SOLUTIONS ⇒ EXPONENTIAL GROWTH REVIEW

1. $\{(1,2),(2,4),(3,6),(4,8),\ldots\}$

↳

x	y
1	2
2	4
3	6
4	8

- x-values increase by 1.
- y-values increasing by $+2$, therefore these coordinates are linear.

2. $\{(0,1),(1,5),(2,25),(3,125),\ldots\}$

↳

x	y
0	1
1	5
2	25
3	125

- x-values increase by 1.
- y-values are increasing by a factor of 5 ($\times 5$), therefore these coordinates are exponential.

3. $\{(-1,3), (0,6), (1,9), (2,12), \ldots\}$

↳

x	y
-1	3
0	6
1	9
2	12

- x-values increase by 1.
- y-values are increasing by +3, therefore these coordinates are linear.

2a) (3, 12, 48, 192, 768, 3072)

Common ratio is 4.

b) (6, 3, 3/2, 3/4, 3/8, 3/16)

Common ratio is 1/2

c) (7, -21, 63, -189, 567, -1701)

Common ratio is -3.

3a) $y = 2^x$ Common ratio is 2.
b) $y = 5^x$ Common ratio is 5.
c) $y = (0.6)^x$ Common ratio is 0.6.
d) $y = 100(0.6)^x$ Common ratio is 0.6.
e) $y = 200(1.02)^x$ Common ratio is 1.02.

4. Investment: \$1500 Interest: 5% per year.

a) $y = 1500(1.05)^x$

b) After 7 years $\Rightarrow x = 7$

$y = 1500(1.05)^7$
$y = \$2110.65$

5.

$$y = \underset{\substack{\downarrow \\ \text{Initial} \\ \text{Value}}}{1000}\,(\underset{\substack{\downarrow \\ \text{Doubles.}}}{2})^{\frac{x}{1}} \rightsquigarrow \text{Time it takes to double.}$$

a). After 4 min $\Rightarrow$ $x = 4$.

$$\begin{aligned} y &= 1000(2)^{4} \\ &= 1000(16) \\ &= 16000 \end{aligned}$$

16 000 bacteria after 4 min.

b) After 6 min => $x = 6$.

$$y = 1000(2)^6$$
$$= 1000(64)$$
$$= 64\,000.$$

64 000 bacteria after 6 min.

c) After 10 min => $x = 10$.

$$y = 1000(2)^{10}$$
$$= 1000(1024)$$
$$= 1\,024\,000$$

1 024 000 bacteria after 10 min.

6. $A = P\left(\frac{1}{2}\right)^{\frac{t}{92}}$

a) Initial Amount => $P = 7g$
Time => $t = 40h$

$A = 7\left(\frac{1}{2}\right)^{\frac{40}{92}}$

$A = 7(0.5)^{\frac{40}{92}}$

$A \approx 5.2g$

b) According to the equation given, the half-life of radon-222 is 92 hours.

7. EQUATION: $y = 225\,000(1.10)^{\frac{x}{5}}$

WHEN $x = 12$ $y = 225\,000(1.10)^{\frac{12}{5}}$

$y = \$282\,829.67$

8. a) 5^{-2}
$= \frac{1}{5^2}$
$= \frac{1}{25}$

b) 7^0
$= 1$

c) 4^{-1}
$= \frac{1}{4^1}$
$= \frac{1}{4}$

d) 2^{-3}
$= \frac{1}{2^3}$
$= \frac{1}{8}$

e) $\left(\frac{1}{2}\right)^{-4}$
$= 2^4$
$= 16$

f) $(-12)^0$
$= 1$

g) $\left(-\frac{2}{5}\right)^{-4}$
$= \left(-\frac{5}{2}\right)^4$
$= \frac{625}{16}$

h) -8^0
$= -1$

i) $\left(-\frac{4}{3}\right)^{-3}$
$= \left(-\frac{3}{4}\right)^3$
$= \frac{-27}{64}$

j) $2^{-2} + 3^{-2}$

$= \frac{1}{2^2} + \frac{1}{3^2}$

$= \frac{1}{4} + \frac{1}{9}$

$= \frac{9}{36} + \frac{4}{36}$

$= \frac{13}{36}$

k) $\left(\frac{3m}{n^2}\right)^{-3}$

$= \left(\frac{n^2}{3m}\right)^3$

$= \frac{n^6}{27m^3}$

9a) $3^{x+2} = 3^{2x+6}$

$$x + 2 = 2x + 6$$
$$2 - 6 = 2x - x$$
$$-4 = x$$

b) $2^{2x+2} = 16^{x+6}$

$$2^{2x+2} = (2^4)^{x+6}$$
$$2^{2x+2} = 2^{4x+24}$$
$$2x + 2 = 4x + 24$$
$$2 - 24 = 4x - 2x$$
$$\frac{-22}{2} = \frac{\cancel{2}x}{\cancel{2}}$$
$$-11 = x$$

c) $2^{2x+2} = 16(2^x)$

$$2^{2x+2} = (2^4)(2^x)$$
$$2^{2x+2} = 2^{4+x}$$
$$2x + 2 = 4 + x$$
$$2x - x = 4 - 2$$
$$x = 2$$

d) $8^x + 24 = 88$

$$8^x = 88 - 24$$
$$8^x = 64$$
$$8^x = 8^2$$
$$x = 2$$

e) $3^{x-2} = \frac{27^{2x}}{9^{x-1}}$

$3^{x-2} = \frac{(3^3)^{2x}}{(3^2)^{x-1}}$

$3^{x-2} = \frac{3^{6x}}{3^{2x-2}}$

$3^{x-2} = 3^{6x-(2x-2)}$

$3^{x-2} = 3^{6x-2x+2}$

$3^{x-2} = 3^{4x+2}$

$x-2 = 4x+2$

$-2-2 = 4x-x$

$\frac{-4}{3} = \frac{\cancel{3}x}{\cancel{3}}$

$\frac{-4}{3} = x$

f) $(4^{x})(2^{x+3}) = 16^{2x-5}$

$$(2^{2})^{x}(2^{x+3}) = (2^{4})^{2x-5}$$
$$(2^{2x})(2^{x+3}) = 2^{8x-20}$$
$$2^{2x+x+3} = 2^{8x-20}$$
$$2^{3x+3} = 2^{8x-20}$$
$$3x+3 = 8x-20$$
$$3+20 = 8x-3x$$
$$\frac{23}{5} = \frac{\cancel{5}x}{\cancel{5}}$$
$$\frac{23}{5} = x$$

10. a) $25^{1/2}$

$= (\sqrt{25})^1$

$= (5)^1$

$= 5$

b) $125^{\frac{2}{3}}$

$= (\sqrt[3]{125})^2$

$= (5)^2$

$= 25$

c) $(-64)^{\frac{3}{2}}$

$= (\sqrt{-64})^3$

NO REAL SOLUTION

d) $-81^{\frac{3}{4}}$

$= -(\sqrt[4]{81})^3$

$= -(3)^3$

$= -27$

e) $\left(\frac{8}{27}\right)^{-\frac{3}{2}}$

$= \left(\frac{27}{8}\right)^{\frac{2}{3}}$

$= \left(\sqrt[3]{\frac{27}{8}}\right)^2$

$= \left(\frac{3}{2}\right)^2$

$= \frac{9}{4}$

11. a) $\sqrt{17}$

$= 17^{\frac{1}{2}}$

b) $(\sqrt[6]{8})^2$

$= 8^{\frac{2}{6}}$

c) $\frac{1}{2\sqrt{x}}$

$= \frac{1}{2x^{\frac{1}{2}}}$

d) $(\sqrt{x})^4$

$= (x^{\frac{1}{2}})^4$

$= x^{\frac{4}{2}}$

$= x^2$

e) $(\sqrt[4]{25})^3$

$= 25^{\frac{3}{4}}$

12. The graph of $y = b^x$ represents exponential <u>growth</u> when $b > 1$.

13. The graph of $y = b^x$ represents exponential <u>decay</u> when $0 < b < 1$.

14. $y = b^x$

Domain: $\{x \in R\}$

Range: $\{y \mid y > 0, y \in R\}$

y-int: $y = 1$ <u>OR</u> $(0, 1)$

Location of Horizontal Asymptote: $y = 0$ <u>OR</u> x-axis.

15. a) $3^4 = 81$

$\hookrightarrow \log_3 81 = 4$

b) $3^{-3} = \frac{1}{27}$

$\hookrightarrow \log_3\left(\frac{1}{27}\right) = -3$

16. a) $\log_3 27 = 3$

$\hookrightarrow 3^3 = 27$

b) $\log_8 2 = \frac{1}{3}$

$\hookrightarrow 8^{\frac{1}{3}} = 2$

17. a) $\log_3 x = 5$

$3^5 = x$

$243 = x$

b) $\log_2 (x-3) = 2$

$2^2 = x-3$

$4 = x-3$

$4+3 = x$

$7 = x$

c) $\log_3 (-x+1) = 6$

$3^6 = -x+1$

$729 = -x+1$

$729 - 1 = -x$

$728 = -x$

$-728 = x$

18. a) $\log_2 8$

$x = \log_2 8$

$2^x = 8$

$2^x = 2^3$

$x = 3$

b) $\log_5 \frac{1}{25}$

$x = \log_5 \frac{1}{25}$

$5^x = \frac{1}{25}$

$5^x = \frac{1}{5^2}$

$5^x = 5^{-2}$

$x = -2$

c) $\log_{\frac{1}{3}} \frac{1}{243}$

$x = \log_{\frac{1}{3}} \frac{1}{243}$

$\frac{1}{3}^x = \frac{1}{243}$

$\frac{1}{3}^x = \frac{1}{3^5}$

$(3^{-1})^x = 3^{-5}$

$3^{-1x} = 3^{-5}$

$\frac{-1x}{-1} = \frac{-5}{-1}$

$x = 5$

19. $2\log_5 3 + \log_5 6 - \log_5 27$

$= \log_5 3^2 + \log_5 6 - \log_5 27$

$= \log_5 9 + \log_5 6 - \log_5 27$

$= \log_5 (9 \cdot 6) - \log_5 27$

$= \log_5 54 - \log_5 27$

$= \log_5 \left(\frac{54}{27}\right)$

$= \log_5 2$

20. $\frac{1}{2}\log_2 16 + \log_2 8 - \log_2 4$

Evaluate each term separately:

$\frac{1}{2}\log_2 16 = x$

$\log_2 16^{\frac{1}{2}} = x$

$\log_2 4 = x$

$2^x = 4$

$2^x = 2^2$

$x = 2$

$\log_2 8 = x$

$2^x = 8$

$2^x = 2^3$

$x = 3$

$\log_2 4 = x$

$2^x = 4$

$2^x = 2^2$

$x = 2$

Therefore we have:

$2 + 3 - 2$

$= 5 - 2$

$= 3$