SOLUTIONS => LAW OF COSINES WORKSHEET
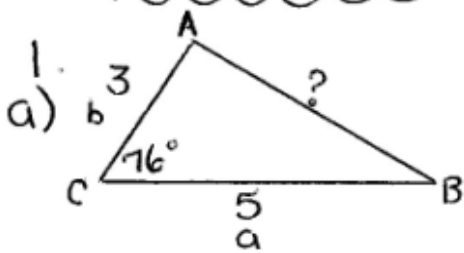
1.

a)

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$c^2 = (5)^2 + (3)^2 - 2(5)(3) \cos 76°$$
$$c^2 = 25 + 9 - 30(0.2419)$$
$$c^2 = 34 - 7.2577$$
$$c^2 = 26.7423$$
$$c = 5.2$$

b)

$$r^2 = p^2 + q^2 - 2pq \cos R$$
$$r^2 = (6)^2 + (8)^2 - 2(6)(8) \cos 120°$$
$$r^2 = 36 + 64 - 96(-0.5000)$$
$$r^2 = 100 - (-48)$$
$$r^2 = 100 + 48$$
$$r^2 = 148$$
$$r = 12.2$$

c)

A

?
c

12
b

135°

C   10
     a
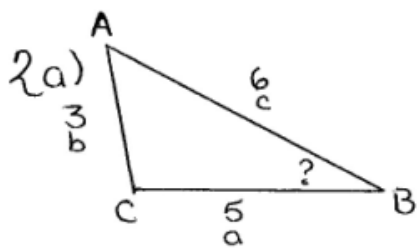
B
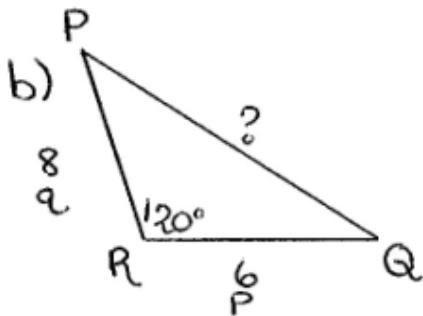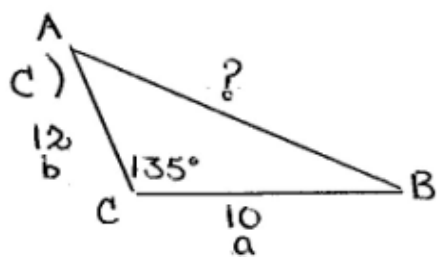
$c^2 = a^2 + b^2 - 2ab \cos C$
$c^2 = (10)^2 + (12)^2 - 2(10)(12) \cos 135°$
$c^2 = 100 + 144 - 240(-0.7071)$
$c^2 = 244 - (-169.7056)$
$c^2 = 244 + 169.7056$
$c^2 = 413.7056$
$c = 20.3$

2a)

A

6
c

3
b

?

C   5
     a

B

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{(5)^2 + (6)^2 - (3)^2}{2(5)(6)}$

$\cos B = \dfrac{25 + 36 - 9}{60}$

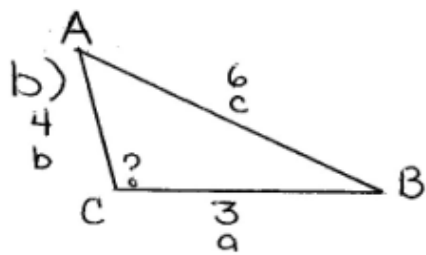$\cos B = \dfrac{52}{60}$

$\cos B = 0.8667$

$B = \cos^{-1}(0.8667)$

$B = 30°$

A

2

b)

A

b) 4 b

6 c

? C 3 a B

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{(3)^2 + (4)^2 - (6)^2}{2(3)(4)}$
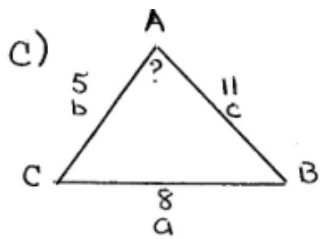
$\cos C = \dfrac{9 + 16 - 36}{24}$

$\cos C = \dfrac{-11}{24}$

$\cos C = -0.4583$

$C = \cos^{-1}(-0.4583)$

$C = 117°$

c)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{(5)^2 + (11)^2 - (8)^2}{2(5)(11)}$$

$$\cos A = \frac{25 + 121 - 64}{110}$$

$$\cos A = \frac{82}{110}$$

$$\cos A = 0.7455$$
$$A = \cos^{-1}(0.7455)$$
$$A = 42°$$

3. $$r^2 = p^2 + q^2 - 2pq \cos R$$ $P = 10$
                                             $q = 12$
    $$r^2 = (10)^2 + (12)^2 - 2(10)(12)\left(\frac{1}{5}\right)$$ $\cos R = \frac{1}{5}$

    $$r^2 = 100 + 144 - 240(0.2000)$$ $r = ?$
    $$r^2 = 244 - 48$$
    $$r^2 = 196$$
    $$r = \sqrt{196}$$
    $$r = 14$$

4. $\cos U = \dfrac{t^2 + v^2 - u^2}{2tv}$ $t = 10$

      $\cos U = \dfrac{(10)^2 + (12)^2 - (14)^2}{2(10)(12)}$ $u = 14$
                                        $v = 12$

      $\cos U = \dfrac{100 + 144 - 196}{240}$ $\cos U = \; ?$

      $\cos U = \dfrac{48}{240}$

      $\cos U = 0.2000$

5a) $p^2 = m^2 + n^2 - 2mn \cos P$ $m = 8$

     $p^2 = (8)^2 + (8)^2 - 2(8)(8)\cos 48°$ $n = 8$

     $p^2 = 64 + 64 - 128(0.6691)$ $<P = 48°$

     $p^2 = 128 - 85.6487$ $p = \; ?$

     $p^2 = 42.4513$

     $p = \sqrt{42.4513}$

     $p = 6.5$

5 b) $\cos P = \dfrac{q^2 + r^2 - p^2}{2qr}$

$\cos P = \dfrac{(3)^2 + (4)^2 - (4)^2}{2(3)(4)}$

$\cos P = \dfrac{9 + 16 - 16}{24}$

$\cos P = \dfrac{9}{24}$

$\cos P = 0.3750$

$P = \cos^{-1}(0.3750)$

$P = 68°$

$p = 4$
$q = 3$
$r = 4$
$<P = ?$

c) $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\cos C = \dfrac{(5)^2 + (7)^2 - (8)^2}{2(5)(7)}$

$\cos C = \dfrac{25 + 49 - 64}{70}$

$\cos C = \dfrac{10}{70}$

$\cos C = 0.1429$

$C = \cos^{-1}(0.1429)$

$C = 82°$

6a) $q^2 = p^2 + r^2 - 2pr \cos Q$ $\qquad$ $p = 4.1$

$q^2 = (4.1)^2 + (6.3)^2 - 2(4.1)(6.3)\cos 53°$ $\quad r = 6.3$

$q^2 = 16.81 + 39.69 - 51.66(0.6018) < Q = 53°$

$q^2 = 56.50 - 31.0898$ $\qquad q = ?$

$q^2 = 25.4102$

$q = \sqrt{25.4102}$

$q = 5.0$

b) $e^2 = d^2 + f^2 - 2df \cos E$ $\qquad$ $d = 9.6$

$e^2 = (9.6)^2 + (8.3)^2 - 2(9.6)(8.3)\cos 126°$ $\quad f = 8.3$

$e^2 = 92.16 + 68.89 - 159.36(-0.5878) < E = 126°$

$e^2 = 161.05 - (-93.6695)$ $\qquad e = ?$

$e^2 = 161.05 + 93.6695$

$e^2 = 254.7195$

$e = \sqrt{254.7195}$

$e = 16.0$

7a) To find u:

$t = 7$
$v = 6$
$\angle u = 43°$

$u^2 = t^2 + v^2 - 2tv\cos u$
$u^2 = (7)^2 + (6)^2 - 2(7)(6)\cos 43°$
$u^2 = 49 + 36 - 84(0.7314)$
$u^2 = 85 - 61.4337$
$u^2 = 23.5663$
$u = \sqrt{23.5663}$
$u = 4.9$

To find $\angle v$:

$\cos V = \dfrac{t^2 + u^2 - v^2}{2tu}$

$\cos V = \dfrac{(7)^2 + (4.9)^2 - (6)^2}{2(7)(4.9)}$

$\cos V = \dfrac{49 + 23.5663 - 36}{68.6}$

$\cos V = \dfrac{36.5663}{68.6}$

$\cos V = 0.5330$
$V = \cos^{-1}(0.5330)$
$V = 58°$

$\angle T = 180° - 43° - 58°$
$\angle T = 79°$

b) To find <J:

$j = 4.5$
$K = 6.3$
$f = 5.8$

$\cos J = \dfrac{f^2 + K^2 - j^2}{2fK}$

$\cos J = \dfrac{(5.8)^2 + (6.3)^2 - (4.5)^2}{2(5.8)(6.3)}$

$\cos J = \dfrac{33.64 + 39.69 - 20.25}{73.08}$

$\cos J = \dfrac{53.08}{73.08}$

$\cos J = 0.7263$

$J = \cos^{-1}(0.7263)$

$J = 43°$

To find <K:

$\cos K = \dfrac{j^2 + f^2 - K^2}{2jf}$

$\cos K = \dfrac{(4.5)^2 + (5.8)^2 - (6.3)^2}{2(4.5)(5.8)}$

$\cos K = \dfrac{20.25 + 33.64 - 39.69}{52.2}$

$\cos K = \dfrac{14.2}{52.2}$

$\cos K = 0.2720$

$K = \cos^{-1}(0.2720)$

$K = 74°$

$<F = 180° - 43° - 74°$

$<F = 63°$

c) To find h:

$h^2 = p^2 + w^2 - 2pw \cos H$
$h^2 = (4.9)^2 + (6.3)^2 - 2(4.9)(6.3)\cos 136°$
$h^2 = 24.01 + 39.69 - 61.74(-0.7193)$
$h^2 = 63.70 - (-44.4096)$
$h^2 = 108.1096$
$h = \sqrt{108.1096}$
$h = 10.4$

p = 49
w = 6.3
H = 136°

To find <P:

$\cos P = \dfrac{h^2 + w^2 - p^2}{2hw}$

$\cos P = \dfrac{(10.4)^2 + (6.3)^2 - (4.9)^2}{2(10.4)(6.3)}$

$\cos P = \dfrac{108.16 + 39.69 - 24.01}{131.04}$

$\cos P = \dfrac{123.84}{131.04}$

$\cos P = 0.9451$
$P = \cos^{-1}(0.9451)$
$P = 19°$

<W = 180° - 136° - 19°
<W = 25°