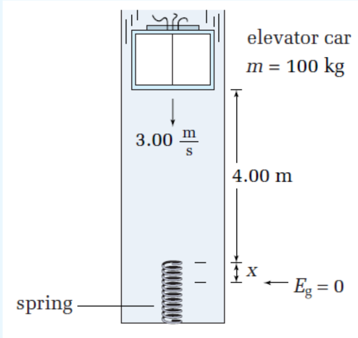
**MODEL PROBLEM**

## Vertical Elastic Collisions

**A freight elevator car with a total mass of 100.0 kg is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a spring constant of $8.000 \times 10^3$ N/m. By how much will the spring be compressed when the car reaches zero velocity?**




### Frame the Problem

- Initially, the car is in *motion* and therefore has *kinetic energy*. It also has *gravitational potential energy*.
- As the car begins to *fall*, the *gravitational potential energy* transforms into *kinetic energy*. When the elevator hits the spring, the elevator *slows*, losing *kinetic energy*, and the spring compresses, gaining *elastic potential energy*.
- When the elevator comes to a complete *stop*, it has *no kinetic* or *gravitational potential energy*. All of the energy is now stored in the spring in the form of *elastic potential energy*.
- Since all of the motion is in a downward direction, define "down" as the positive direction for this problem.
- Choose the lowest level reached by the elevator as the reference level for gravitational potential energy

$$E_{T_0} = E_{T_f} \Longrightarrow \Delta E_T = 0$$

$$\Delta E_k + \Delta E_g + \Delta E_e = 0$$

$$\left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2\right) + \left(mgh_f - mgh_0\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_0^2\right)$$

$$0 - \tfrac{1}{2}(100)(3)^2 + 0 - (100)(9.81)(4+x) + \tfrac{1}{2}(8000)x^2 + 0$$

$$-450 - 3924 - 981x + 4000x^2 = 0$$

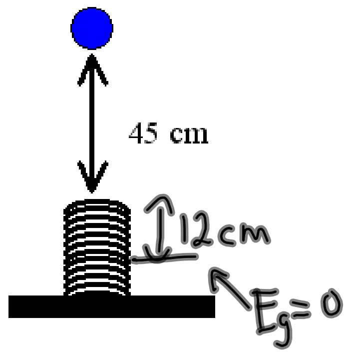
$$x^2 - 0.24525x - 1.0935 = 0$$

$$x = \frac{-(-0.245) \pm \sqrt{(0.245)^2 + 4(1)(1.09)}}{2}$$

$$x = \frac{0.245 \pm 2.102}{2}$$

$$x = 1.17\,\text{m} \quad \text{or} \quad -0.9285\,\text{m}$$

A ball of mass 6.0 kg is dropped from a height of 45 cm above a spring and compresses the spring by 12 cm. What is the spring constant of the spring? (k = 4700 N/m)




$$\Delta E_T = 0$$

$$\cancel{\Delta E_k}^{\to 0} + \Delta E_g + \Delta E_e = 0$$

$$mg(0) - mg(0.45 + 0.12) + \tfrac{1}{2}k(0.12)^2 - 0 = 0$$

$$-(6)(9.81)(0.57) + \tfrac{1}{2}k(0.12)^2 = 0$$

$$\frac{1}{2}k(0.12)^2 = 33$$

$$k = 4700 \text{ N/m}$$

# 7.2 Conservation of Total Energy

## LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but it can be transformed from one form to another or transferred from one object to another. The total energy of an isolated system, including all forms of energy, always remains constant.

## CONSERVATION OF TOTAL ENERGY

The work done by nonconservative forces is the difference of the final mechanical energy and the initial mechanical energy of a system.

$$W_{nc} = E_{final} - E_{initial}$$

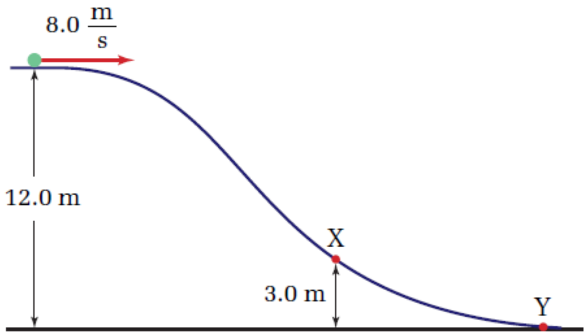
$W_{nc} = \Delta E_T$

| Quantity | Symbol | SI unit |
|---|---|---|
| work done by nonconservative forces | $W_{nc}$ | J (joule) |
| mechanical energy of the system after the process | $E_{final}$ | J (joule) |
| mechanical energy of the system before the process | $E_{initial}$ | J (joule) |

**Unit Analysis**

All of the units are joules.

**19.** A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg. Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does $1.22 \times 10^3$ J of work on the snow on the way to point X.




$$W_{nc} = \Delta E_T$$

* $W_c = -1220\,J$ as friction removes energy/lost.

$$W_{nc} = \Delta E_k + \Delta E_g$$

$$W_{nc} = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2\right) + \left(mgh_f - mgh_0\right)$$

$$-1220 = \left(\tfrac{1}{2}(70)v_f^2 - \tfrac{1}{2}(70)(8)^2\right) + \left((70)(9.81)(3) - (70)(9.81)(12)\right)$$

$$-1220 = 35v_f^2 - 2240 + 2060 - 8240$$

$$-1220 = 35v_f^2 - 8420$$

$$7200 = 35v_f^2$$

$$\boxed{14.3\,m/s = v_f}$$

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A 0.75 kg mass, initially at rest, is pulled by a applied force of 8.5 N a distance of 12 m. If the final velocity is 9.5 m/s what is the force of friction?

$$W_{nc} = \Delta E_T$$

*Energy lost by friction $= \Delta E_K$

$$W_c = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$v_2$ = actual speed $= 9.5$ m/s

$v_1$ = speed if no friction

Assuming no friction

$$W = \Delta E_K$$

$$Fd = \frac{1}{2}mv_1^2$$

$$(8.5)(12) = \frac{1}{2}(0.75)v_1^2$$

$$16.5 \text{ m/s} = v_1$$

Friction

$$W_{nc} = -F_f d$$

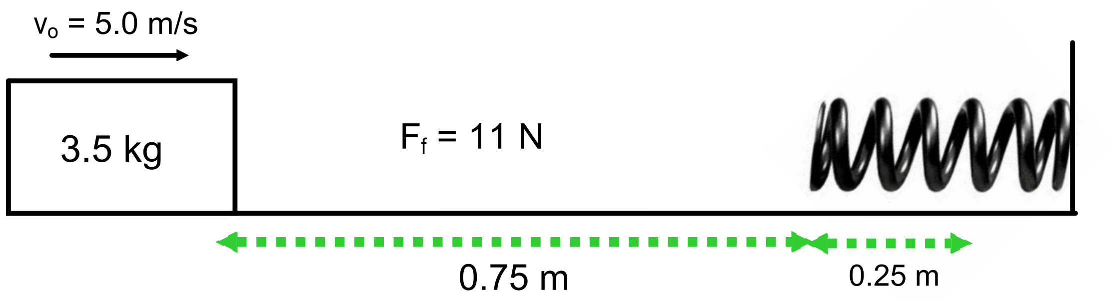
- $$-F_f d = \frac{1}{2}(0.75)(9.5)^2 - \frac{1}{2}(0.75)(16.5)^2$$

$$-F_f(12) = 33.8 - 102.1$$

$$-F_f(12) = -68.25$$

$$F_f = 5.7 \text{ N}$$

# Conservation of Total Mechanical Energy




a) Find k. (Ans: k = 1050 N/m)

$$W_{nc} = \Delta E_T$$

$$W_{nc} = \Delta E_k + \Delta E_e$$

$$-F_f d = \cancel{\tfrac{1}{2} m v_f^2}^{\,0} - \tfrac{1}{2} m v_o^2 + \tfrac{1}{2} k x_f^2 - \cancel{\tfrac{1}{2} k x_o^2}^{\,0}$$

$$-(11)(1) = -\tfrac{1}{2}(3.5)(5)^2 + \tfrac{1}{2} k (0.25)^2$$

$$-11 = -43.75 + 0.03125\,k$$

$$1050 \text{ N/m} = k$$

b) How fast will the mass be moving as it leaves the spring? (4.14 m/s)

$$-F_f d = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_o^2 + \cancel{\tfrac{1}{2} k x_f^2}^{\,0} - \cancel{\tfrac{1}{2} k x_o^2}^{\,0}$$

$$-(11)(1.25) = \tfrac{1}{2}(3.5) v_f^2 - \tfrac{1}{2}(3.5)(5)^2$$

$$-13.75 = 1.75 v_f^2 - 43.75$$

$$30 = 1.75 v_f^2$$

$$4.14 \text{ m/s} = v_f$$

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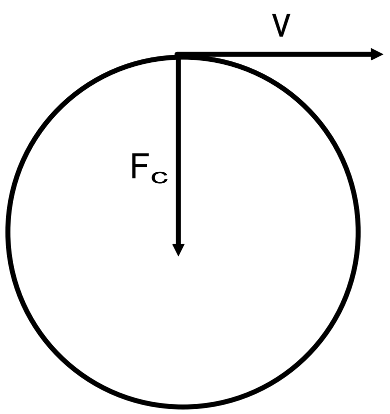
# Circular Motion and W-E-P

The concepts of work, energy, and power apply as normal; be mindful of equations governing circular motion and the direction of all the forces.




$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

A block starts from rest and is subjected to a force over a distance of 5.2 m. After the object is free of the above force, a 375 N force can turn the object through a 90° corner with a radius of 0.75 m. How great was the initial force? ($F_o$ = 27 N)

$F_c = 375$

$r = 0.75$

$$W = \Delta E_K$$

$$F_o d = E_{Kf} - E_o$$

$$F_o d = \frac{1}{2} m v_f^2 - \cancel{\frac{1}{2} m v_o^2} \rightarrow 0$$

$$F_c = \frac{m v_f^2}{r} \rightarrow v_f^2 = \frac{F_c r}{m}$$

So, $$F_o d = \frac{1}{2} \cancel{m} \left( \frac{F_c r}{\cancel{m}} \right)$$

$$F_o (5.2) = \frac{1}{2} (375)(0.75)$$

$$F_o = 27 \text{ N}$$

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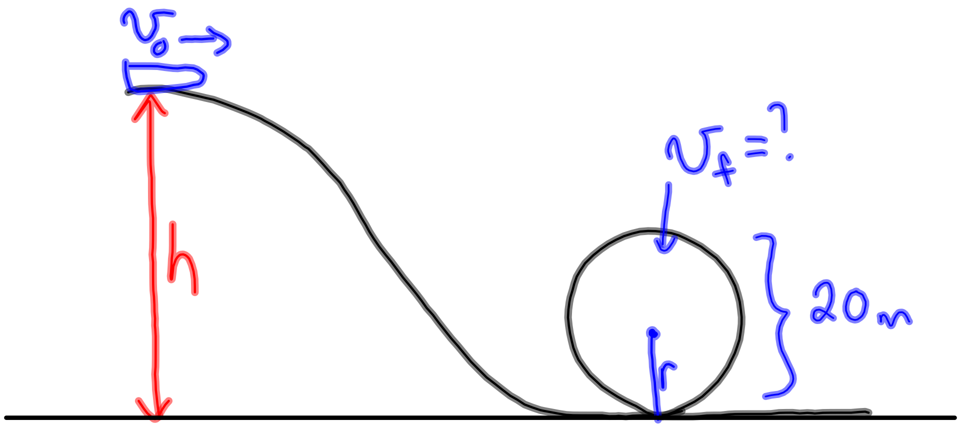
A frictionless roller coaster is made with one circular loop. If the car starts from rest, what is the minimum height of the roller coaster for the car to survive a 10 m radius loop? (25 m)




@ top of circle $\cancel{N}^{\to 0} + W = \frac{mv^2}{r}$

$$mg = \frac{mv^2}{r}$$

$rg = v^2$ ← min vel to survive loop

$$v_f = \sqrt{rg} = \sqrt{(10)(9.81)} = 9.9\,m/s$$

$$\Delta E_T = 0$$

$$(E_{kf} - \cancel{E_{k0}}^{\to 0}) + (E_{gf} - E_{g0}) = 0$$

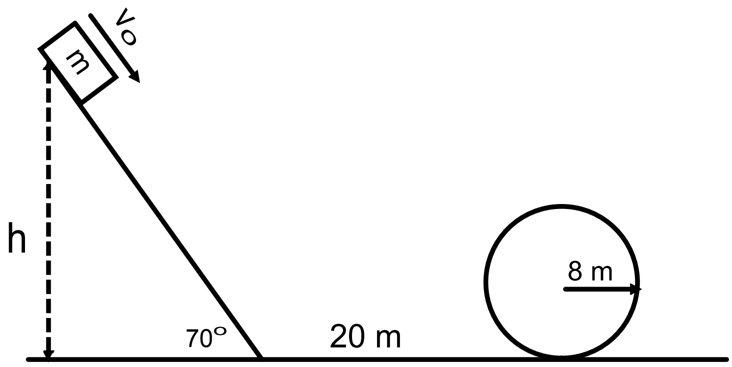
$$\tfrac{1}{2}\cancel{m}v_f^2 + \cancel{m}gh_f - \cancel{m}gh_0 = 0$$

$$\tfrac{1}{2}(9.9)^2 + (9.81)(20) - (9.81)h_0 = 0$$

$$49.05 + 196.2 = 9.81\,h_0$$

$$\boxed{25\,m = h_0}$$

A roller coaster is made with one circular loop. The average force of friction between the car and the track is 450 N. If the 125 kg car starts from rest, what is the minimum height of the roller coaster for the car to survive a 8.00 m radius loop? (22.7 m)

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Read & Review:
Pages 280 - 309

Practice:
Pg 296 #s 9 - 14 (springs)
Pg 298 #s 15 - 17 (falling on spring)
Pg 308 #s 19 - 24 (friction)

Textbook: Page 329, PFU #21-23

Page 332 # 38, 39, 40, 41, 44 - 51, 54

Handout:
Circular Motion # 54, 56, 57, 59-61.
Springs # 10, 12.
Combination Problems # 53-57, 59.

Work - Energy - Power Review (Chapter 6): Pg 275 #s 15 - 39

Energy Transformations - Simulations

Energy In and Out of a System

Downhill Skiing

Roller Coaster

Regents Simulations

Roller Coaster

Sled

Pendulum

Dart