METHOD #2   Completing the Square

Example #1

Solve the quadratic equation: $x^2 + 4x - 10 = 0$

We cannot factor this as in the previous examples since there are no 2 integers that add to 4 and multiply to -10. We use a method called completing the square. This method was introduced earlier when we had to convert from the general form of a quadratic function to the standard or transformational form.

To use the method of completing the square, you must ensure that the coefficient of the squared term is 1.

$$x^2 + 4x = 10$$

$$(x^2 + 4x + 4) = 10 + 4$$

$$(x + 2)^2 = 14$$

$$\sqrt{(x+2)^2} = \sqrt{14}$$

$$x + 2 = \pm\sqrt{14}$$

$$x + 2 = +\sqrt{14} \qquad x + 2 = -\sqrt{14}$$

$$x = -2 + \sqrt{14} \qquad x = -2 - \sqrt{14}$$

$$x = -2 \pm \sqrt{14}$$

To complete the above example:

Step 1 - Move the constant term to the right-hand side $\qquad\qquad\qquad$ $\mathbf{x^2 + 4x = 10}$

Step 2 – Make sure that the coefficient of the squared term is 1. If it is not 1, divide every term by the coefficient of the squared term.

The coefficient of the squared term is 1 in this example, so we can skip this step!

Step 3 – Complete the square on the left side(as normal), by taking half of the middle coefficient and squaring it. Don't forget to add this result to the right side as well. $\qquad$ $x^2 + 4x + 4 = 10 + 4$

Step 4 – Factor the left side(as normal), by taking the square root of the first term, the sign from the middle term and the square root of the last term. $\qquad\qquad$ $(x + 2)^2 = 14$

Step 5 – Since we are now trying to solve for "x", we first need to take the square root of both sides and then move the constant term to the other side. $\qquad\qquad$ $(x + 2) = \pm\sqrt{14}$
$$x = -2 \pm\sqrt{14}$$

***In some cases, you will need to simplify the radical. In other cases, the radical may "disappear" leaving you with only integers.

Example #2

Solve the quadratic equation: $3x^2 - 8x - 12 = 0$

Step 1: $3x^2 - 8x = 12$

Step 2: $x^2 - \dfrac{8}{3}x = 4$

Step 3: $x^2 - \dfrac{8}{3}x + \dfrac{16}{9} = 4 + \dfrac{16}{9}$

Step 4: $\left(x - \dfrac{4}{3}\right)^2 = \dfrac{36}{9} + \dfrac{16}{9}$

$\left(x - \dfrac{4}{3}\right)^2 = \dfrac{52}{9}$

Step 5: $\left(x - \dfrac{4}{3}\right) = \pm\sqrt{\dfrac{52}{9}}$

$\left(x - \dfrac{4}{3}\right) = \pm\dfrac{\sqrt{52}}{3}$

$x = \dfrac{4}{3} \pm \dfrac{\sqrt{52}}{3}$ (WE NEED TO SIMPLIFY $\sqrt{52}$)

$x = \dfrac{4}{3} \pm \dfrac{\sqrt{4 \times 13}}{3}$

$x = \dfrac{4}{3} \pm \dfrac{2\sqrt{13}}{3}$ or $\dfrac{4 \pm 2\sqrt{13}}{3}$

$$\frac{3x^2}{3} - \frac{8x}{3} - \frac{12}{3} = 0$$

$$x^2 - \frac{8}{3}x - 4 = 0$$

$$x^2 - \frac{8}{3}x = 4$$

$$x^2 - \frac{8}{3}x + \frac{64}{36} = 4 + \frac{64}{36}$$

$$x^2 - \frac{8}{3} + \frac{16}{9} = \frac{52}{9}$$

$$\sqrt{\left(x - \frac{4}{3}\right)^2} = \sqrt{\frac{52}{9}}$$

$$x - \frac{4}{3} = \pm\sqrt{\frac{52}{9}}$$

$$x = \frac{4}{3} \pm \sqrt{\frac{52}{9}}$$

$$\sqrt{\frac{52}{9}} = \frac{\sqrt{52}}{3} = \frac{\sqrt{4 \times 13}}{3}$$

$$= \frac{2\sqrt{13}}{3}$$

$$x = \frac{4}{3} \pm \frac{2\sqrt{13}}{3}$$