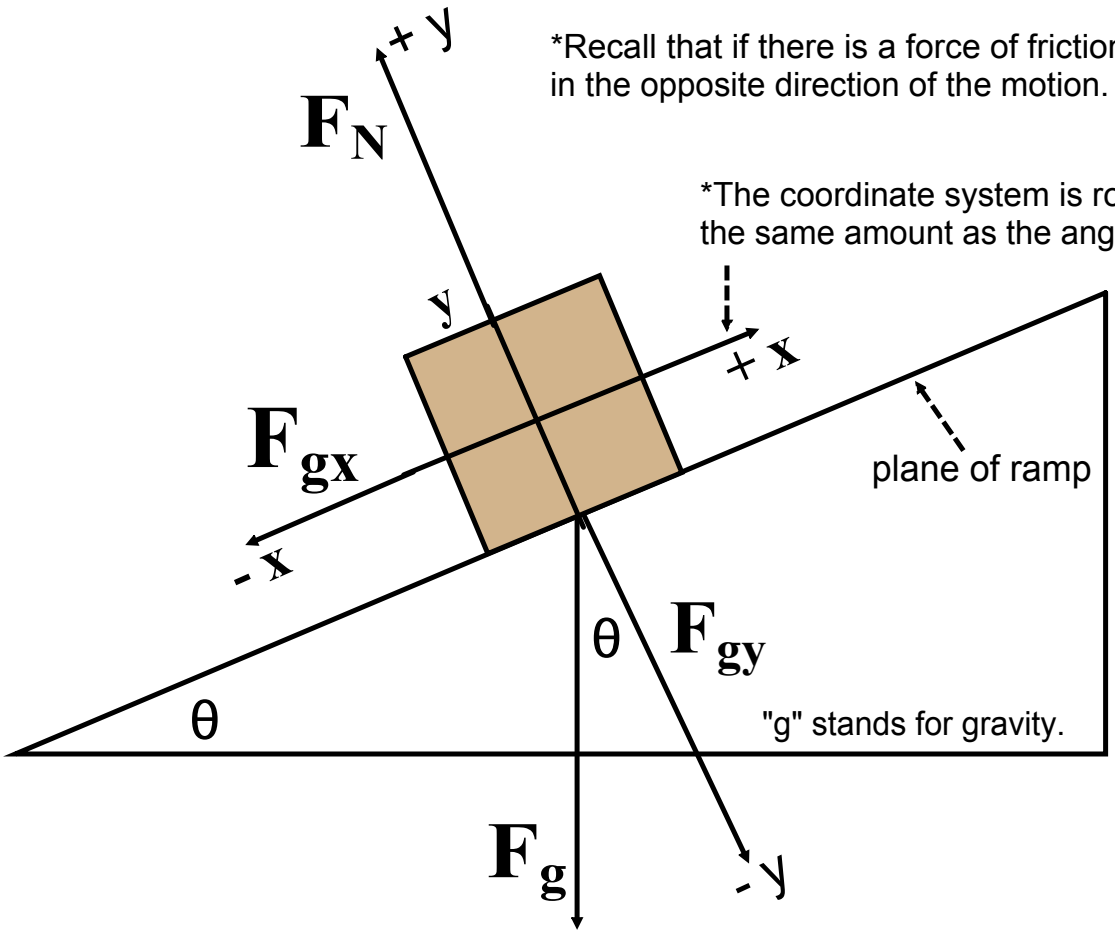
Type III - Inclined Planes, Hills, Ramps

(printed copy for students)

*Recall that if there is a force of friction it acts in the opposite direction of the motion.

*The coordinate system is rotated the same amount as the angle θ.



F_{gy} and F_g are separated by θ because of two similar triangles.

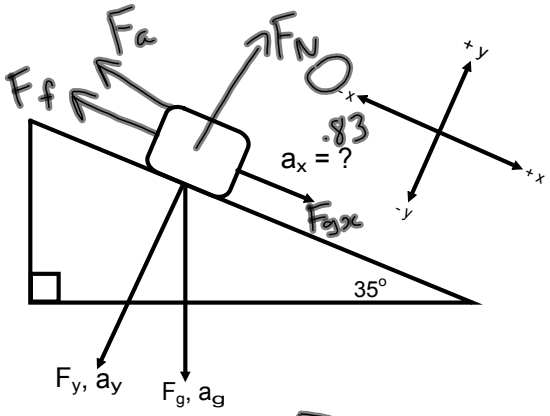
$F_{gx} = F_g \sin \theta$ ⟵ component parallel to the plane.

$F_{gy} = F_g \cos \theta$ ⟵ component perpendicular to the plane.

NOTE! The *sin* and *cos* have switched places. This will only happen when dealing with objects on a ramp.

NOTE FURTHER! Every ***F*** in the above diagram can be replaced with an ***a*** for acceleration.

1. A 55 kg block is sliding down an incline. The coefficient of kinetic friction is 0.13 and the incline makes an angle of 35° with the ground. What applied force up the ramp is necessary so the block accelerates with a magnitude of 0.83 m/s^2? down the ramp?



$F_{netx} = F_{gx} + F_f + F_a$

$F_{netx} = ma_x$

$F_{netx} = 55(0.83)$

$= 45.7\ N$

$F_f = \mu F_N$

Ramp $\rightarrow |F_N| = |F_{gy}|$

$F_f = 0.13\ F_{gy} = (0.13)(F_g \cos 35)$

$F_f = (55)(9.81)\cos 35^\circ \times 0.13$

(F_g)

$F_f = 57.5\ N$

$F_{gx} = F_g \sin 35^\circ$

$= (55)(9.81)\sin 35$

$= 309\ N$

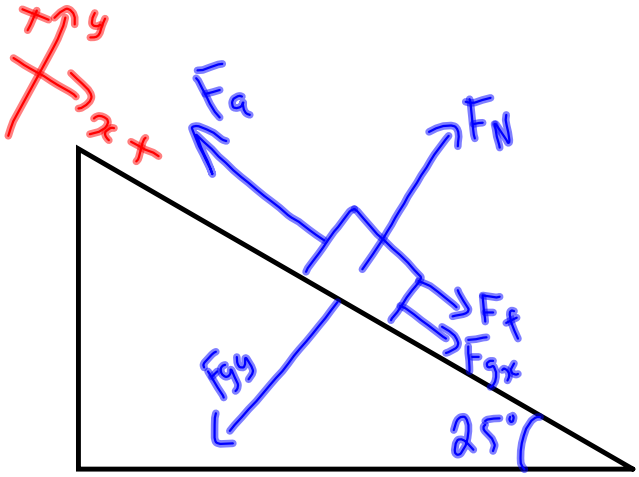
$F_{netx} = F_{gx} + F_f + F_a$

$45.7 = 309 + (-57.5) + F_a$

★ $-206\ N = F_a$

↑ up the ramp

2. What applied force is necessary for a person to pull a 30 kg object up a ramp at a constant velocity? The ramp makes an angle of 25° with the ground and the coefficient of kinetic friction is 0.12.



$F_{netx} = F_a + F_f + F_{gx}$

$F_{netx} = 0$

$F_f = \mu F_N$

$F_f = 0.12(30)(9.81)\cos 25$

$= 32\ N$

$F_{gx} = F_g \sin\theta$

$= (30)(9.81)\sin 25$

$= 124\ N$

$F_{netx} = F_{gx} + F_f + F_a$

$0 = 124\ N + 32\ N + F_a$

$-156\ N = F_a$

up the ramp

3. An inclined ramp is to be used to slide down an object at a constant speed. The coefficient of kinetic friction is 0.16. What angle should the ramp make with the ground for this to happen?

$\theta = ?$

$a_g = 9.81\ m/s^2$

$F_{net_x} = 0$

$= F_f + F_{gx}$

$a_{net_x} = a_f + a_{gx}$

$a_f = \mu a_N$

$a_f = \mu a_{gy}$

$a_f = \mu a_g \cos\theta$

$a_{gx} = a_g \sin\theta$

$a_{net_x} = -\mu a_g \cos\theta + a_g \sin\theta$

$0 = -0.16(9.81)\cos\theta + 9.81\sin\theta$

$0 = -0.16\cos\theta + \sin\theta$

$-\sin\theta = -0.16\cos\theta$

$\frac{-\sin\theta}{-\cos\theta} = 0.16$

$\tan\theta = 0.16$

$\theta = \tan^{-1}(0.16)$

$\theta = 9.1^\circ$

Sample Problems - Inclined Planes

Handout

1. A trunk weighing 562 N is resting on a plane inclined at 30.0° from the horizontal. Find the components of the trunk's weight parallel and perpendicular to the plane.

2. A 562 N trunk is placed on a frictionless plane inclined at 30.0° from the horizontal. Find the magnitude and direction of the trunk's acceleration.

3. A worker places a large plastic waste container with a mass of 84 kg on the ramp of a loading dock. The ramp makes an angle of 22° with the horizontal. The worker turns to pick up another container before pushing the first one up the ramp. If the coefficient of static friction is 0.47, will the crate slide down the ramp?

4. A 1975 kg car is rolling down a hill inclined at an angle of 15°. What is the acceleration of the car? Neglect friction.

5. A skier coasts down a 3.5° slope at a constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope.

6. You slide a 325 N trunk up a 20.0° inclined plane with a constant velocity by exerting a force of 211 N parallel to the inclined plane.

   a) What is the sum of your applied force, friction and the parallel component of the trunk's weight? Justify your answer.

   b) What is the magnitude and direction of the force of friction?

   c) What is the coefficient of friction?

#5 $\theta = 3.5°$

$\mu = ?$

$F_{net\,x} = 0\,N$

$F_f = \mu F_N \longrightarrow a_f = \mu a_N$

$$\mu = \frac{a_f}{a_N}$$

$F_{net\,x} = 0 = a_f + a_{gx}$

$0 = a_f + a_g \sin 3.5$

$\therefore \quad |a_f| = |a_g \sin 3.5|$

$F_{net\,y} = 0 = a_N + a_{gy}$

$0 = a_N + a_g \cos 3.5$

$|a_N| = |a_g \cos 3.5|$

$$\mu = \frac{a_f}{a_N} = \frac{a_g \sin 3.5}{a_g \cos 3.5} = \tan 3.5°$$

$\mu = 0.061$

Inclined Plane - Solutions

1. $F_{gx} = 281\ N$, $F_{gy} = 487\ N$

2. $a_{gx} = 4.905\ m/s^2$ down the ramp

3. $N_o, F_f > F_g$ $F_f = 359\ N$; $F_{gx} = 309\ N$

4. $a_x = 2.53\ m/s^2$

5. $\mu = 0.061$

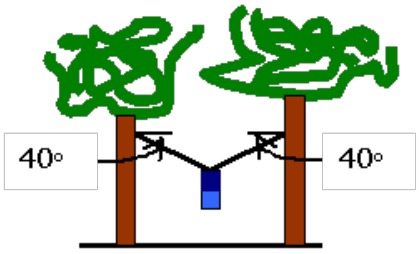
6. a) $F_{netx} = 0\ N$
   b) $|F_f| = 100\ N$ down the ramp
   c) $\mu = 0.32$

Handout

Physics 122/121

Handout: Problems I, II and III

1. On a camping trip you stretch a rope between two trees and hang your backpack from the middle of it to keep it safe from bears. The mass of your backpack is 36.0 kg and each half of the rope makes an angle of 40.0° with the horizontal.
   a) Find the amount of weight supported by each half of the rope.
   b) Find the magnitude of the tension in each rope.



2. A 2.5 kg brick is pulled at a constant speed across a table by a cord that makes an angle of 20° with the horizontal. There is 7.0 N of force in the cord.
   a) Calculate the force of friction between the brick and the table.
   b) Calculate the normal force.

3. Joey moves a 26 kg wagon at a constant speed by pushing on the handle that makes an angle, theta, with the horizontal. Joey exerts a force of 54 N on the handle and the force of friction on the wagon is 34 N.
   a) Calculate the angle the handle of the wagon makes with the horizontal.
   b) What is the magnitude of the normal force acting on the wagon?

4. A 10 N block is held motionless on a frictionless inclined plane which makes an angle of 30° with the horizontal. What force would be needed to hold the block in position?

5. An object weighing 600 N is pulled up a frictionless incline at a constant speed using a rope. If the incline makes an angle of 42.0° with the horizontal, what is the magnitude of the force that is applied to the rope?

6. A 10 kg object, starting from rest, slides down a frictionless incline with a constant acceleration of 2.0 m/s^2. What angle does the incline make with the horizontal?

7. An object with a mass of 7.2 kg is allowed to slide from rest down an inclined plane. The plane makes an angle of 30° with the horizontal and is 65 m long. The coefficient of friction between the plane and the object is 0.45. What is the velocity of the object at the bottom of the plane?

8. A piano is accelerating down a ramp that is inclined at an angle of 38.5° above the horizontal. The acceleration is 4.62 m/s^2. What is the coefficient of friction between the piano and the ramp?

Answers

1. a)Each half of the rope supports half of the weight of the backpack, 176 N.
   b)The tension in each rope is 274 N.

2. a)The force of friction is 6.6 N, in a direction opposite to the motion of the brick.
   b)The magnitude of the normal force is 22 N.

3. a)The handle makes an angle of 51° with the horizontal.
   b)The normal force is 3.0×10^3 N, up.

4. A 5.0 N force exerted up the incline would be needed.

5. It is 401 N.

6. The incline makes an angle of 12°.

7. The velocity of the object is –12 m/s.

8. The coefficient of friction is 0.19.

Warm Up

A 120 N weight is pushed to obtain constant speed along the ground. The applied force is 80 N at an angle of 40^{o} to the horizontal. What is the coefficient of ~~static~~ friction? (0.35)

Kinetic

Warm up

① $120N = F_g$ $\quad F_{netx} = 0N$

$F_a = 80N \text{ @ } 40°$ $\quad F_{nety} = 0N$

$\mu = ?$

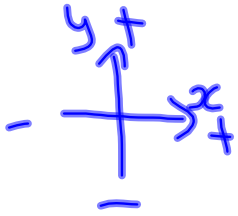
$\mu = \frac{F_f}{F_N}$

Find F_f

$F_{netx} = F_f + F_{ax}$

$0 = F_f + 80\cos 40°$

$F_f = -61N$



Find F_N

$F_{nety} = F_N + F_g + F_{ay}$

pushing (downward)

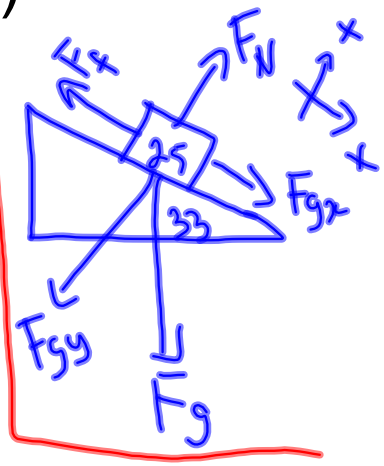
$0 = F_N - 120 - 80\sin 40$

$F_N = 171N$

$\mu = \frac{61}{171} = 0.35$

A 25 kg box is placed on an incline 33° to the horizontal. The coefficient of kinetic friction is 0.38. Find the acceleration of the box. (2.2 m/s^2 down the ramp)

$F_{net_x} = \sum \text{Forces} = m a_{net_x}$



$F_{net_x} = F_f + F_{gx}$

$F_{net_x} = F_f + (25)(9.81)\sin 33$

Find F_f

$F_f = \mu F_N$

$F_f = \mu F_{gy} = (0.38)(25)(9.81)\cos 33°$

$F_f = 78\ N$ ← magnitude

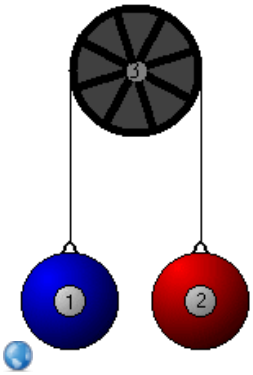
$F_{net_x} = -78\ N + 134\ N$

$= 56\ N$

$a_{net_x} = \frac{F_{net_x}}{m} = \frac{56\ N}{25\ kg} = 2.2\ m/s^2$

Multiple Masses and Finding Net Force

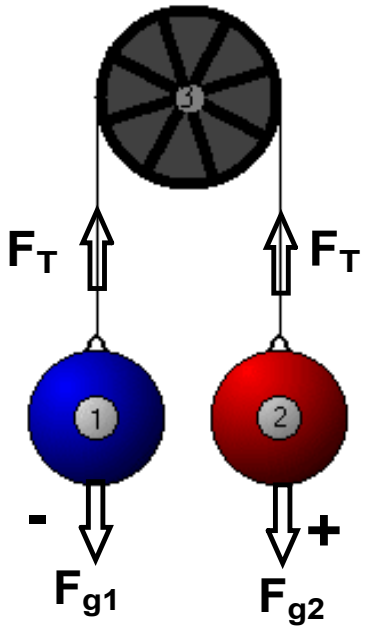
Chapter 10.2 of text:
Read Pg 478 - 489
Problems Pg 485 #s 19 - 22, Pg 488 #s 24 - 28



This is an example of a system where there are multiple masses, the Atwood machine.

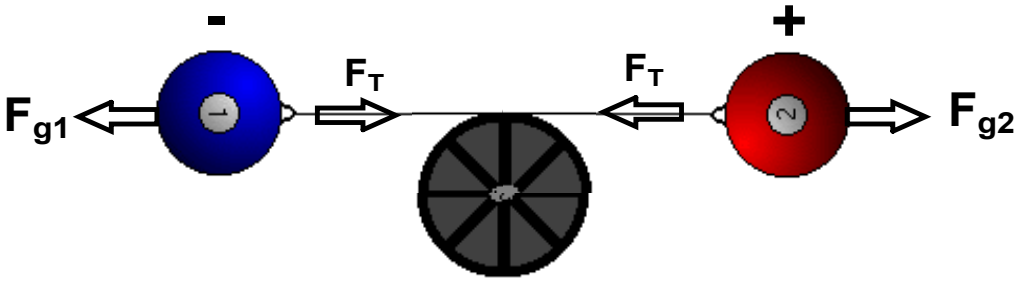
We will apply the concept of forces to determine the resulting acceleration.

Define the Direction of Forces

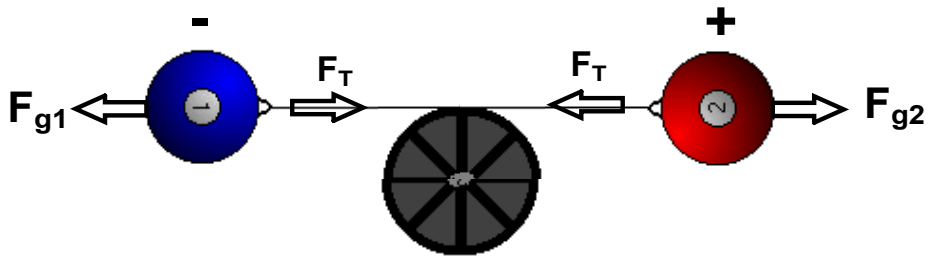


Our problems will not include friction and the pulley will be massless.

You may find it easier to picture, or draw, the system horizontally.



Acceleration of the Masses: Newton's 2nd Law



$$F_{net} = ma$$

$$F_{net} = \sum Forces$$

$$m = \sum masses\ that\ accelerate$$

$$F_{g1} + F_T + F_T + F_{g2} = (m_1 + m_2)a$$

$$-m_1 g + F_T + (-F_T) + m_2 g = (m_1 + m_2)a$$

$$m_2 g - m_1 g = (m_1 + m_2)a$$

$$(m_2 - m_1)g = (m_1 + m_2)a$$

☆ $$\vec{a} = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

To Find Tension:

$$F_{g1} + F_T = m_1 a$$

or

$$F_{g2} + F_T = m_2 a$$

☆ How would the formula for acceleration change if positive was to the left?