## WRITING EQUATIONS TO REPRESENT LINEAR GRAPHS

In your earlier courses, you learned how to write equations of linear functions. Basically, there were two methods you were shown.

1. **Slope-Intercept Method:** If you are given the slope, m, and the y-intercept, b, then the equation of the line is given by: $y = mx + b$.
2. **Slope-Point Method:** If you are given the slope, m, and any point, $(x_1, y_1)$ on the line, the equation can be obtained from using: $m = \frac{y - y_1}{x - x_1}$.

* OR $y - y_1 = m(x - x_1)$

# Examples:

**Equation of line segment from A to B:**

slope = 10 and y-intercept is 0 therefore, **$y = 10x$**

**Equation of line segment from B to C:**

slope = 4 and a point on the line segment is (7 , 40 ).

$y - y_1 = m(x - x_1)$

$y - 40 = 4(x - 7)$

$y - 40 = 4x - 28$

**$y = 4x + 12$**

# APPLICATIONS OF AVERAGE RATES OF CHANGE

## EXAMPLE 1:

A man takes a trip from Town A to Town B which can be described by the function $f(t) = .03t^2 + 8.2t + 245$. Find the average speed of the trip between **3** h and 7 h. Consider the distance to be in kilometers.

## SOLUTION:

We evaluate the function at 3 h and at 7 h. We subtract these values and then divide by the change in time.

$$\mathbf{f(t) = .03t^2 + 8.2t + 245}$$

$$f(7) = .03(7)^2 + 8.2(7) + 245 = 303.87 \text{ km}$$
$$f(3) = .03(3)^2 + 8.2(3) + 245 = 269.87 \text{ km}$$

$$\text{average speed} = \frac{(303.87 - 269.87)km}{(7-3)h} = \frac{34.0km}{4h} = 8.5\,km/h$$

**EXAMPLE 2:**

The national debt of the United States from 1980 to 1998 can be estimated using the function

$f(t) = 8.9t^2 - 42.3t + 474.7$ ; where t is the time in years past 1980 and f(t) is the debt in billions of dollars.

a) Find the average rate of change of the debt between 1980 and 1990.

b) Find the average rate of change of the debt between 1990 and 1998.

c) Draw a graph of the function using a table of values.

d) State when the debt was increasing and when it was decreasing.

**SOLUTION:**

a) Reading the question carefully tells us that **t** is the number of years past 1980.

Therefore, we will calculate f(0) for 1980 and f(10) for 1990.

$\mathbf{f(t) = 8.9t^2 - 42.3\ t + 474.7}$

$f(10) = 8.9(10)^2 - 42.3(10) + 474.7 = 941.7$ billion

$f(0) = 8.9(0)^2 - 42.3(0) + 474.7 = 474.7$ billion

$$\text{average rate of change} = \frac{(941.7 - 474.7)\mathit{billion}}{(10-0)y} = 46.7\,\mathit{billion\ per\ year}$$

b) Between 1990 and 1998, **t** will be 10 and 18

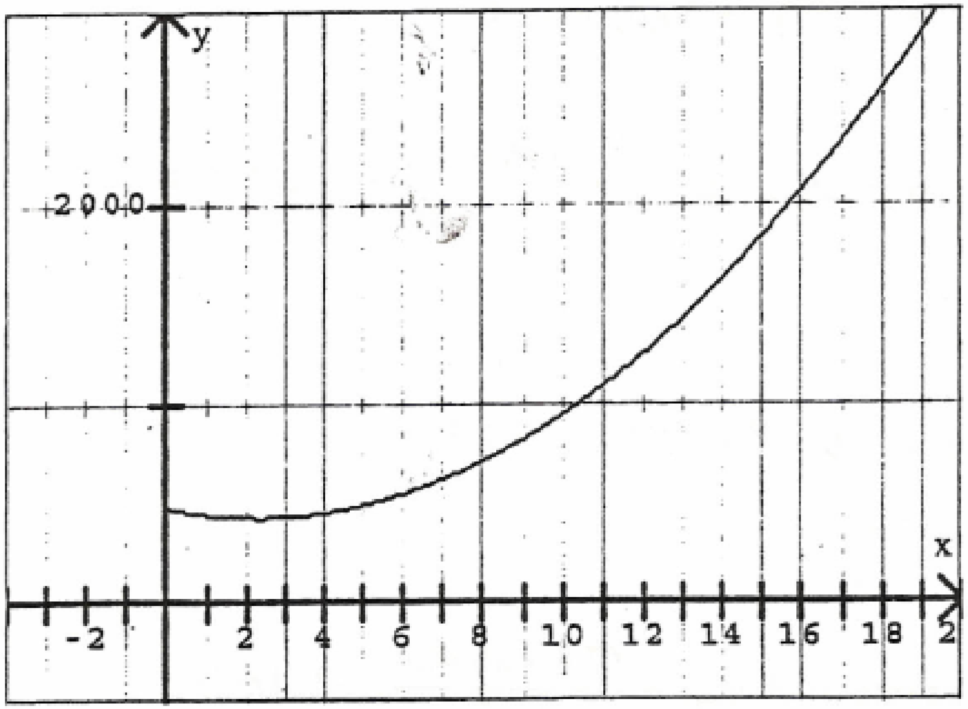
$f(18) = 8.9(18)^2 - 42.3(18) + 474.7 = 2596.9$ billion

$f(10) = 941.7$ billion

$$\text{average rate of change} = \frac{(2569.9 - 941.7)\mathit{billion}}{(18-10)y} = 206.9\,\mathit{billion\ per\ year}$$

c)

| x | y |
|---|---|
| 0.00 | 474.70 |
| 1.00 | 441.30 |
| 2.00 | 425.70 |
| 3.00 | 427.90 |
| 4.00 | 447.90 |
| 5.00 | 485.70 |
| 6.00 | 541.30 |
| 7.00 | 614.70 |
| 8.00 | 705.90 |
| 9.00 | 814.90 |
| 10.00 | 941.70 |
| 11.00 | 1086.30 |
| 12.00 | 1248.70 |
| 13.00 | 1428.90 |
| 14.00 | 1626.90 |
| 15.00 | 1842.70 |
| 16.00 | 2076.30 |
| 17.00 | 2327.70 |
| 18.00 | 2596.90 |
| 19.00 | 2883.90 |
| 20.00 | 3188.70 |



d) From the table of values and the graph, it can be seen that the debt decreases from 1980 to 1982. The slope of the secant line joining 1980 to 1982 would be negative. After 1982, the debt increases. The slope of the secant line joining any two points after 1982 would be positive.

**EXAMPLE 3:**

A ball is thrown vertically upward and its path is described by the formula

**$f(t) = -4.9t^2 + 20t + 6$,** where t is in seconds and f(t) is the height in metres.

a) Find the average rate of change of the height from 0 to 2 s.

b) Find the average rate of change of the height from 2 to 4 s.

**SOLUTION:**

**a)** **$f(t) = -4.9t^2 + 20t + 6$**

$f(2) = -4.9(2)^2 + 20(2) + 6 = 26.4$ m

$f(0) = -4.9(0)^2 + 20(0) + 6 = 6$ m

$$\text{average rate of change} = \frac{(26.4 - 6)m}{(2 - 0)s} = 10.2\, m/s$$

b) $f(4) = -4.9(4)^2 + 20(4) + 6 = 7.6$ m

$f(2) = 26.4$ m

$$\text{average rate of change} = \frac{(7.6 - 26.4)m}{(4 - 2)s} = -9.4\, m/s$$