SOLUTIONS => EXERCISE 8

1. $\log 7 + \log 10$

$= \log(7 \cdot 10)$

$= \log 70$

(Remember => Base 10)

2. $\log 18 - \log 36$

$= \log\left(\frac{18}{36}\right)$

$= \log \frac{1}{2}$

3. $3\log_2 7 - 2\log_2 3$

$= \log_2 7^3 - \log_2 3^2$

$= \log_2 343 - \log_2 9$

$= \log_2 \frac{343}{9}$

4. $3\log_6 15 + \frac{3}{2}\log_6 7$

$= \log_6 15^3 + \log_6 (7)^{3/2}$

$= \log_6 (15^3 \cdot 7^{3/2})$

5. $\log_5 48 - \log_5 12 + \log_5 4$

$= \log_5 \left(\frac{48}{12}\right) + \log_5 4$

$= \log_5 4 + \log_5 4$

$= \log_5 (4 \cdot 4)$

$= \log_5 16$

6. $\frac{1}{2}\log_a x - \frac{2}{3}\log_a y$

$= \log_a x^{1/2} - \log_a y^{2/3}$

$= \log_a \frac{x^{1/2}}{y^{2/3}}$

7. $\log x + 2\log y - \frac{1}{3}\log z - 5\log m$

$= \log x + \log y^2 - \log z^{1/3} - \log m^5$

$= \log x \cdot y^2 - \log z^{1/3} - \log m^5$

$= \log \frac{xy^2}{z^{1/3}m^5}$

8. $\log_5 \frac{xyz}{mw}$

$= \log_5 x + \log_5 y + \log_5 z - \log_5 m - \log_5 w$

9. $\log x^{1/2} y^3 z^5$

$= 1/2 \log x + 3 \log y + 5 \log z$

10. $\log \frac{36^{\frac{1}{3}} \cdot 8^{\frac{1}{2}}}{13^{\frac{2}{5}}}$

$= 1/3 \log 36 + 1/2 \log 8 - 2/5 \log 13$

11. $\log_7 \frac{x^{\frac{1}{4}} y^{\frac{2}{7}}}{z^{\frac{6}{5}}}$

$= 1/4 \log_7 x + 2/7 \log_7 y - 6/5 \log_7 z$

12. $\log_3 9 + \log_4 64 + \log_4\left(\frac{1}{16}\right) + \log_3\left(\frac{1}{9}\right) + \log_{10}\left(\frac{1}{10}\right)$

Evaluate each term separately:

$\log_3 9 = x$

$3^x = 9$

$3^x = 3^2$

$x = 2$

$\log_4 64 = x$

$4^x = 64$

$4^x = 4^3$

$x = 3$

$\log_4\left(\frac{1}{16}\right) = x$

$4^x = \frac{1}{16}$

$4^x = \frac{1}{4^2}$

$4^x = 4^{-2}$

$x = -2$

$\log_3\left(\frac{1}{9}\right) = x$

$3^x = \frac{1}{9}$

$3^x = \frac{1}{3^2}$

$3^x = 3^{-2}$

$x = -2$

$\log_{10}\left(\frac{1}{10}\right) = x$

$10^x = \frac{1}{10}$

$10^x = 10^{-1}$

$x = -1$

Therefore we have:

$2 + 3 + (-2) + (-2) + (-1)$

$= 2 + 3 - 2 - 2 - 1$

$= 0$

13. $\log_2 32 - \log_2\left(\frac{1}{32}\right) + \log_4 8 - \log_8 16$

Evaluate each term:

$\log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$x = 5$

$\log_2\left(\frac{1}{32}\right) = x$

$2^x = \frac{1}{32}$

$2^x = \frac{1}{2^5}$

$2^x = 2^{-5}$

$x = -5$

$\log_4 8 = x$

$4^x = 8$

$(2^2)^x = (2^3)$

$2^{2x} = 2^3$

$2x = 3$

$x = 3/2$

$\log_8 16 = x$

$8^x = 16$

$(2^3)^x = (2^4)$

$2^{3x} = 2^4$

$\frac{3x}{3} = \frac{4}{3}$

$x = 4/3$

Therefore we have:

$5 - (-5) + 3/2 - 4/3$

$= 5 + 5 + \frac{3}{2} - \frac{4}{3}$

$= \frac{10}{1} + \frac{3}{2} - \frac{4}{3}$

$= \frac{60}{6} + \frac{9}{6} - \frac{8}{6}$

$= \frac{69}{6} - \frac{8}{6}$

$= \frac{61}{6}$

14. $3\log_2 4 + 2\log_3 9 + \log(0.1) - \log_3\left(\frac{1}{9}\right)$

Evaluate each term:

$3\log_2 4 = x$ $2\log_3 9 = x$ $\log(0.1) = x$ $\log_3\left(\frac{1}{9}\right) = x$

$\log_2 4^3 = x$ $\log_3 9^2 = x$ $10^x = 0.1$ $3^x = \frac{1}{9}$

$\log_2 64 = x$ $\log_3 81 = x$ $10^x = 10^{-1}$ $3^x = \frac{1}{3^2}$

$x = -1$ $3^x = 3^{-2}$

$2^x = 64$ $3^x = 81$ $x = -2$.

$2^x = 2^6$ $3^x = 3^4$

$x = 6$ $x = 4$

Therefore we have:

$6 + 4 + (-1) - (-2)$

$= 6 + 4 - 1 + 2$

$= 11$