

Graph of the Quadratic Function $f(x) = x^2$

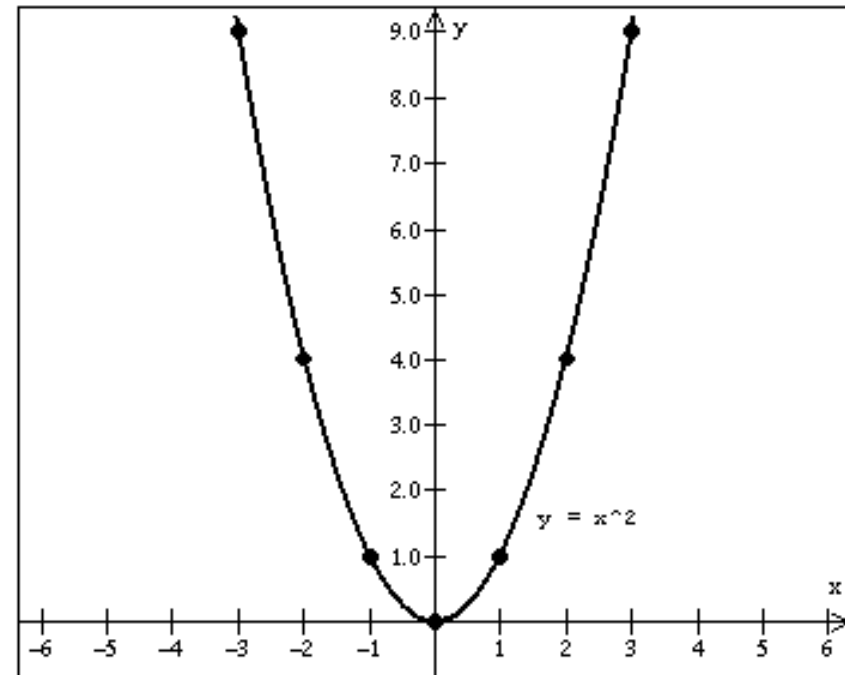
Here are some points on the curve $y = x^2$:

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Notice that the vertex is at $(0, 0)$.

Notice that the graph “opens up”, and therefore has a minimum value of $(0, 0)$.

Notice that the y -axis (eqn: $x = 0$) is the axis of symmetry.



The graph of every quadratic function can be obtained by transforming the graph of $y = x^2$ with:

1. a vertical shift,
2. a horizontal shift,
3. a reflection about the x -axis,
4. and/or a vertical stretch or compression

Graph of the Quadratic Function $f(x) = x^2 + k$ (a vertical shift):

Adding a constant to the graph of $y = x^2$ has the effect of shifting the graph vertically (up/down).

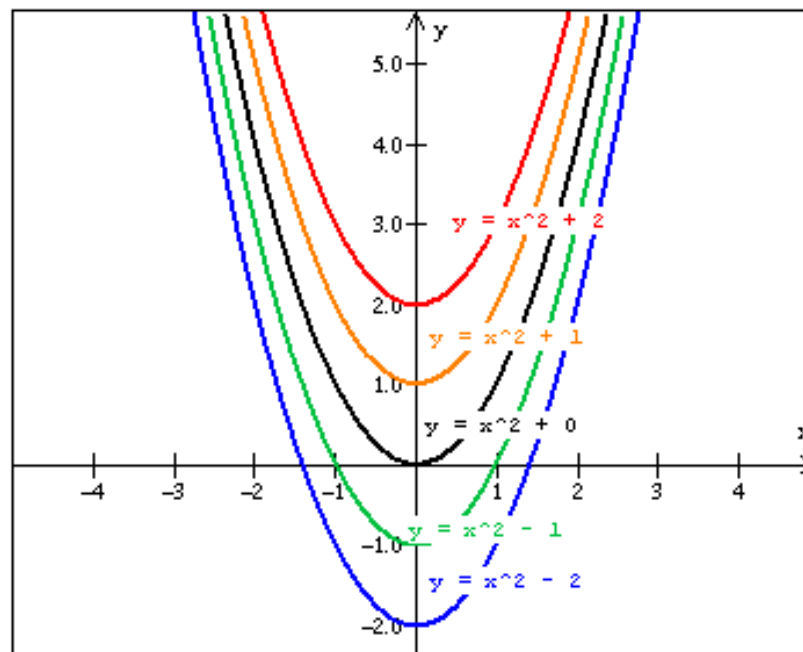
The graph of $y = x^2 + k$ is the graph of $y = x^2$ shifted *up* by k units when k is a positive number.

The graph of $y = x^2 + k$ is the graph of $y = x^2$ shifted *down* by k units when k is a negative number.

Notice that the vertex is at $(0, k)$.

Notice that the graph still “opens up”, and has a minimum value.

Notice that the y -axis (eqn: $x = 0$) is still the axis of symmetry.



Graph of the Quadratic Function $f(x) = (x-h)^2$ (a horizontal shift):

Adding a constant to the x variable in $y = x^2$ before squaring has the effect of shifting the graph horizontally (left/right).

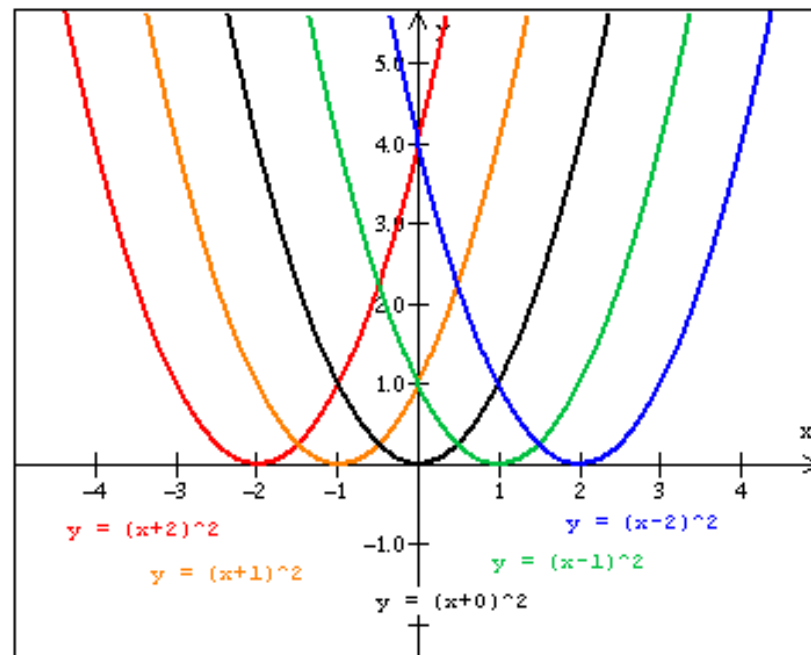
The graph of $y = (x - h)^2$ is the graph of $y = x^2$ shifted **right** by h units when h is a positive number.

The graph of $y = (x - h)^2$ is the graph of $y = x^2$ shifted **left** by h units when h is a negative number.

Notice that the vertex is at $(h, 0)$.

Notice that the graph still “opens up”, and has a minimum value.

Notice that the vertical line $x = h$ is the new axis of symmetry.



Graph of the Quadratic Function $f(x) = ax^2$ (a reflection about the x -axis, and/or a vertical stretch):

Multiplying the x variable by a constant in $y = x^2$ after squaring can reflect the graph about the x -axis, or produce a vertical stretching of the graph.

The graph of $y = ax^2$ is the graph of $y = x^2$ stretched vertically upwards when a is a positive number greater than 1.

The graph of $y = ax^2$ is the graph of $y = x^2$ compressed vertically upwards when a is a positive number less than 1.

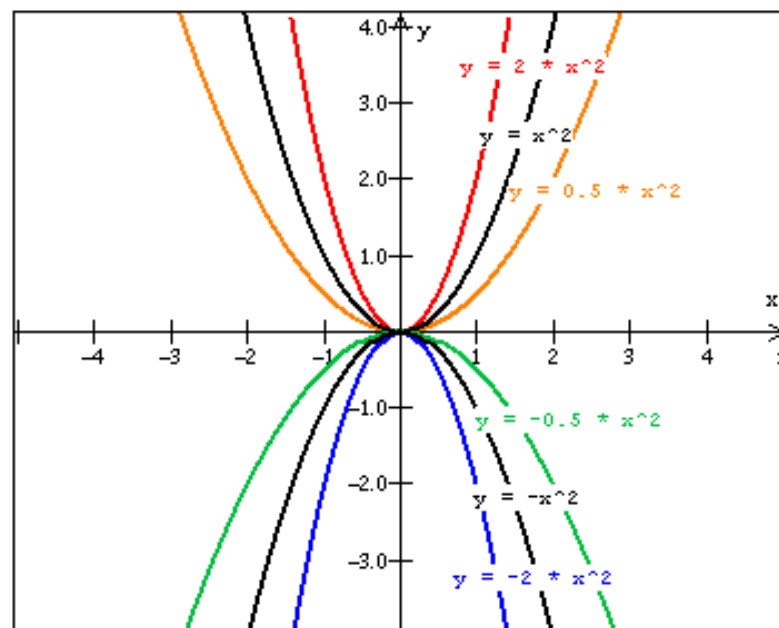
The graph of $y = ax^2$ is the graph of $y = x^2$ reflected about the x -axis and stretched vertically downwards when a is a negative number less than -1.

The graph of $y = ax^2$ is the graph of $y = x^2$ reflected about the x -axis and compressed vertically downwards when a is a positive number greater than -1.

Notice that the vertex is at $(0, 0)$.

Notice that the graph “opens up” when $a > 0$ and “opens down” when $a < 0$.

Notice that the vertical line $x = 0$ is still the axis of symmetry.



Writing Quadratic Equations

General Form	Standard Form	Transformational Form
$y = ax^2 + bx + c$ where $a \neq 0$	$y = a(x - h)^2 + k$ where $a \neq 0$	$\frac{1}{a}(y - k) = (x - h)^2$
a = the vertical stretch(stretch factor) +/- = whether graph opens up or down or has a maximum or a minimum.	a = the vertical stretch(stretch factor) +/- = whether graph opens up or down or has a maximum or a minimum.	a = the vertical stretch(stretch factor) +/- = whether graph opens up or down or has a maximum or a minimum.
c = the value of the y-coordinate of the y-intercept.	Not able to determine the y-intercept.	Not able to determine the y-intercept.
Not able to determine vertex.	(h, k) gives us the coordinates of the vertex of the parabola.	(h, k) will give us the coordinates of the vertex of the parabola.
Not able to determine the equation of the axis of symmetry.	$x = h$ gives us the equation of the axis of symmetry.	$x = h$ gives us the equation of the axis of symmetry.

NOTE:

Horizontal Shift

The h value in the standard/transformational form of the quadratic equation stands for the horizontal shift. This transformation means that you **shift EVERY point** to the left or right h units. Remember the following:

- If h is **positive**, you will shift the graph to the **right**.
- If h is **negative**, you will shift the graph to the **left**.

This can get a little tricky, as the standard/transformational form of the quadratic equation throws a wrench into things. The “ h ” is contained within $(x - h)^2$, therefore you will have to be very careful in your thinking!

If the sign in front of the “ h ” is negative, the graph will actually be shifted to the right and if the sign in front of the “ h ” is positive, the graph will actually be shifted to the left.

Vertical Shift

The vertical shift is represented by the variable k in the standard/transformational form of the quadratic equation. The following rules apply:

- If the value of k is **positive**, you shift EVERY point on the graph **UP** k units.
- If the value of k is **negative**, you shift EVERY point on the graph **DOWN** k units.

These rules are very easy to follow for the standard form of the quadratic equation. In transformational form, however, the “ k ” is contained within $(y - k)$. Therefore, if the sign in front of the “ k ” is negative, the graph will actually be shifted upward and if the sign in front of the “ k ” is positive, the graph will actually be shifted downward.