Circular Motion Formulae

$$v = \frac{2\pi r}{T}$$

$$v = 2\pi r f$$

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$a_c = 4\pi^2 r f^2$$

$$F_c = m a_c$$

$$F_c = \frac{m v^2}{r}$$

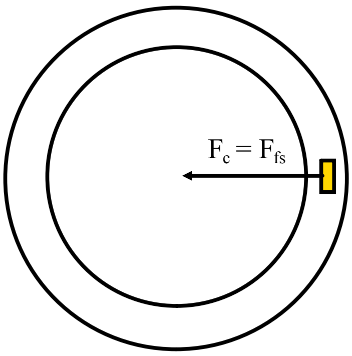
$$F_c = \frac{m 4\pi^2 r}{T^2}$$

$$F_c = m 4\pi^2 r f^2$$

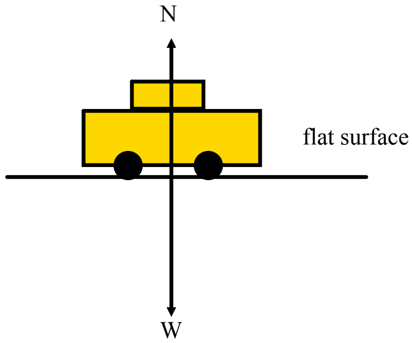
$v = \sqrt{rg\mu}$ (horizontal) unbanked

$v = \sqrt{rg\tan\theta}$ (banked)

Unbanked Curves and Centripetal Force



When a car travels around a flat curve (unbanked curve), the centripetal force keeping the car on the curve comes from the static friction between the road and the tires. (It is static and not kinetic friction because the tires are not slipping with respect to the radial direction.) If static friction is insufficient given the speed and radius of the turn, the car will skid off the road.



$$F_c = F_{fs}$$

$$\frac{mv^2}{r} = \mu_s N$$

$$\frac{mv^2}{r} = \mu_s W$$

$$\frac{\cancel{m}v^2}{r} = \mu_s \cancel{m} g$$

$$v^2 = \mu_s g r$$

$$\boxed{v = \sqrt{rg\mu_s}}$$

v -> speed (m/s) μ_s -> coefficient of static friction r -> radius of curve (m) g = 9.80 m/s²

$$\frac{km}{h} \xrightarrow{\div 3.6} \frac{m}{s}$$

Example:

If the maximum speed at which a car can safely navigate an unbanked turn of radius 50.0 m is 21.0 m/s, what is the coefficient of static friction? ($\mu_s = 0.900$)

$$v = \sqrt{rg\mu_s}$$

$$21 = \sqrt{(50)(9.81)\mu}$$

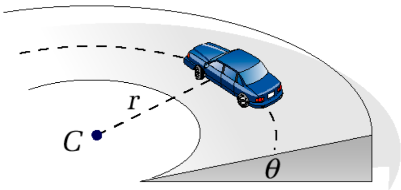
$$21^2 = (50)(9.81)\mu_s$$

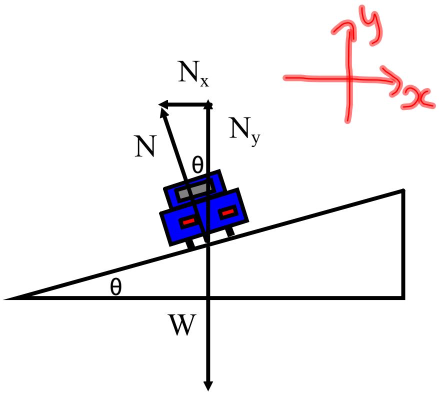
$$\frac{441}{490.5} = \mu_s \rightarrow \boxed{0.90}$$

Banked Curves and Centripetal Force

The reliance on friction can be eliminated completely for a given speed if a curve is banked at an angle relative to the horizontal.

** Assume a friction-free banked curve.





We'll need two equations to derive a formula for this type of problem.

$F_c = N_x$ $\quad\quad$ $W = N_y$

$$\boxed{\frac{mv^2}{r} = N\sin\theta} \quad (1) \qquad\qquad \boxed{mg = N\cos\theta} \quad (2)$$

$$\frac{\dfrac{mv^2}{r} = N\sin\theta}{mg \quad N\cos\theta} \quad \begin{matrix}(1)\\(2)\end{matrix}$$

$$\frac{v^2}{gr} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{v^2}{gr} = \tan\theta$$

$$v^2 = gr\tan\theta$$

$$\boxed{v = \sqrt{rg\tan\theta}}$$

Example:

The turns in a track have a maximum radius at the top of 316 m and are banked steeply. If a car travels at a speed of 43 m/s, what is the angle of the curve with respect to the horizontal? Assume the turn is frictionless. (31)