Motion Equations

Remember that everything is a vector (sign of the variable depends on its direction) except time and that the change in time can never be negative!

Kinematics – Mathematical Analysis & Projectile Motion

Symbol	Quantity (Unit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)
$\Delta\vec{x}$	Horizontal displacement (m)	$\vec{v}_{fx}$	Final velocity x-direction (m/s)	$\vec{g}$	9.81 (m/s^2; surface of the Earth)
$\Delta\vec{y}$	Vertical displacement (m)	$\vec{v}_{fy}$	Final velocity y-direction (m/s)	$\vec{g}_x$	x-component of g
$\vec{x}_o$	Initial horizontal position (m)	$\vec{v}_{avg}$	Average velocity (m/s)	$\vec{g}_y$	y-component of g
$\vec{y}_o$	Initial vertical position (m)	$\vec{v}$	Velocity (m/s)	t	time (s; refers to a time interval)
$\vec{x}_f$	Final horizontal position (m)	$\lvert\vec{v}\rvert$	Velocity magnitude (m/s)	θ	Angle (degrees, °)
$\vec{y}_f$	Final vertical position (m)	v_x	x-component velocity (m/s)	Δ	Change in (final - initial)
$\vec{v}_f$	Final velocity (m/s)	v_y	y-component velocity (m/s)		
$\vec{v}_o$	Initial velocity (m/s)	$\lvert\vec{a}\rvert$	Acceleration magnitude (m/s^2)		
$\vec{v}_{ox}$	Initial velocity x-direction (m/s)	$\vec{a}_x$	x-component acceleration (m/s^2)		
$\vec{v}_{oy}$	Initial velocity y-direction (m/s)	$\vec{a}_y$	y-component accelerations (m/s^2)		

$\vec{v}_{avg} = \frac{\Delta\vec{x}}{t}$	$\Delta\vec{x} = \vec{v}_{ox}t + \frac{1}{2}\vec{a}_x t^2$	$\Delta\vec{y} = \vec{v}_{oy}t + \frac{1}{2}\vec{a}_y t^2$	${v_{fy}}^2 = {v_{oy}}^2 + 2a_y\Delta\vec{y}$	$v_{oy} = v\sin\theta$	$\lvert\vec{v}\rvert = \sqrt{{v_{fx}}^2 + {v_{fy}}^2}$
$\vec{v}_{avg} = \frac{v_f + v_o}{2}$	$\vec{a} = \frac{\vec{v}_f - \vec{v}_o}{t}$	${v_{fx}}^2 = {v_{ox}}^2 + 2a_x\Delta\vec{x}$	$v_{ox} = v\cos\theta$	$\theta = \tan^{-1}\left\lvert\frac{v_y}{v_x}\right\rvert$	

Dynamics – Forces, Impulse, Torque, Momentum, & Circular Motion*

Symbol	Quantity (Unit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)
$\vec{F}_{net}$	Net force (N)	$\vec{F}_T$	Force of Tension (N)	$\vec{J}$	Impulse (N·s)
$\vec{F}_A$	Force applied (N)	$\vec{F}_N$	Normal Force (N)	$\vec{p}$	Momentum (kg·m/s)
$\vec{F}_g$	Force of gravity (N)	$\vec{F}_c$	Centripetal Force (N)	$\vec{p}_{oT}$	Initial total momentums (kg·m/s)
$\vec{F}_f$	Force of friction (N)	m	Mass (kg)	$\vec{p}_{fT}$	Final total momentums (kg·m/s)
$\vec{F}_s$	Restoring Force (N)	μ	Coefficient of friction (no units)	f	Frequency (Hz)
$\vec{a}_c$	Centripetal Acceleration (m/s^2)	T	Period (s)	r	Circular & Orbital Radius (m)
τ	Torque ($N \cdot m$)	$F_\perp$	Perpendicular Component of Force (N)	v	Circular Speed (m/s)
k	Spring Constant (N/m)				

*All variables that are vectors have an x and y component. The equations below are generalized and you have to remember to analyze (calculate) each dimension independently if the problem warrants.

$\vec{F}_{net} = \sum Forces$	$\sum F = \sum m \times a$	$\vec{F}_{net} = m\vec{a}$	$\vec{F}_g = m\vec{g}$	$\vec{F}_f = \mu\vec{F}_N$	$F_s = -kx$	$\vec{p} = m\vec{v}$	$\vec{J} = \Delta\vec{p}$ $\vec{F}t = m\Delta\vec{v}$	$\vec{p}_{oT} = \vec{p}_{fT}$
$v = \frac{2\pi r}{T}$	$a_c = \frac{v^2}{r}$	$F_c = \frac{mv^2}{r}$	$f = \frac{1}{T}$	$v = \sqrt{rg\mu_s}$	$v = \sqrt{rg\tan\theta}$	$\tau = rF_\perp$	$\vec{\tau}_{net} = \sum Torques$	
$\vec{J} = \vec{F}t$ $\vec{J}$ = Area under F - t curve	$T = 2\pi\sqrt{\frac{m}{k}}$							

Motion Examples

A car accelerates from zero to 35 m/s in 7.3 seconds.
a) What is the average acceleration?
b) What distance was covered during the acceleration?

a) $v_0 = 0\ m/s$ $v_f = 35\ m/s$

$t = 7.3\ s$ $a = ?$

$$a = \frac{v_f - v_0}{t} = \frac{35 - 0}{7.3} = \boxed{4.8\ m/s^2}$$

b) $\Delta x = ?$

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$= (0)(7.3) + \frac{1}{2}(4.8)(7.3)^2$$

$$\boxed{\Delta x = 128\ m}$$

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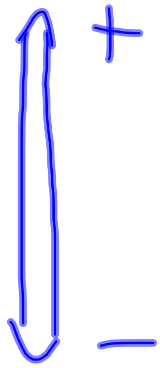
Standing near the edge of a cliff a baseball is launched straight up with a velocity of 15 m/s. The ball is in the air for a total of 4.5 s before it hits the ground at the bottom of the cliff. Find the height of the cliff (magnitude of the acceleration of gravity, g = -9.8 m/s^2).

$v_{0y} = 15\ m/s$

$t = 4.5\ s$

$g = -9.8\ m/s^2$



$\Delta y = ?$

$$\Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$= (15)(4.5) + \tfrac{1}{2}(-9.8)(4.5)^2$$

$$= 67.5\ m - 99.2\ m$$

$$\Delta y = -31.7\ m$$

$$h = -31.7\ m$$

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