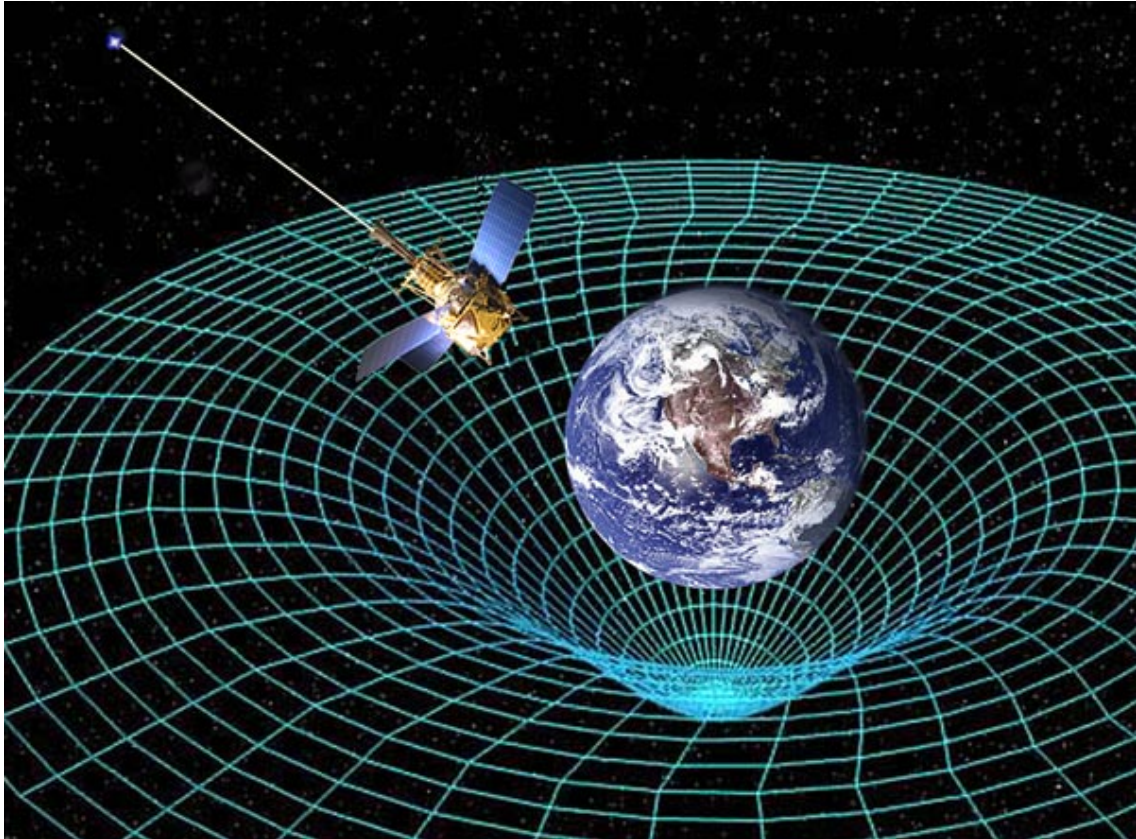
Brahe believed in the geocentric theory of planetary motion - Earth as the center of the universe.

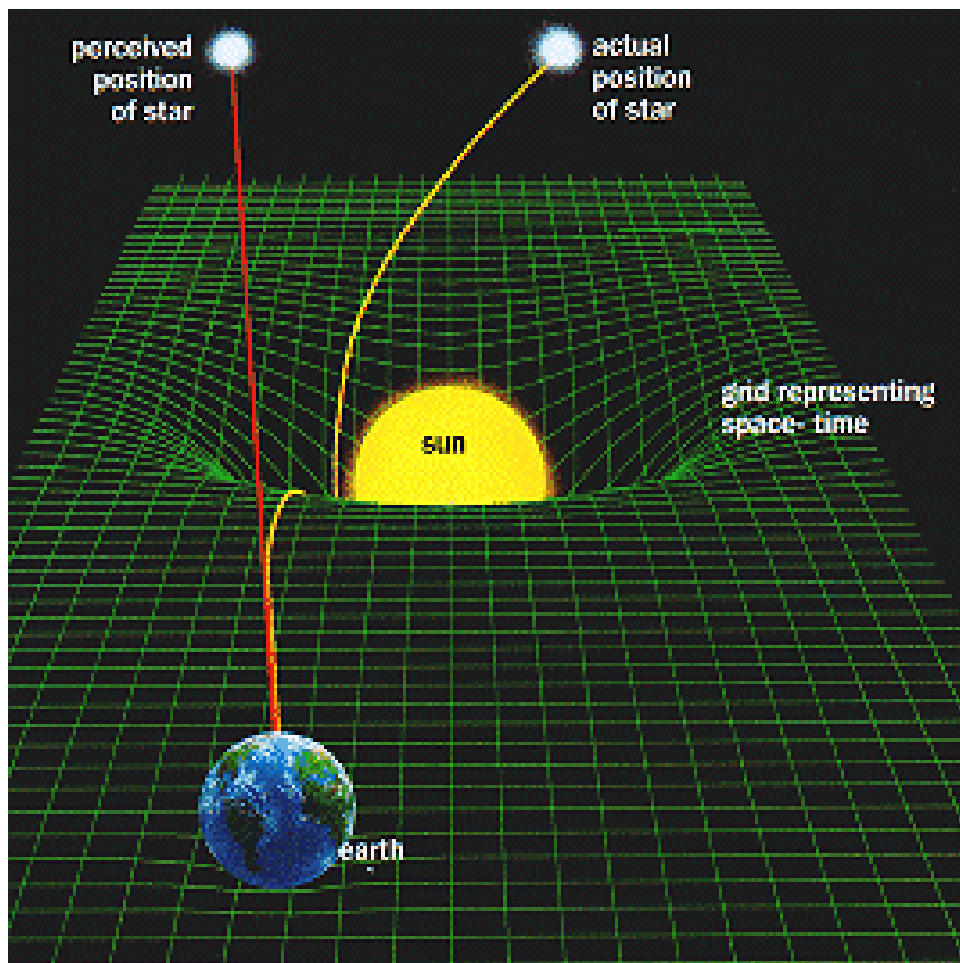


2. Johannes Kepler (1571-1630)

Kepler believed in the heliocentric theory of planetary motion - Sun as the center of the universe.

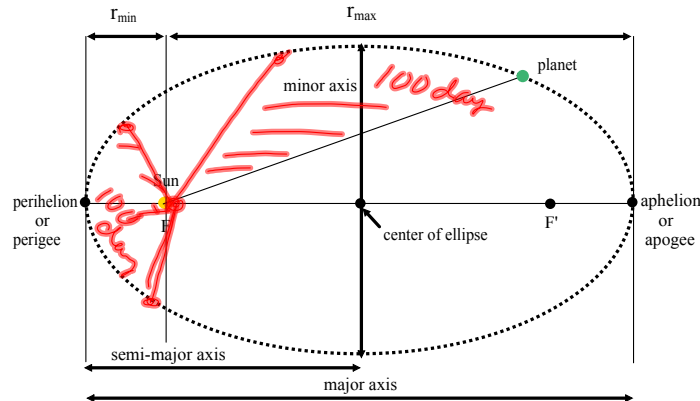






Kepler's Three Laws of Planetary Motion

1. Law of Orbits - The paths of the planets are ellipses with the center of the Sun at one focus.



eccentricity - a quantity indicating how non-circular an ellipse is
 - based on a scale from 0 to 1 where a value nearer 1 implies a higher degree of non-circularity

2. Law of Areas - An imaginary line from the sun to a planet sweeps out equal areas in equal time intervals. Therefore, planets move faster when closest to the sun and slowest when farthest away.

Simulation



3. Law of Periods - The ratio of the square of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun.

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

T_A -> period of object A (s, min, h, etc)

T_B -> period of object B (s, min, h, etc)

r_A -> mean orbital radius (m, km, etc)

r_B -> mean orbital radius (m, km, etc)

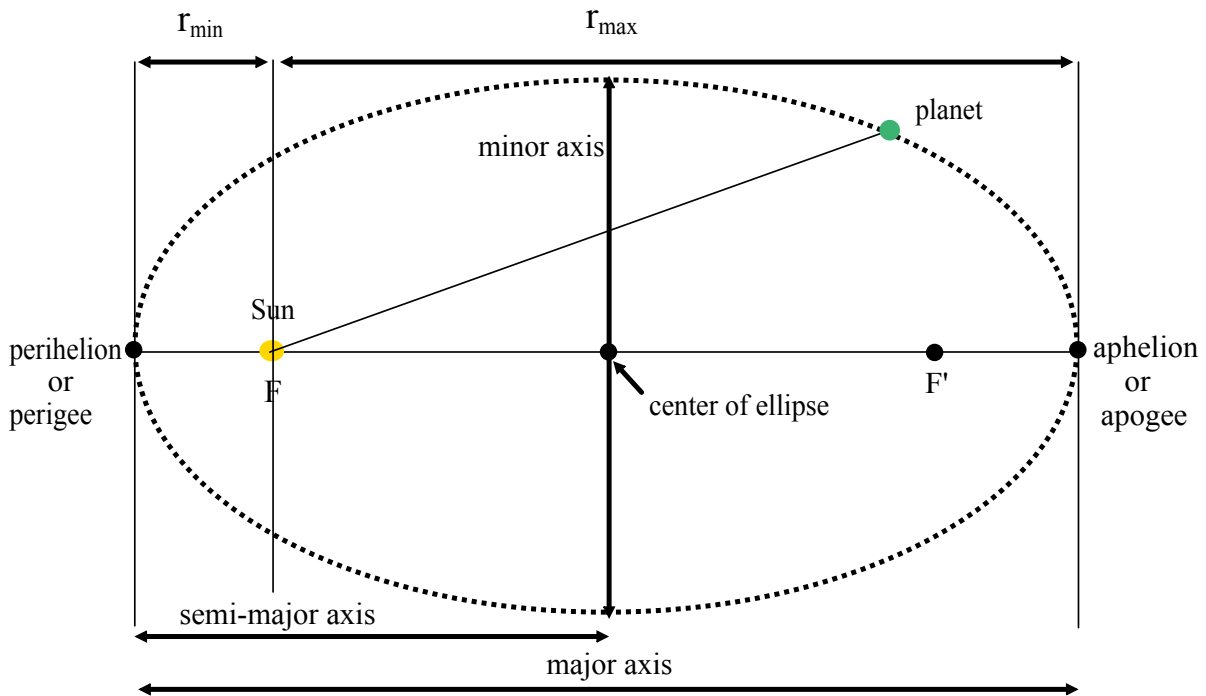
NOTE: The first two laws apply to each planet, satellite or moon. The third relates to the motion of two objects which are orbiting the same central body. The central body does not have to be the sun.

NOTE: One astronomical unit (AU) is the average distance from Earth to the Sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

NOTE: Physical data can be found on Page 573 and 955.

Elliptical Orbit



Example:

Galileo discovered four moons of Jupiter. Io, which he measured to be 4.2 units from the center of Jupiter, has a period of 1.8 days. He measured the radius of Ganymede's orbit as 10.7 units. Use Kepler's third law to find the period of Ganymede. (7.3 days)

Example:

The fourth moon of Jupiter, Callisto, has a period of 16.7 days. Find its average distance from Jupiter using the same units Galileo used.
(19 units)

Example:

On average, Mars is 1.52 AU. Predict the time required for Mars to circle the sun in Earth days. (684 days)

Example:

Uranus requires 84 years to circle the sun. Find Uranus' orbital radius as a multiple of Earth's orbital radius. ($19r_E$) AU

Universal Law of Gravitation

Newton -> explained the motion of the planets

-> a force of attraction between the sun and planets keep the planets traveling along their elliptical paths

$$\textcircled{1} \quad F \propto m_1 m_2$$

* Force of attraction is directly proportional to the product of the masses of the objects.

$$\textcircled{2} \quad F \propto 1/r^2 \quad F = \frac{m_1 m_2}{r^2}$$

* Gravitation force is inversely proportional to the distance squared.

$$\Rightarrow F = Gm_1 m_2 / r^2$$

F = gravitational force (N)

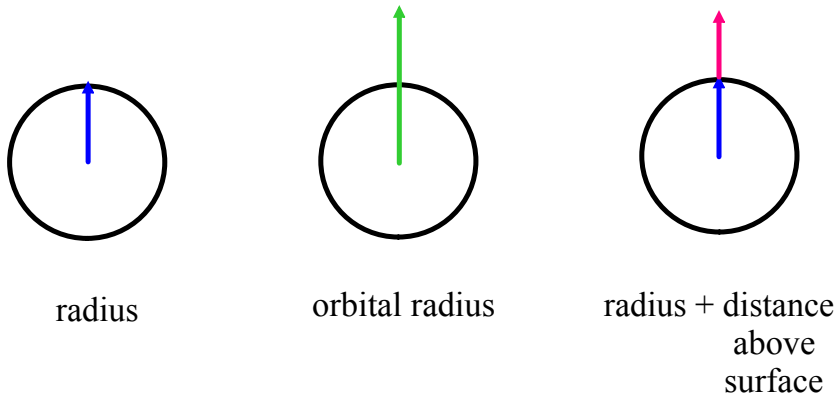
m_1 = mass of object 1 (kg)

m_2 = mass of object 2 (kg)

r = distance between masses (m)

G = universal gravitational constant

($6.672 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$)



Proportionality Questions

Universal Gravitation Handout

Text Pages 580 # 1 - 8

Read
Pg. 580, 583

By what factor will the force of gravity on a person change if the mass of this person doubles?

$$F \propto m_1 m_2$$

Force doubled

$$F \propto \frac{1}{r^2}$$

By what factor will the force of gravity on a person change if the person orbits a distance three times the Earth's radius?

$$\frac{1}{r^2} \quad \begin{array}{l} 9 \text{ times} \\ \text{less} \end{array}$$

$$F_{\text{new}} = \frac{2}{9} \times 900 \text{ N} \\ (= 200 \text{ N})$$

$$F = \frac{G M_1 M_2}{r^2}$$

Sun's Mass
 $1.989 \times 10^{30} \text{ kg}$
Radius = $6.96 \times 10^8 \text{ m}$

$$M_E = 5.974 \times 10^{24} \text{ kg}$$

$$r = 1.5 \times 10^{11} \text{ m} \Rightarrow (1 \text{ AU})$$

$$F = \frac{6.672 \times 10^{-11} (5.974 \times 10^{24}) (1.989 \times 10^{30})}{(1.5 \times 10^{11})^2}$$

$$F = \frac{7.928 \times 10^{44}}{(1.5 \times 10^{11})^2}$$

$$= 3.52 \times 10^{22} \text{ N}$$

$$\text{Moon: } 7.349 \times 10^{22} \text{ kg}$$

$$\text{Diameter: } 3476 \text{ km} = 3.476 \times 10^6 \text{ m}$$

$$\text{Earth-Moon Dist} = 384400 \text{ km}$$

$$= 3.844 \times 10^8 \text{ m}$$

$$\text{Sun Mass} = 1.989 \times 10^{30} \text{ kg}$$

$$\text{Sun Radius} = 6.96 \times 10^8 \text{ m}$$

$$\text{Proton Mass} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Electron Mass} = 9.109 \times 10^{-31} \text{ kg}$$

Force of Gravity between Earth & Sun: 3.52×10^{22} N

" " " " Earth & Moon: 1.97×10^{20} N

Tidal Force: Change in gravitation force with distance (Force Gradient)

-4.70×10^{11} N/m ^{Sun}

$$F = G M_E M_P (r^{-2})$$

-1.03×10^{12} N/m
Moon

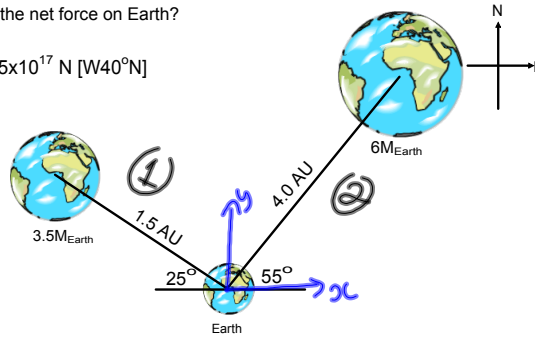
$$\frac{dF}{dr} = -\frac{2 M_E M_P}{r^3}$$

Tides are caused by the Moon because the change in force is greater per unit distance than that of the Sun.

Gravitational Force as a Vector

What is the net force on Earth?

$$F = 1.65 \times 10^{17} \text{ N } [W40^\circ N]$$



$$F_1 = \frac{G(3.5 M_E)(M_E)}{(1.5 \text{ AU})^2} \quad F_2 = \frac{G(6 M_E)(M_E)}{(4.0 \text{ AU})^2}$$

$$= \frac{G 3.5 M_E^2}{2.25} \quad = 0.375 G M^2$$

$$= 1.56 G M^2$$

$$F_{1x} = -1.56 G M^2 \cos 25^\circ \quad F_{2x} = 0.375 G M^2 \cos 55^\circ$$

$$= -1.41 G M^2 \quad = 0.215 G M^2$$

$$F_{1y} = +1.56 G M^2 \sin 25^\circ \quad F_{2y} = 0.375 G M^2 \sin 55^\circ$$

$$= +0.695 G M^2 \quad = 0.307 G M^2$$

$$F_x = F_{1x} + F_{2x} \quad F_y = F_{1y} + F_{2y}$$

$$= -1.41 G M^2 + 0.215 G M^2 \quad = 0.695 + 0.307$$

$$= -1.195 G M^2 \quad = 1.002 G M^2$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$F_{\text{net}} = \sqrt{(-1.195 G M^2)^2 + (1.002 G M^2)^2}$$

$$= \sqrt{1.428 G^2 M^4 + 1.004 G^2 M^4}$$

$$= \sqrt{2.432 G^2 M^4}$$

$$= 1.56 G M^2 \quad \theta = \tan^{-1}\left(\frac{1.002 G M^2}{1.195 G M^2}\right)$$

$$\theta = 40^\circ$$

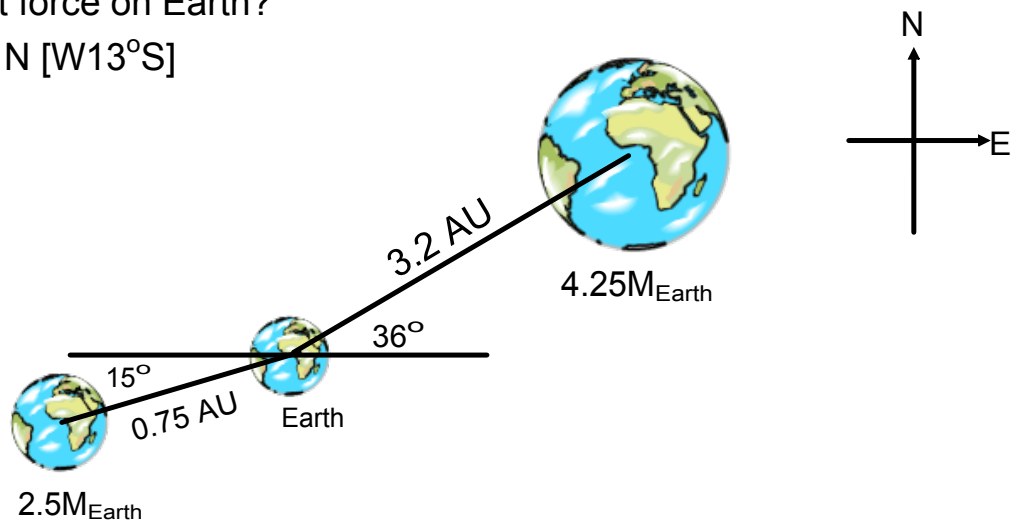
$$F_{\text{(Newtons)}} = \frac{1.56 (6.672 \times 10^{-11}) (5.974 \times 10^{24})^2}{(1.5 \times 10^{11} \text{ m})^2}$$

$$= \frac{3.71 \times 10^{39}}{(1.5 \times 10^{11})^2}$$

$$\vec{F} = 1.65 \times 10^{17} \text{ N } [W 40^\circ N]$$

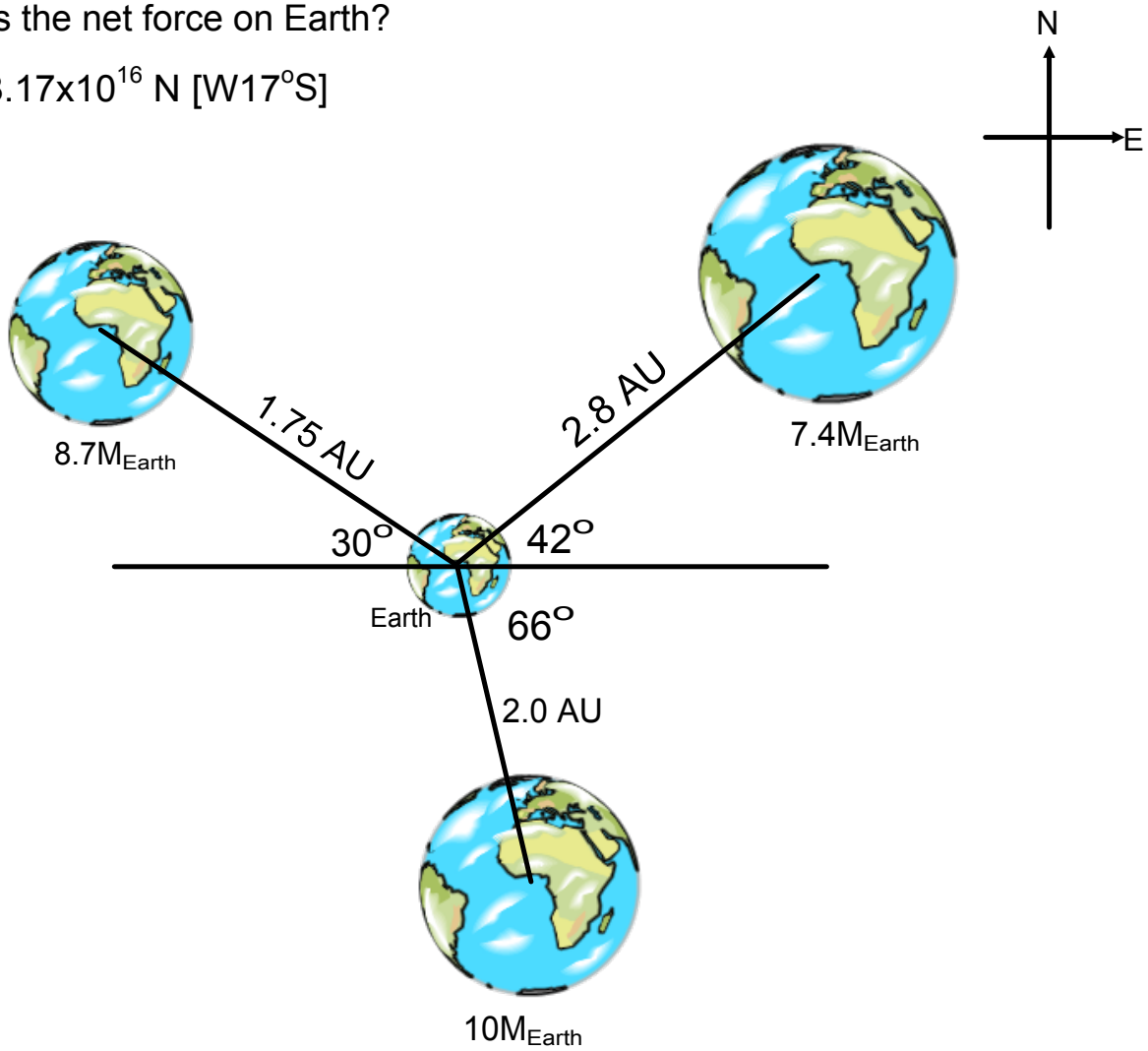
What is the net force on Earth?

$$F = 4.3 \times 10^{17} \text{ N [W}13^\circ\text{S]}$$



What is the net force on Earth?

$$F = 8.17 \times 10^{16} \text{ N [W}17^\circ\text{S]}$$



What is the net force on Earth?

$$F = 1.03 \times 10^{18} \text{ N [W}83^\circ\text{N]}$$

