ANSWERS => INSTANTANEOUS RATE OF CHANGE

1. $y = 7x^3 + 6x^2 - 2x + 1$ at $x = 4$

When $x = 3.9$

$y = 7(3.9)^3 + 6(3.9)^2 - 2(3.9) + 1$
$= 7(59.319) + 6(15.21) - 7.8 + 1$
$= 415.233 + 91.26 - 7.8 + 1$
$= 499.693$

When $x = 4.1$

$y = 7(4.1)^3 + 6(4.1)^2 - 2(4.1) + 1$
$= 7(68.921) + 6(16.81) - 8.2 + 1$
$= 482.447 + 100.86 - 8.2 + 1$
$= 576.107$

$$IROC = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{576.107 - 499.693}{4.1 - 3.9}$$
$$= \frac{76.414}{0.2}$$
$$= 382.07$$

2. $s = -5t^2 - 2t + 2$ at $t = 10$

When $x = 9.9$.

$$
\begin{aligned}
S &= -5(9.9)^2 - 2(9.9) + 2 \\
&= -5(98.01) - 19.8 + 2 \\
&= -490.05 - 19.8 + 2 \\
&= -507.85
\end{aligned}
$$

When $x = 10.1$

$$
\begin{aligned}
S &= -5(10.1)^2 - 2(10.1) + 2 \\
&= -5(102.01) - 20.2 + 2 \\
&= -510.05 - 20.2 + 2 \\
&= -528.25
\end{aligned}
$$

$$
\begin{aligned}
\text{IROC} &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{-528.25 - -507.85}{10.1 - 9.9} \\
&= \frac{-20.4}{0.2} \\
&= -102
\end{aligned}
$$

3. $h = -4.9t^2 + 100t$

a) If $t = 0$

$$h = -4.9(0)^2 + 100(0)$$
$$= -4.9(0) + 0$$
$$= 0 + 0$$
$$= 0$$

$(0, 0)$

If $t = 3$

$$h = -4.9(3)^2 + 100(3)$$
$$= -4.9(9) + 300$$
$$= -44.1 + 300$$
$$= 255.9$$

$(3, 255.9)$

$$AROC = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{255.9 - 0}{3 - 0}$$
$$= \frac{255.9}{3}$$
$$= 85.3 \, m/s$$

b) If $t = 3$

$h = 255.9$

$(3, 255.9)$

If $t = 4$.

$h = -4.9(4)^2 + 100(4)$

$= -4.9(16) + 400$

$= -78.4 + 400$

$= 321.6$

$(4, 321.6)$

$$AROC = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{321.6 - 255.9}{4 - 3}$$

$$= \frac{65.7}{1}$$

$$= 65.7 \text{ m/s.}$$

c) $h = -4.9t^2 + 100t$ at $t = 1$

When $t = 0.9$

$$h = -4.9(0.9)^2 + 100(0.9)$$
$$= -4.9(0.81) + 90$$
$$= -3.969 + 90$$
$$= 86.031\text{ m}$$

When $t = 1.1$

$$h = -4.9(1.1)^2 + 100(1.1)$$
$$= -4.9(1.21) + 110$$
$$= -5.929 + 110$$
$$= 104.071\text{ m}$$

$$\text{IROC} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{104.071 - 86.031}{1.1 - 0.9}$$
$$= \frac{18.04}{0.2}$$
$$= 90.2\ \text{m/s}$$

4. $h = -4.9t^2 + 39.2t + 5$

a) $h = -4.9t^2 + 39.2t + 5$

$h - 5 = -4.9t^2 + 39.2t$

$h - 5 = -4.9(t^2 - 8t)$

$h - 5 - 78.4 = -4.9(t^2 - 8t + 16)$

$h - 83.4 = -4.9(t^2 - 8t + 16)$

$h - 83.4 = -4.9(t-4)^2$

$h = -4.9(t-4)^2 + 83.4$. (Standard Form)

Vertex $(4, 83.4)$

The maximum height was 83.4 m

The projectile reached the maximum height at 4 seconds.

b) $h = -4.9t^2 + 39.2t + 5$

$a = -4.9 \quad b = 39.2 \quad c = 5$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-39.2 \pm \sqrt{(39.2)^2 - 4(-4.9)(5)}}{2(-4.9)}$$

$$= \frac{-39.2 \pm \sqrt{1536.64 + 98}}{-9.8}$$

$$= \frac{-39.2 \pm \sqrt{1634.64}}{-9.8}$$

$$= \frac{-39.2 \pm 40.4307}{-9.8}$$

$$= \frac{-79.6307}{-9.8} \text{ or } \frac{1.2307}{-9.8}$$

$$= 8.126 \text{ or } \cancel{-0.126}$$

↳ Time cannot be negative!

★ The projectile hit the ground after 8.126 s.

c) The total time of flight found in part (b) is not equal to exactly twice the time to reach the maximum height found in part (a) because the projectile is not starting at a height of zero, but rather at the height of the cannon (5m)

d) $h = -4.9t^2 + 39.2t + 5$ at $t = 2$

When $t = 1.9$:

$h = -4.9(1.9)^2 + 39.2(1.9) + 5$

$= -4.9(3.61) + 74.48 + 5$

$= -17.689 + 74.48 + 5$

$= 61.791\,m$

When $t = 2.1$:

$h = -4.9(2.1)^2 + 39.2(2.1) + 5$

$= -4.9(4.41) + 82.32 + 5$

$= -21.609 + 82.32 + 5$

$= 65.711\,m$

$$IROC = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{65.711 - 61.791}{2.1 - 1.9}$$

$$= \frac{3.92}{0.2}$$

$$= 19.6\ m/s.$$

e) The instantaneous rate of change of the height should be zero at the time the projectile reaches its maximum height. (The object "stops" for an instant)