$$\frac{-12x^{-7}y^{2}}{36x^{9}y^{-5}z^{5}}$$

$$-\frac{1\,y^{2}\,y^{5}}{3x^{7}x^{9}\,z^{5}} = \frac{-y^{7}}{3x^{16}z^{5}}$$

SOLVING EXPONENTIAL EQUATIONS

To solve any type of an equation, we must find the value(s) of the unknown quantities such that the right-hand side equals the left-hand side.

For example, for the simple equation: $3x = 12$

We divide both sides by 3: $x = 4$

The solution is, then, x=4. To check:

LHS		RHS
$3x$		12
$3(4)$		12
12	=	12

Thus we have shown that 4 is the solution for our equation.

In earlier sections we solved quadratic equations by employing a number of strategies: factoring, completing the square, and using the quadratic formula.

An ***exponential equation*** is an equation in which the unknown appears in the exponent. Some examples of exponential equations are:

- $5^{3x} = 123$
- $17^{(2x-12)} = 14^{11x}$
- $15^{x+3} + 114 = 12 - 2^{7x}$

Up until now we have not had the means to solve any of these equations. Over the next few sections, we will use methods to solve any type of exponential equation. We start by looking at a method that will help us to solve a certain type of equation. This type of equation is one in which **both sides of the equation can be written as powers of the same base.**

Examples: **Solve the following exponential equations.**

1. $3^{3x+1} = 3^{12}$

The general method of solving these equations is to try to get both sides of the equations written as powers to the same base. Obviously, the base on each side is 3. **Since the bases are the same, the exponents must be the same!**

So: $3x + 1 = 12$

$3x = 12 - 1$

$3x = 11$

$x = \dfrac{11}{3}$

2. $15^{2(x+4)} = 1$

Recall that any base raised to the power of zero equals 1. Thus, we can write the right-hand side as 15^0, then follow the same steps as in example 1.

$15^{2(x+4)} = 1$

$15^{2(x+4)} = 15^0$

$2(x + 4) = 0$

$2x + 8 = 0$

$2x = -8$

$x = -4$

3. $3^{4x} = 27$

The bases are not the same. However, we can see that we can write the right-hand side as a power of 3.

$$3^{4x} = 3^3$$

$$4x = 3$$

$$x = \frac{3}{4}$$

4. $16^{3x+3} = 8^{2x}$

The common base here is 2, as this is the base in which both 16 and 8 can be written.

$(2^4)^{3x+3} = (2^3)^{2x}$ Use the rules of exponents to simplify. Raising a power to a power, we **multiply** the powers.

$2^{12x+12} = 2^{6x}$

$12x + 12 = 6x$

$12x - 6x = -12$

$6x = -12$

$x = -2$

5. $4^{x+4} = 16 \cdot 4^{3x-3}$

$4^{x+4} = (4^2)4^{3x-3}$ When multiplying terms with common bases, **add** the exponents.

$4^{x+4} = 4^{2+3x-3}$

$x + 4 = 3x - 1$

$-2x = -5$

$x = \frac{5}{2}$

6.

$$13^x + 200 = 369$$

$$13^x = 369 - 200$$

$$13^x = 169$$

$$13^x = 13^2$$

$$x = 2$$

7. $5(3^{2x-4}) = 405$ Divide by 5.

$$3^{2x-4} = \frac{405}{5} = 81$$

$$3^{2x-4} = 3^4$$

$$2x - 4 = 4$$

$$2x = 8$$

$$x = 4$$

8. $\left(\frac{1}{9}\right)^{x+2} = 3^{2x+5}$ Recall that $\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$(3^{-2})^{x+2} = 3^{2x+5}$

$3^{-2x-4} = 3^{2x+5}$

$-2x - 4 = 2x + 5$

$-9 = 4x$

$x = \frac{-9}{4}$

9. $$\frac{1}{81}\cdot\left(\frac{1}{27}\right)^{2x-5}=3^{4x+1}\cdot\left(\frac{1}{3}\right)^{x}$$

$$(3^{-4})(3^{-3})^{2x-5}=(3^{4x+1})(3^{-1})^{x}$$

$$(3^{-4})(3^{-6x+15})=(3^{4x+1})(3^{-x})$$

$$3^{-4-6x+15}=3^{4x+1-x}$$

$$-4-6x+15=4x+1-x$$

$$10=9x$$

$$x=\frac{10}{9}$$