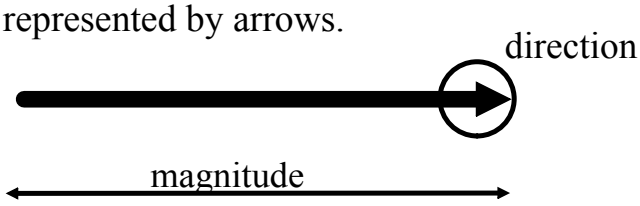
Physics 122/121

Applications of Vectors

VECTOR REVIEW

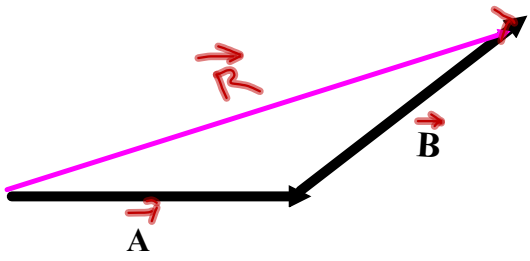
Vector quantities have both magnitude and direction. Some vector quantities are velocity, force, acceleration and momentum.

Vectors are represented by arrows.

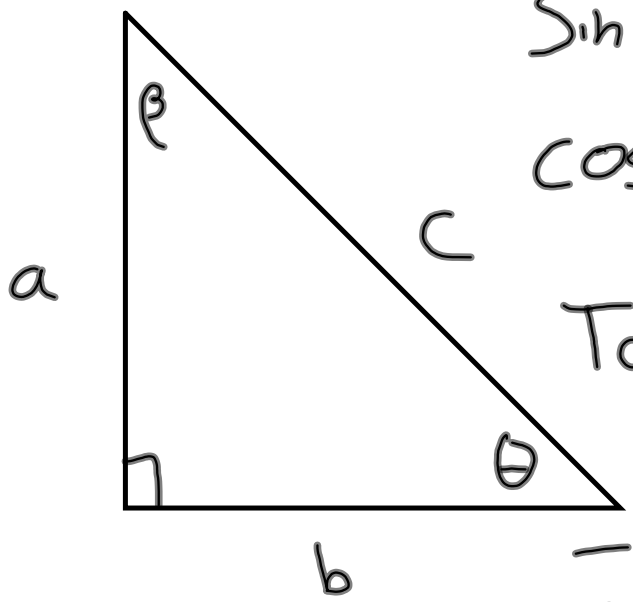


Graphical Methods of Adding Vectors

1. Tip-to-tail Method



R - resultant (sum of vectors)



$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

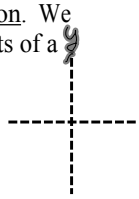
$$\tan\theta = \frac{a}{b}$$

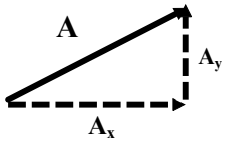
$$\sin\beta = \frac{b}{c} \qquad \tan\beta = \frac{b}{a}$$

$$\cos\beta = \frac{a}{c}$$

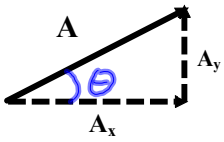
Components of a Vector (Chapter 10)

A vector can be expressed as the sum of two other vectors, called the components of the vector. The process of finding the components of a vector is called vector resolution. We will always be finding the perpendicular components of a vector.





Use trigonometric ratios to determine the magnitudes of the components. The arrows of the components show their directions.

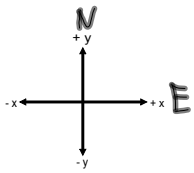


$A_y = A \sin\theta$

$A_x = A \cos\theta$

$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

Ex: Find the components of the following:



a) 95 km [E39°N]

North: $95 \sin 39 = 60\ km$

East: $95 \cos 39 = 74\ km$

b) 112 m/s [E77°S]

$y\ comp = -112 \sin 77° = -109\ m/s$

$x\ comp = +112 \cos 77 = 25\ m/s$

c) 1575 m [W22°S]

$y_{comp} = -1575 \sin 22 = -590\ m$

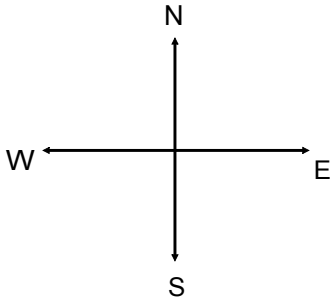
$x_{comp} = -1575 \cos 22 = -1460\ m$

Adding Vectors Using Perpendicular Components

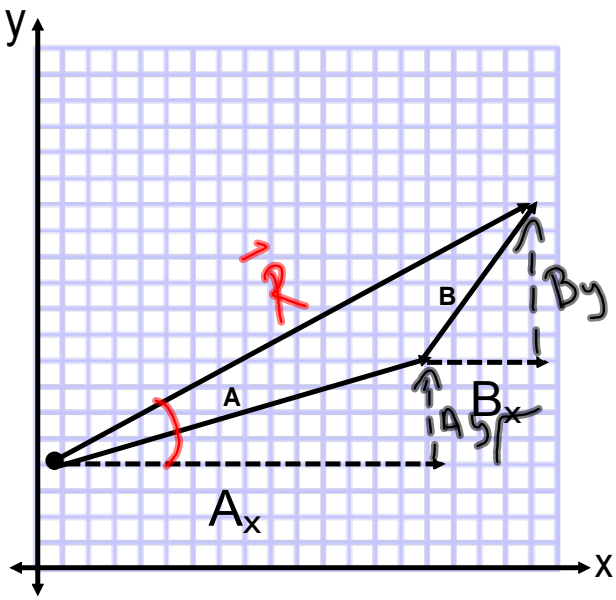
1. Resolve each vector into its perpendicular components.
2. Add corresponding vector components.

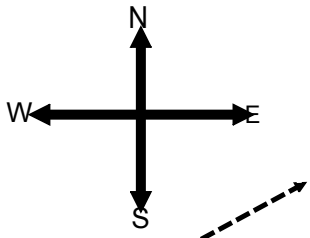
$$\mathbf{R_x} = \mathbf{A_x} + \mathbf{B_X}$$
$$\mathbf{R_y} = \mathbf{A_y} + \mathbf{B_y}$$

3. Sketch $\mathbf{R_x}$ and $\mathbf{R_y}$ tip-to-tail.
4. Use the Law of Pythagoras and a trig ratio to determine the magnitude and direction of the resultant.



Consider the two vectors A and B.





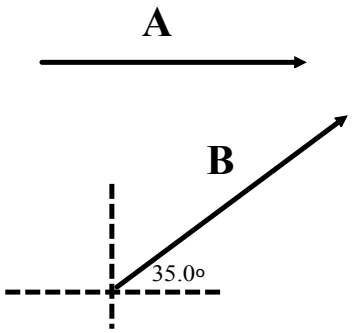
$$R_y = A_y + B_y$$

$$R_x = A_x + B_x$$

$$R^2 = R_y^2 + R_x^2$$

$$\theta = \tan^{-1}\left|\frac{R_y}{R_x}\right|$$

Example - Find the resultant of 1.60 km, east and 3.40 km, E35.0° N

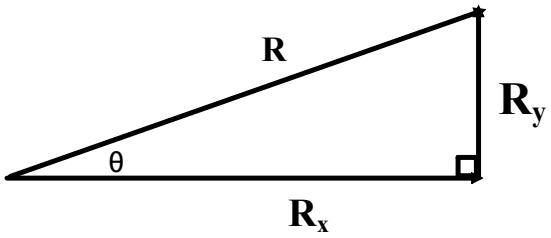


$\mathbf{A}_x = +1.60$ km
$\mathbf{A}_y = 0$ km

$\mathbf{B}_x = (3.40 \text{ km})(\cos 35.0^\circ)$
$\mathbf{B}_x = +2.785$ km

$\mathbf{B}_y = (3.40 \text{ km})(\sin 35.0^\circ)$
$\mathbf{B}_y = +1.95$ km

$\mathbf{R}_x = 1.60 \text{ km} + 2.785 \text{ km} = 4.385 \text{ km}$
$\mathbf{R}_y = 0 \text{ km} + 1.950 \text{ km} = 1.950 \text{ km}$



$$R = \sqrt{(4.385)^2 + (1.950)^2}$$

$R = 4.80$ km

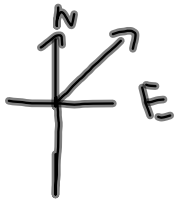
$$\tan\theta = \frac{R_y}{R_x}$$

$\theta = 24.0^\circ$

$\mathbf{R} = 4.80$ km, E24.0°N

$A_x = 6.4$

$A_y = 7.7$



$\vec{A} = 10$ $\quad \theta = \text{Tan}^{-1}\left|\frac{A_y}{A_x}\right| = \text{Tan}^{-1}\left(\frac{7.7}{6.4}\right)$

$\theta = 50°$

$\vec{A} = 10 \text{ E } 50° \text{ N}$

C:

$C_x = -3.09$

$C_y = +9.51$

$|\vec{C}| = 10$

$\theta = \text{Tan}^{-1}\left(\left|\frac{9.51}{3.09}\right|\right) = 72°$

$\vec{C} = 10 \text{ [W } 72° \text{ N]}$

Addition and Subtraction of Vectors

If **A** = 56 km [E29°N], **B** = 44 km [E81°S], **C** = 65 km [W45°S] find:

a) 2A + C

$A_N = 56 \sin 29 = 27$ km

$A_E = 56 \cos 29 = 49$ km

$C_N = -65 \sin 45 = -46$ km

$C_W = -65 \cos 45 = -46$ km

East comp

$R_E = 2A_E + C_E$

$= 2(49) + (-46)$

$R_E = 52$ km

North

$R_N = 2A_N + C_N$

$= 2(27) + (-46)$

$= 8$ km

$R^2 = R_N^2 + R_E^2$

$R = \sqrt{(8)^2 + (52)^2}$

$R = 53$ km

$\theta = \tan^{-1}\left|\frac{R_N}{R_E}\right| = \tan^{-1}\left|\frac{8}{52}\right|$

$= 8.7°$

$\vec{R} = 53$ km [E 8.7° N]

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If **A** = 56 km [E29°N], **B** = 44 km [E81°S], **C** = 65 km [W45°S] find:

b) 3A - C - 2B

$B_E = 44\cos 81 = 6.9 \text{ km}$

$B_N = -44\sin 81 = -43 \text{ km}$

$R_E = 3A_E - C_E - 2B_E$

$= 3(49) - (-46) - 2(6.9)$

$= 179.2 \text{ km}$

$R_N = 3A_N - C_N - 2B_N$

$= 3(27) - (-46) - 2(-43)$

$= 81 + 46 + 86$

$= 213 \text{ km}$

$R^2 = R_N^2 + R_E^2 = (213)^2 + (179)^2$

$R = \sqrt{77410} = 278 \text{ km}$

$\theta = \tan^{-1}\left(\frac{213}{179}\right) = \tan^{-1}\left(\left|\frac{R_N}{R_E}\right|\right)$

$\theta = 50°$

$\vec{R} = 278 \text{ km } [E\,50°N]$

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Vector Addition and Subtraction of Vector Components - Look at the Worksheet

Part II

a) $\vec{R} = 19.8\ [E\ 59°\ N]$

b) $\vec{R} = 19\ [E\ 44°\ S]$

c) $\vec{R} = 16\ [W\ 82°\ N]$

d) $\vec{R} = 20.8\ [W\ 87°\ S]$