Geometric Sequences

Definition:

A geometric sequence is a pattern of numbers in which each term is found by "multiplying" the preceding term by the same amount.

64, 32, 16, 8, 4, 2, 1 . . .

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-32 -16 -8 -4 -2 -1

Example 1:

$r = \frac{15}{5} = 3$

(5, 15, 45, 135, …)

$r = \frac{135}{45} = 3$

➡ **Each term is 3 times the preceding term.**

Example 2:

$r = \frac{45}{135} = \frac{1}{3}$

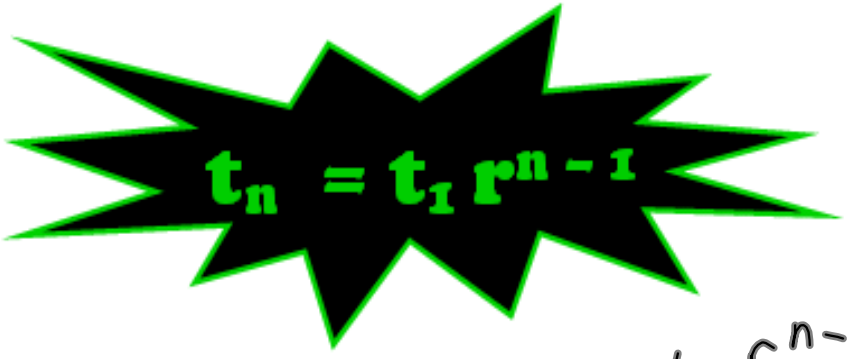
(135, 45, 15, 5, …)

➡ **Each term is $\frac{1}{3}$ of the preceding term.**

$r = \frac{5}{15} = \frac{1}{3}$

Equation:

The following equation can be used to write the function of a geometric sequence...



$t_n = t_1 r^{n-1}$

t_1 = first term

r = common multiplier

n = the value of n

Example 1:

Find t_{15} for the sequence {-2, 6, -18, 54,...}

Solution:

$$r = \frac{6}{-2} = -3$$

$t_1 = -2$

$r = -3$

$t_n = t_1 r^{n-1}$

$t_{15} = (-2)(-3)^{15-1}$

$t_{15} = (-2)(-3)^{14}$

$t_{15} = (-2)(4\ 782\ 969)$

$t_{15} = -9\ 565\ 938$

Example 2:

Find the number of terms "n" for the following sequence:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, \frac{1}{256}$

$r = \frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$

Solution:

$t_1 = \frac{1}{2}$

$r = \frac{1}{2}$

$t_n = \frac{1}{256}$

$t_n = t_1 r^{n-1}$

$\frac{1}{256} = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{256} \div \frac{1}{2} = \frac{1}{2}^{n-1}$

$\frac{1}{256} \times \frac{2}{1} = \frac{1}{2}^{n-1}$

$\frac{2}{256} = \frac{1}{2}^{n-1}$

$\frac{1}{128} = \frac{1}{2}^{n-1}$

We need to make the "bases" the same!

$\left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^{n-1}$

Now that the "bases" are equal ...

$7 = n - 1$

$7 + 1 = n$

$8 = n$

2^x ← Base

Don't forget that (any number)^0 = 1!!!