

LAW OF COSINES

$$\mathbf{a^2 = b^2 + c^2 - 2bc \cos A}$$

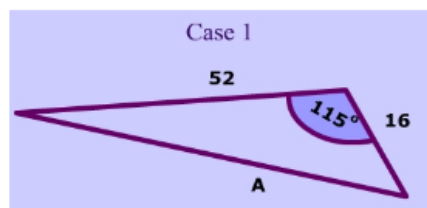
$$\mathbf{b^2 = a^2 + c^2 - 2ac \cos B}$$

$$\mathbf{c^2 = a^2 + b^2 - 2ab \cos C}$$

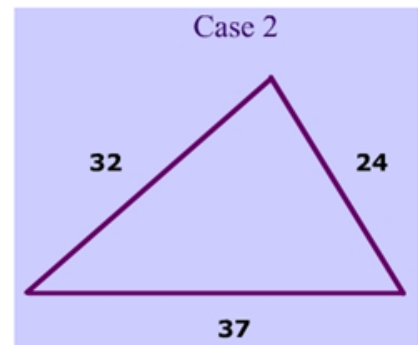
When will you use the Law of Cosines?

You will use the Law of Cosines when:

- A) you need to find a missing side and you are given the other two sides and the angle in between them (the included angle).**



- B) you need to find an angle measure when all three side lengths are given.**



★ For case (B) the formula can be rearranged in the following forms:

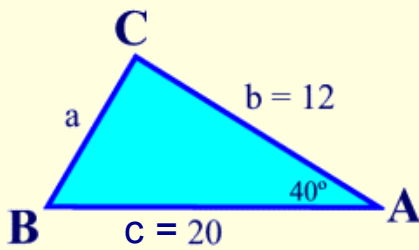
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

★ You may need to combine both the **Law of Sines** and the **Law of Cosines** in the same question 😊

Example 1: In $\triangle ABC$, side $b = 12$, side $c = 20$ and $m\angle A = 40^\circ$. Find side a to the nearest integer.



Since the only known angle is A, we use the version of the Law of Cosines dealing with angle A.

This problem involves all three sides but only one angle of the triangle. This fits the profile for the Law of Cosines.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (12)^2 + (20)^2 - 2(12)(20) \cos 40^\circ$$

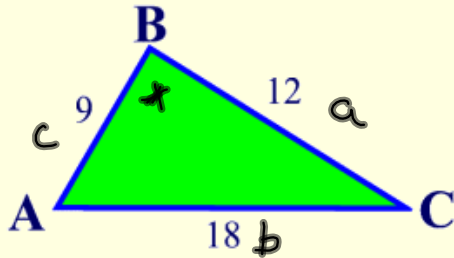
$$a^2 = 144 + 400 - 480(0.7660)$$

$$a^2 = 176.3$$

$$a = \sqrt{176.3}$$

$$= 13.3$$

Example 2: Find the largest angle, to the nearest tenth of a degree, of a triangle whose sides are 9, 12 and 18.



In a triangle, the largest angle is opposite the largest side.
We need to find $\angle B$.

Use the Law of Cosines:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$18^2 = 9^2 + 12^2 - 2(9)(12) \cos B$$

$$324 = 81 + 144 - 216 \cos B$$

$$324 - 81 - 144 = -216 \cos B$$

$$99 = -216 \cos B$$

$$\frac{99}{-216} = \cos B$$

$$-0.4583 = \cos B$$

$$117^\circ = \angle B$$