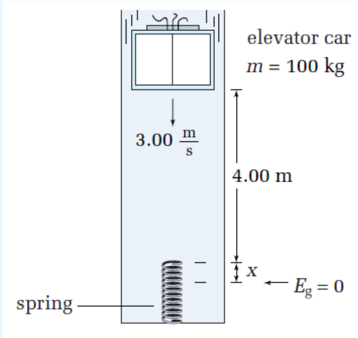
MODEL PROBLEM

Vertical Elastic Collisions

A freight elevator car with a total mass of 100.0 kg is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a spring constant of 8.000×10^3 N/m. By how much will the spring be compressed when the car reaches zero velocity?



Frame the Problem

- Initially, the car is in *motion* and therefore has *kinetic energy*. It also has *gravitational potential energy*.
- As the car begins to *fall*, the *gravitational potential energy* transforms into *kinetic energy*. When the elevator hits the spring, the elevator *slows*, losing *kinetic energy*, and the spring compresses, gaining *elastic potential energy*.
- When the elevator comes to a complete *stop*, it has *no kinetic* or *gravitational potential energy*. All of the energy is now stored in the spring in the form of *elastic potential energy*.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.
- Choose the lowest level reached by the elevator as the reference level for gravitational potential energy

$$E_{T_0} = E_{T_f} \Longrightarrow \Delta E_T = 0$$

$$\Delta E_k + \Delta E_g + \Delta E_e = 0$$

$$\left(\tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2\right) + \left(m g h_f - m g h_0\right) + \left(\tfrac{1}{2} k x_f^2 - \tfrac{1}{2} k x_0^2\right)$$

$$0 - \tfrac{1}{2}(100)(3)^2 + 0 - (100)(9.81)(4 + x) + \tfrac{1}{2}(8000)x^2 + 0$$

$$-450 - 3924 - 981x + 4000x^2 = 0$$

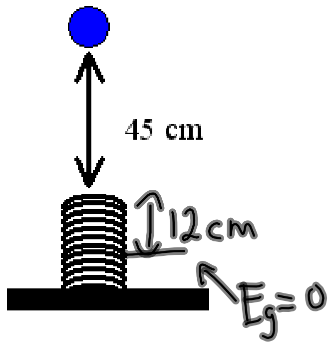
$$x^2 - 0.24525x - 1.0935 = 0$$

$$x = \frac{-(-0.245) \pm \sqrt{(0.245)^2 + 4(1)(1.09)}}{2}$$

$$x = \frac{0.245 \pm 2.102}{2}$$

$$x = 1.17\text{m} \quad \text{or} \quad -0.9285\text{ m}$$

A ball of mass 6.0 kg is dropped from a height of 45 cm above a spring and compresses the spring by 12 cm. What is the spring constant of the spring? (k = 4700 N/m)



$$\Delta E_T = 0$$

$$\cancelto{0}{\Delta E_k} + \Delta E_g + \Delta E_e = 0$$

$$mg(0) - mg(0.45 + 0.12) + \tfrac{1}{2}k(0.12)^2 - 0 = 0$$

$$-(6)(9.81)(0.57) + \tfrac{1}{2}k(0.12)^2 = 0$$

$$\frac{1}{2}k(0.12)^2 = 33$$

$$k = 4700\ \text{N/m}$$

7.2 Conservation of Total Energy

LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but it can be transformed from one form to another or transferred from one object to another. The total energy of an isolated system, including all forms of energy, always remains constant.

CONSERVATION OF TOTAL ENERGY

The work done by nonconservative forces is the difference of the final mechanical energy and the initial mechanical energy of a system.

$$W_{nc} = \Delta E_T$$

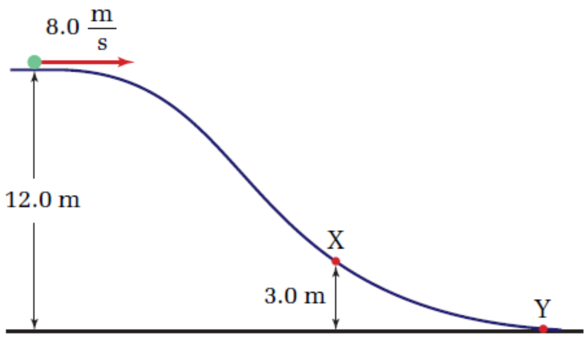
$$W_{nc} = E_{final} - E_{initial}$$

Quantity	Symbol	SI unit
work done by nonconservative forces	W_{nc}	J (joule)
mechanical energy of the system after the process	E_{final}	J (joule)
mechanical energy of the system before the process	$E_{initial}$	J (joule)

Unit Analysis

All of the units are joules.

19. A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg. Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does 1.22×10^3 J of work on the snow on the way to point X.



$$W_{nc} = \Delta E_T$$

* $W_c = -1220$ J as friction removes energy/lost.

$$W_{nc} = \Delta E_k + \Delta E_g$$

$$W_{nc} = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2\right) + (mgh_f - mgh_0)$$

$$-1220 = \left(\tfrac{1}{2}(70)v_f^2 - \tfrac{1}{2}(70)(8)^2\right) + \left((70)(9.81)(3) - (70)(9.81)(12)\right)$$

$$-1220 = 35v_f^2 - 2240 + 2060 - 8240$$

$$-1220 = 35v_f^2 - 8420$$

$$7200 = 35v_f^2$$

$$\boxed{14.3\text{ m/s} = v_f}$$

•

A 0.75 kg mass, initially at rest, is pulled by a applied force of 8.5 N a distance of 12 m. If the final velocity is 9.5 m/s what is the force of friction?

$$W_{nc} = \Delta E_T$$

*Energy lost by friction = ΔE_K

$$W_c = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

v_2 = actual speed = 9.5 m/s

v_1 = speed if no friction

Assuming no friction

$$W = \Delta E_K$$

$$Fd = \frac{1}{2}mv_1^2$$

$$(8.5)(12) = \frac{1}{2}(0.75)v_1^2$$

$$16.5\ \text{m/s} = v_1$$

Friction

$$W_{nc} = -F_f d$$

- $$-F_f d = \frac{1}{2}(0.75)(9.5)^2 - \frac{1}{2}(0.75)(16.5)^2$$

$$-F_f(12) = 33.8 - 102.1$$

$$-F_f(12) = -68.25$$

$$F_f = 5.7\ \text{N}$$