ZERO AND NEGATIVE EXPONENTS

We have been examining the exponential function $y = b^x$. You will notice that we have always used the natural (counting) numbers as the exponents, $x = \{1, 2, 3, \ldots\}$. We now look at the cases where the exponent is either zero, or a negative.

The basic rules of exponents are:	$a^5 \times a^6 = a^{5+6} = a^{11}$
	$\frac{a^{12}}{a^5} = a^{12-5} = a^7$
	$(a^{11})^5 = a^{5 \times 11} = a^{55}$

Zero Exponents

Consider: $\frac{a^4}{a^4} = a^{4-4} = a^0$

But we know that if we divide anything by itself the answer is 1.

$$\frac{a^4}{a^4} = 1$$

It follows, then, that $a^0 = 1$

Negative Exponents

Consider: $\frac{a^2}{a^8} = a^{2-8} = a^{-6}$

But: $\frac{a^2}{a^8} = \frac{a \times a}{a \times a \times a \times a \times a \times a \times a \times a}$ which reduces to $\frac{1}{a^6}$.

It follows, then, that $a^{-6} = \frac{1}{a^6}$,

and, in general $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

Examples

Evaluate the following without calculators.

1. $7^0 = 1$

2. $1546^0 = 1$

3. $2^0 + 4^0 - 15^0 = 1 + 1 - 1 = 1$

4. $-8^0 = -1$ The exponent applies to the base immediately below it. -8^0 can be thought of as $(-1)(8^0) = (-1)(1) = -1$.

5. $(-8)^0 = 1$ The brackets indicate the number -8 is raised to the zero power. Thus the answer is 1. $[(-1)(8)]^0 = 1$.

6. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

7. $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

8. $\left(\frac{2}{3}\right)^{-4} = \frac{2^{-4}}{3^{-4}} = \frac{\frac{1}{2^4}}{\frac{1}{3^4}} = \frac{1}{2^4} x \frac{3^4}{1} = \frac{3^4}{2^4} = \frac{81}{16}$ That's a lot of work! Notice that we can save some time if we realize that the result is found **by taking the reciprocal of the original fraction and changing the exponent from a negative to a positive.**

9. $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{2} = \frac{25}{9}$

10. $2^{-1} + 3^{-1} = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

11. $2^{-1} \times 3^{-1} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

12. $(5x^{-2}y^{-5})^{0} = 1$

13. $7x^{-4}y^{5} = \frac{7y^{5}}{x^{4}}$ **Note: Only the base that is raised to the negative power is moved from the top to the bottom of the fraction. The exponent of the base 7 is 1.**

14. $\dfrac{120x^{-4}y^{7}z^{-8}}{-10x^{5}y^{-5}} = \dfrac{-12y^{7}y^{5}}{x^{5}x^{4}z^{8}} = \dfrac{-12y^{12}}{x^{9}z^{8}}$ **Note: Reduce the fraction as always. Only the bases that are raised to negative exponents are moved, either from the numerator of the fraction to the denominator, or from the denominator to the numerator. You must then simplify the resulting expression by using the rules of exponents.**

COMMON ERROR! The negative sign in front of the number 10 is <u>not an exponent</u>. Therefore the 10 does not move up to the numerator and become + 10!

15. $a^{-3} + b^{-5} = \dfrac{1}{a^{3}} + \dfrac{1}{b^{5}} = \dfrac{b^{5} + a^{3}}{a^{3}b^{5}}$

16. $(x + y)^{-2} = \dfrac{1}{(x+y)^{2}} = \dfrac{1}{x^{2} + 2xy + y^{2}}$ **Note:** $(x+y)^{-2} \neq (x^{-2} + y^{-2})$!

17. $(a^{-3} - b^{-3})^{2} = \left(\dfrac{1}{a^{3}} - \dfrac{1}{b^{3}}\right)^{2} = \left(\dfrac{b^{3} - a^{3}}{a^{3}b^{3}}\right)^{2} = \dfrac{b^{6} - 2a^{3}b^{3} + a^{6}}{a^{6}b^{6}}$