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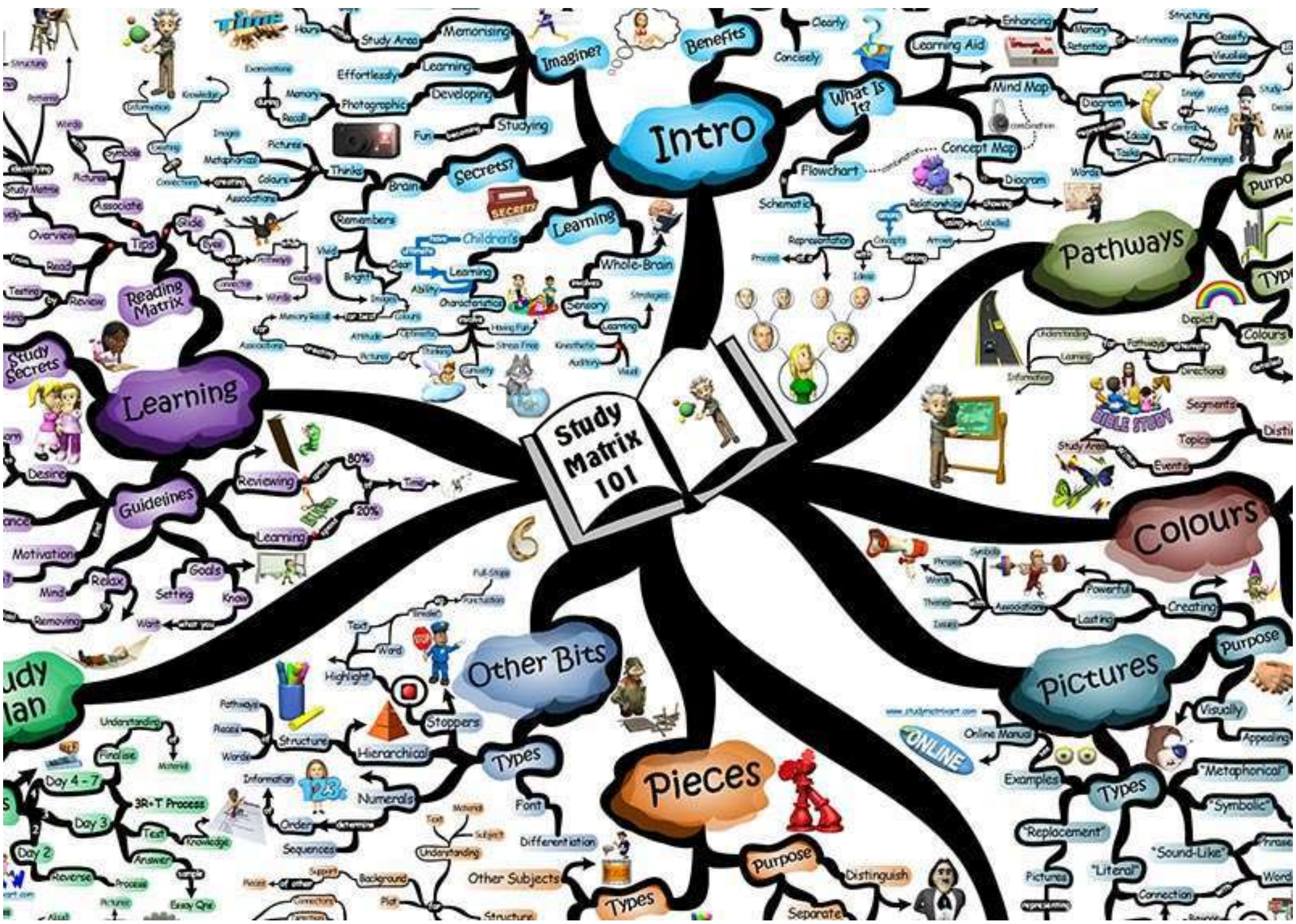
Physics 112/122 Study Guide

2012 – 2013



Mr. P. MacDonald

Motion

Physics

Big Picture

Everything is in motion. For example, we are all moving around the sun. This means that when we talk about motion, we must look at it relative to something else. The motion of objects through space is one of the first subjects of study for early physicists, but it took a very long time before motion was fully understood. To describe motion, we use rates such a velocity, speed, and acceleration. Throughout this study guide, assume negligible air resistance.

Key Terms

Displacement: The distance an object has moved from its starting position. SI units: m

Speed: The rate at which an object covers distance (distance/time). SI units: m/s

Velocity: Speed in a given direction. SI units: m/s

Acceleration: The rate at which an object's velocity is changing (velocity/time). Like velocity, acceleration is a vector. SI units: m/s^2

Projectile Motion: The non-linear path an object takes when moving under the constant acceleration of gravity.

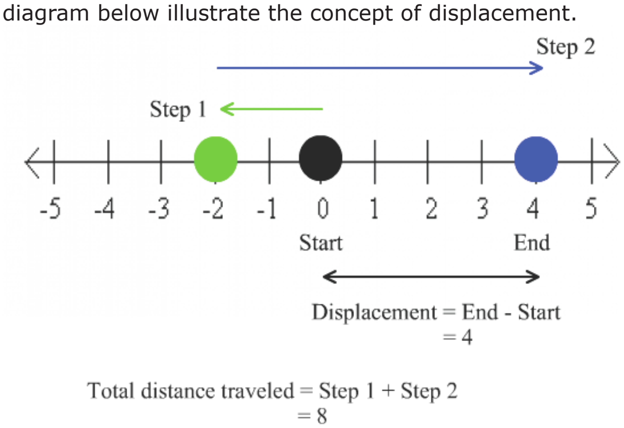
Free Fall: Objects falling due to gravity only. Example: an apple falling from a tree is in free fall.

Describing Motion

Displacement vs. Distance

Displacement and distance both measure a change in position. Displacement is a vector quantity, while distance is a scalar.

If an object moves in one direction and then returns to its original position, the total distance will be a positive number and the total displacement will be 0. The diagram below illustrate the concept of displacement.



Speed vs. Velocity

Speed is a scalar quantity that measures how fast an object is moving. In comparison, **velocity** is a vector quantity. Velocity is speed with a direction. Objects moving at the same speed can have different velocities if they are moving in different directions.

For example, we can say a car is moving at the speed of 25 mph, but if we say the car is moving at 25 mph to the north, then we are stating its velocity.

Acceleration

Usually we use "acceleration" to mean speeding up and "deceleration" to mean slowing down. In physics, **acceleration** is a vector quantity that could refer to speeding up, slowing down, or even changing direction.

Careful: positive acceleration does not always mean speeding up and negative acceleration does not always means slowing down.

Think of an acceleration as a push. We can define positive acceleration as a push to the right and negative acceleration as a push to the left.

If an object has a negative velocity, meaning that the object is moving towards the left, then a positive acceleration pushing the object to the right will cause the object to slow down.

- If the velocity and acceleration are in the same direction, the object speeds up. If not, the object slows down.

Instantaneous vs. Average

There are two types of speed, velocity, and acceleration: instantaneous and average.

Instantaneous refers to a single moment in time, while average refers to over a period of time.

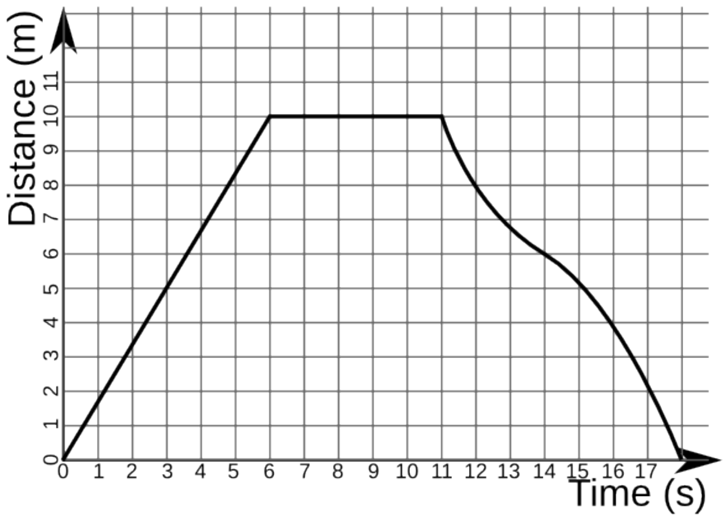
Notes

Motion cont.

Graphs of Motion

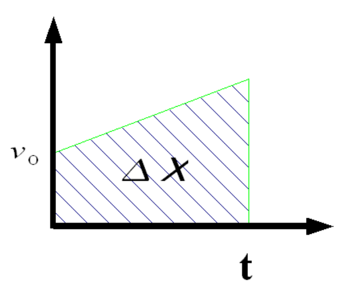
Distance vs. Time

Below is a graph of distance versus time. The slope $\left(\frac{\Delta y}{\Delta x}\right)$ of the curve represents the speed. As we can see, until t = 6 s, the object is moving at a constant speed of $\frac{5\text{ m}}{3\text{ s}}$. After t = 6 s, its speed is 0, meaning the object is at rest. Then, at 11 s, the object starts to move in the opposite direction. After 17 s, it has returned to its original position, resulting in a total displacement of 0.



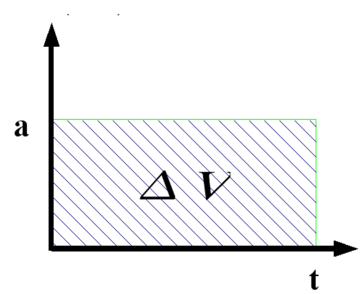
Velocity vs. Time

Below is a graph of velocity versus time. The slope represents acceleration. If the velocity curve is a line (has a constant slope), we know that there is a constant acceleration. The area under the velocity curve is equal to the change in displacement.



Acceleration vs. Time

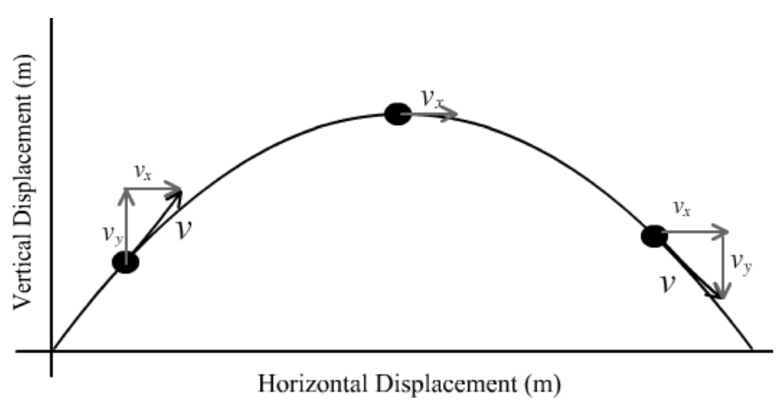
Similarly, for acceleration, the area under the acceleration curve is equal to the change in velocity. Below is a graph of acceleration versus time.



Projectile Motion

Projectile motion is a type of two-dimensional motion where an object moves horizontally (in the x-axis) and vertically (in the y-axis). The x- and y-components of any object's velocity vector are completely independent of each other. The diagram on the right shows the path and velocity vectors of an object in projectile motion.

- At the top of its flight, when the object stops rising and is about to fall, the vertical speed of an object is zero.
- Air resistance is ignored, so the horizontal component of velocity stays constant.
- The vertical component of velocity does not stay constant due to gravity - the object is in **free fall** along the vertical direction.



Motion Problem Guide

Physics

General Guidelines

- Always define a coordinate system for the problem! Define the positive and negative directions.
- Make sure your units make sense.

Important Equations

- All equations are only valid for constant acceleration
- In one dimension, vectors can only point in two directions, typically labeled + and -. The one dimensional vectors below can be treated like scalars with the signs indicating direction.

$v_{avg} = \frac{\Delta x}{\Delta t}$	x - displacement
$a_{avg} = \frac{\Delta v}{\Delta t}$	v - velociy
$v = v_o + at$	v_o - initial velocity
$\Delta x = v_o t + \frac{1}{2}at^2$	a - acceleration
$v^2 = {v_o}^2 + 2a\Delta x$	t - time

Δ - change in (final value - initial value)

Sample Problem

Two cities are 33 miles apart by freeway, where the speed limit is 65 mph. If it took you 45 minutes to get from one city to the other, what was your average speed?

Solution

Since we know the distance and the time, we can easily find the speed, which is just distance over time.

$$\text{speed} = \frac{\text{total distance}}{t}$$

$$\text{speed} = \frac{33 \text{ mi}}{.75 \text{ hr}}$$

$$\text{speed} = 44 \text{ mph}$$

Free Fall

An object thrown into the air is in free fall when it is moving up and falling down. At the maximum height, the object's velocity is 0 as it changes direction. If the object is at the same height before and after it is thrown, the time it takes to reach the maximum height is exactly half the total time the object spends in the air. The path up and the path down is identical but in opposite directions. Acceleration is always 9.8 m/s^2.

When we solve free fall problems, we usually look at only one direction (either the journey up or the journey down) so that there is one less variable to deal with. We can do this because we know the velocit at the top is 0.

Equations to use: $v = v_o + at$, $v^2 = v_o^2 + 2a\Delta y$

Example

A tennis ball is hit up into the air and spends a total of 8 seconds in the air. How high does it reach before coming back down?

Solution

1. Determine what is given and what needs to be found.
2. Set up the equation $v = v_o + at$ and solve for v_o.
3. Set up the equation $v^2 = v_o^2 + 2a\Delta y$.
4. Plug in values and solve for y!

Given: t = 8 seconds
Find: y = ??

Let $+y$ be upward and $-y$ be the downward direction. In this example, we're going to use the ball on its way up, so t = 4 seconds, v = 0 m/s, and a = -9.8 m/s^2.

$v = v_o + at$	$v^2 = v_o^2 + 2ay$
$v_o = v - at$	$y = \frac{v^2 - v_o^2}{2a}$
$v_o = ??$	$y = ??$
$v_o = .8 \text{ m/s}^2 \cdot 4 \text{ s}$	$y = \frac{-(39.2 \text{ m/s})^2}{-2 \cdot 9.8 \text{ m/s}^2}$
$v_o = 39.2 \text{ m/s}$	$y = 78.4 \text{ m}$

So the ball reaches a height of 78 m.

Motion Problem Guide cont.

Projectile Motion

In projectile motion, the acceleration in the y-direction is always -9.8 m/s², while the acceleration in the x-direction is 0. If the object is thrown at an angle, the initial velocity must be broken up into x- and y-components ($v_x = v\cos\theta$, $v_y = v\sin\theta$). Like in freefall, the velocity in the y-direction at the top of the object's path is 0 m/s.

In some questions, the object is thrown at a specific angle "above the horizontal" or "below the horizontal." In the example and diagram below, you can see what "above the horizontal" means. "Below the horizontal" is similar, only v_y and acceleration are both in the same direction.

Equations to use:
$$v = v_o + at$$
$$\Delta x = v_o t + \frac{1}{2}at^2$$

Example

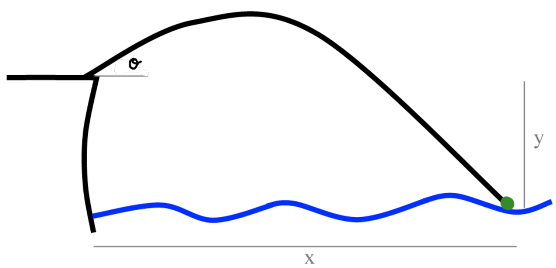
A man standing on a cliff throws a rock at 25° above the horizontal with an initial velocity of 15 m/s. If the rock is in the air for 5 seconds, how tall is the cliff, and how far into the ocean does it travel?

Solution

1. Determine what is given and what needs to be found.
2. Draw and label a diagram.
3. Set up the equation $\Delta x = v_o t + \frac{1}{2}at^2$ for the x-and y-directions.
4. Plug in values and solve!

Let the initial positions x_o and y_o be 0 so that $\Delta x = x - x_o = x$ and $\Delta y = y - y_o$.

Given: v_o = 15 m/s, t = 5 seconds, and θ = 15°
Find: x and y



x - direction

$$x = v_{o,x}t + \frac{1}{2}a_x t^2$$
$$v_{o,x} = v_o \cos\theta$$
$$a_x = 0 \text{ m/s}^2$$
$$x = 15 \text{ m/s} \cdot \cos(15^\circ)$$
$$x = 68.0 \text{ m}$$

y - direction

$$y = v_{o,y}t + \frac{1}{2}a_y t^2$$
$$v_{o,y} = v_o \sin\theta$$
$$a_y = -9.8 \text{ m/s}^2$$
$$y = 15 \text{ m/s} \cdot \sin(15^\circ) - \frac{1}{2} \cdot 9.8 \text{ m/s}^2 \cdot (5 \text{ s})^2$$
$$y = -90.8 \text{ m}$$

So the cliff is 91 m tall, and the rock travels 68 m.

In this problem, we end up with a negative y value that represents the height of the cliff because y = 0 is at the top of the cliff.

Example

If an object is launched into the air with speed at an angle on flat ground, how long will the object be in the air?

Solution

Since we know that the object's final vertical displacement is zero, we can think of its path as a parabola. We can find the roots using the displacement equation.

$$\Delta y = v_o t + \frac{1}{2}at^2$$
Start with the equation for vertical displacement.

$$0 = v_y \sin\theta\, t - \frac{1}{2}gt^2$$
Substitute in known values. The sign changed on the second term because we know g is negative.

$$0 = t(v_y \sin\theta - \frac{1}{2}gt)$$
Begin solving for t like we would for any quadratic.

$$t = 0, \; \frac{2v_y \sin\theta}{g}$$
We already know the displacement is 0 at time $t = 0$, so the second solution is our answer.

Newton's Laws

Physics

Big Picture

Newton's laws form the basis for all of mechanics and describe the effects of forces on an object's motion. Newton's laws can be applied to all sorts of problems in mechanics and even some in electrostatics. Anytime a force is involved, Newton's laws will determine the motion of the object the force is acting on.

Key Terms

Inertia: An object's resistance to changes in its motion.

Force: Any push or pull. SI unit: N

Normal Force (N): The reaction force exerted on an object by the surface that is supporting it. SI unit: N

Friction: The force that acts between moving materials. It always acts opposite the direction of the object's motion or applied force (if the object isn't moving). The frictional force f is proportional to the coefficient of friction μ, which depends on the surface the object is moving over, and the normal force **N**. SI unit: N

Coefficient of Static Friction: Coefficient of friction used for objects at rest to determine how much force is required to make it begin moving.

Coefficient of Kinetic Friction: Coefficient of friction used for objects moving across a surface to determine the force resisting the motion.

Tension: Force on an object provided by a wire, string, cable, or similar object.

Newton's Laws

Newton's First Law

Known as law of **inertia**, it states that an object in motion tends to stay in motion and an object at rest tends to stay at rest unless acted on by an external force.

- This means that an object moving with a constant velocity, speed, and direction will continue moving with that velocity. An object that is not moving will not move unless there a force acts on it.
- The value of an object's inertia can be measured. Mass is the measure of an object's inertia while it is at rest.

Newton's Second Law

Explains how the force acting on an object will affect its motion. The acceleration **a** (change in velocity) of an object is directly proportional to the force **F** exerted on it and inversely proportional to the mass m (inertia) of the object. $\mathbf{F} = m\mathbf{a}$

- If there are multiple forces on the object, the acceleration is proportional to the *net* (overall) force.
- Any net force causes an acceleration, but only forces in the direction of motion causes a change in speed.
- Forces applied perpendicular to direction of motion will change the direction of motion, but not the speed.

Newton's Third Law

This law states that for any force exerted by one object on another, the other object exerts an equal force in the opposite direction on the first object. For example, if a person is pushing on a wall, the wall is also pushing back on them with an equal force in the opposite direction.

- This law holds for all situations in which objects come into contact, even when the objects are accelerating.
- While it seems like the third law would prevent the object from moving because the reaction force would cancel the initial force, the two forces act on different objects so there is still a net force on the individual objects.
- Pairs of forces described by the third law must be: the same type of force, exerted on two different objects, and equal in magnitude in opposite directions.

Free Body Diagrams

The forces on an object are often visualized using a free body diagram (FBD). FBDs are useful for looking at the different types of forces acting on an object.

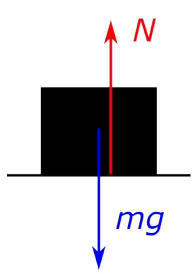
Be careful when assigning signs. For each of the x- and y-components, remember which direction is positive and which is negative. Typically the "downward" direction is assigned as the negative direction, but you do not have to follow this convention as long as your signs are consistent.

Mass on a Table

Gravity is a force that acts on all objects. Near the surface of the Earth, the force due to gravity (F_g) is equal to mg, where g is approximately 9.8 m/s^2.

- mg is what we call the weight of an object. It is not the same thing as mass!

Normal force (**N**) can be understood using Newton's third law. It is the force opposing gravity so that the net force on both objects (the mass and the table) is zero. $N = mg$

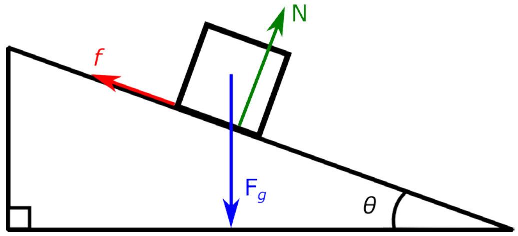


Newton's Laws cont.

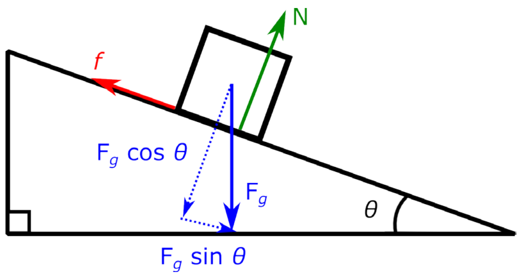
Free Body Diagrams (cont.)

Objects on a Ramp

Another type of problem is an object on a ramp or tilted table. The normal force is still perpendicular to the table, but gravity is always pointing straight down.

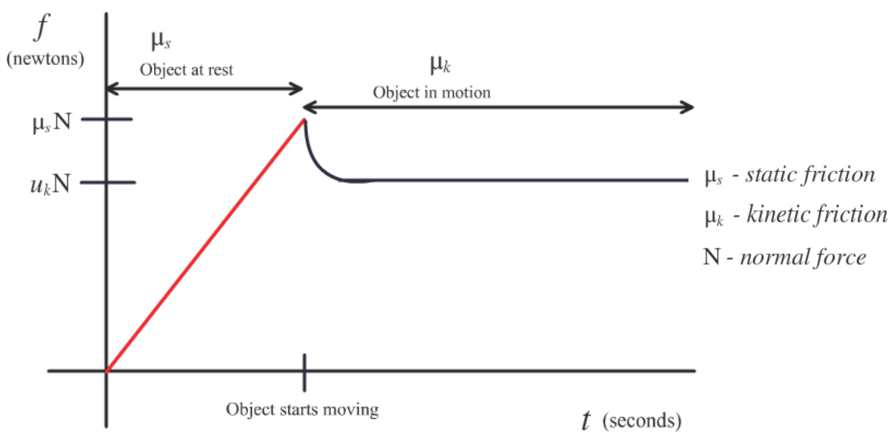


Gravity can be decomposed into its *x*- and *y*-components (relative to the ramp), as shown below. Because of Newton's third law, we know that $N = mg \cos \theta$.



Friction force is always in the opposite direction of motion. In the ramp example, gravity pulls the object down the ramp, so friction acts in the opposite direction (up the ramp), impeding the object's motion.

- There are two friction coefficients: the **coefficient of static friction** (μ_s) and the **coefficient of kinetic friction** (μ_k). The static coefficient is always greater than the kinetic coefficient, which is why it takes less force to keep an object sliding along the ground than it does to make it start moving. The graph below shows the magnitude of the friction force over time while resisting an external force.

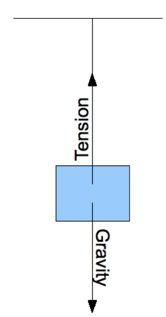


- $\mu_s N$ is the static friction the block needs to overcome before it can move.
- If the block is not moving, we can calculate the magnitude of static friction.
- If the block is moving, use the kinetic coefficient to calculate kinetic friction.

Tension

For objects attached to a string (or similar string-like objects), the force provided by the string is called **tension**. This could be an object hanging from a string or an object being pulled by a string.

- Usually the string is assumed to have no mass - this means that we can assume the string will perfectly transfer energy from one end of the string to the other.
- The total force on a massless string must always be zero. This means that every point along the string feels two equal and opposite tension forces.
- If the string is angled, make sure to decompose the tension force into components.



Newton's Laws Problem Guide

Important Equations

$\mathbf{F} = m\mathbf{a}$	$\mathbf{F}$ - force	m - mass	$\mathbf{a}$ - acceleration
$f = \mu N$	f - force of friction	μ - coefficient of friction	N - normal force

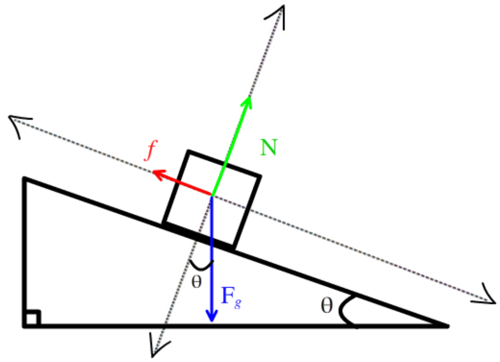
Example

Equilibrium problems (where there is no net force) are the most common type of problem involving Newton's laws.

Example

A block of mass m is at rest on an incline of angle θ. Solve for the the coefficient of static friction (μ) in terms of m, g, and θ.

Solution



The first step in any problem involving Newton's laws is to draw an FBD of the situation and carefully label all the forces involved. The FBD on the right shows all the forces all drawn from a single point instead of from where they're acting so that it's easier to visualize the axes. In these sort of problems it is best to angle the coordinate system so that the x-axis is parallel to the incline. This forces us to break the weight vector into its components. Since the object is in equilibrium, the weight vector's components can be matched to the normal force and friction.

Here is the mathematical process to solve this problem:

$f = \mu N$	start with the equation for friction
$f = \mu mg\cos\theta$	substitute the y-component of the weight vector ($mg\cos\theta$) for N
$mg\sin\theta = \mu mg\cos\theta$	substitute the x-component of the weight vector ($mg\sin\theta$) for f
$\mu = \dfrac{mg\sin\theta}{mg\cos\theta}$	solve for μ
$\mu = \dfrac{\sin\theta}{\cos\theta}$	simplify the result
$\mu = \tan\theta$	

Notes

Harmonic Motion

Physics

Big Picture

Simple harmonic motion (SHM), also known as oscillatory motion, describes the motion of objects that move back and forth along a sine or cosine curve due to a restoring force. In addition, SHM can model the movements or the periodic change of states of all kinds of objects, such as the vibration of springs. When amplitude is not constant over time and there is a net force, other than the restoring force, acting on the system, the object is not considered to be in SHM.

Key Terms

Simple Harmonic Motion (SHM)**:** A symmetrical back and forth motion where the distance traveled in each cycle is constant.

Period: The time it takes for an object in SHM to return to its original state (complete one cycle). SI units: s

Frequency: The number of times an object in SHM returns to its original state (completes a cycle) in a second. SI units: Hz (1 cycle per second).

Amplitude: The greatest distance an object moves away from its equilibrium position. SI units: m

Restoring Force: A force acting on an object that moves it back to its equilibrium position. The force's magnitude and direction varies with its position. The direction will always be toward the equilibrium position and the magnitude generally increases the farther the object is from equilibrium. SI units: N

Simple Harmonic Motion

Simple harmonic motion (SHM) is a special type of back-and-forth motion. Systems that go through SHM:

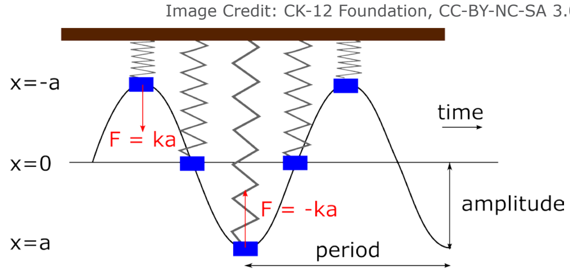
- do not lose any energy (friction assumed to be zero)
- do not have constant acceleration, so we cannot use the three equations for linear motion to describe SHM
- have a **period** (T) and a **frequency** (f) (equal to the inverse of period: $f = \frac{1}{T}$)
- have an **amplitude** (A) that is constant - it is half the total distance traveled in a period
- have zero **restoring force** at the equilibrium point
- have maximum kinetic energy and speed at the equilibrium point

One common example of a system that experiences SHM is a mass attached to a spring.

- The restoring force is the spring force.
- The spring force is described by Hooke's law: $F = -k\Delta x$, where Δx is the displacement.
- $-k$ is the spring constant
- The spring potential energy is: $E_{sp} = \frac{1}{2}k\Delta x^2$.

To the right is a diagram of a spring-mass system in SHM.

Image Credit: CK-12 Foundation, CC-BY-NC-SA 3.0



Another system that experiences SHM is a pendulum. SHM only correctly approximates the motion of a pendulum when the amplitude is much less than the length of the pendulum.

- The restoring force is gravity.
- The period is determined entirely by the length of the pendulum and the acceleration of gravity at that location.

Other Types of Harmonic Motion

In systems going through harmonic motion (not simple harmonic motion), there is a net force other than the restoring force, so the amplitude is not constant. The system is now in either damped or driven harmonic motion:

- Damped harmonic motion: A frictional force causes the amplitude to decease over time.
- Driven harmonic motion: An external force acts on an object during its oscillation.

Important Equations

	$T = \frac{1}{f}$	T - period	f - frequency
Period for mass-spring system in SHM:	$T = 2\pi\sqrt{\frac{m}{k}}$	m - mass	k - spring constant
Period for pendulum in SHM:	$T = 2\pi\sqrt{\frac{l}{g}}$	l - length	g - gravity
	$x(t) = x_o + A\cos(2\pi f(t - t_o))$	x - position	A - amplitude
	$v(t) = -2\pi A\sin(2\pi f(t - t_o))$	v - velocity	t - time

Waves

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Physics

Big Picture

Waves transfer energy between two points in space without transferring any actual matter. For example, when a rock is dropped into a pond, the waves that emanate from the point of impact are transferring the rock's kinetic energy to the edge of the pond. All waves are the result of some sort of vibration. Sound waves result from the macroscopic vibrations of objects, and electromagnetic waves result from the vibrations of electrons in atoms. Similar to simple harmonic motion, the motion of waves can be mathematically modeled by sine and cosine curves.

Key Terms

Note: Some of the terms from the *Simple Harmonic Motion* study guide are also used in the description of waves. The explanations for period, frequency, and amplitude can be found there.

Mechanical Wave: Needs a medium to travel through.

Transverse Wave: Energy is transferred by particles vibrating perpendicular to the direction the wave is traveling.

Longitudinal Wave: Energy is transferred by particles vibrating in the same direction as the wave's motion.

Interference: When two waves of the same type meet, they combine to create a larger or smaller wave.

Destructive Interference: When two waves meet, if one is at the highest point in its vibration (crest) and another is at its lowest (trough), they cancel each other out so no wave appears at this point.

Constructive Interference: If the waves meet when they are both at their crests, their amplitudes will add together so that there appears to be a very large wave at that point.

Beat Frequency: The difference in frequencies when waves with two different frequencies interfere.

Standing Wave: A wave that stays in a constant position. They are the result of interference between two waves traveling in different directions.

Node: Location of complete destructive interference between the the incident (initial) wave and the reflected wave. The wave does not move at a node.

Antinode: Location of complete constructive interference. The wave will have the greatest displacement at an antinode.

Resonance: When an object is shaken or pushed at a frequency that matches its natural frequency.

Doppler Effect: When there is an apparent change in a wave's frequency due to the relative motion of either the source of the wave or the observer.

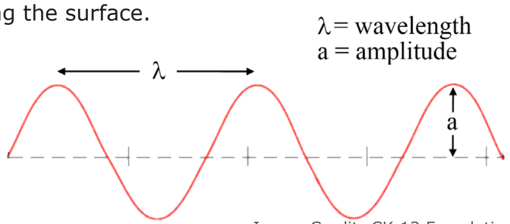
Mechanical Waves

Mechanical waves travel through a substance called a medium.

- Examples include sound waves (travel through air), and seismic waves (travel through the ground).
- Mechanical waves cannot transfer energy if there is no medium between the origin of the wave and its destination.
- Speed of a wave depends on the medium.

Two main types of mechanical waves are **transverse waves** and **longitudinal waves**.

Waves on the surface of water are an example of transverse waves. The water molecules move perpendicular to the surface of the water (up and down) to transfer the energy, while the wave itself moves along the surface.



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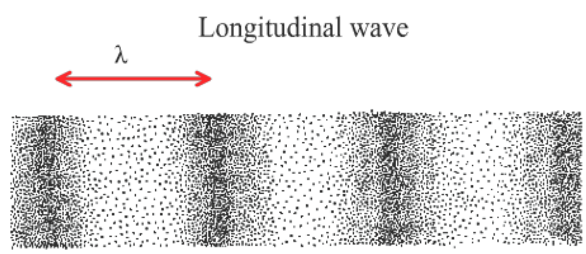
- crest - highest point of the wave
- trough - lowest point of the wave
- amplitude - distance between the equilibrium position and the crest (or trough)
- wavelength - distance between identical positions on two successive waves

Have you ever seen people in a stadium do the wave? The wave travels around the stadium, but the people do not.

We can visualize a longitudinal wave by laying a spring (such as a Slinky) on the ground, stretching it out, then pushing one end of the spring. The compression travels up the spring.

Sound waves are longitudinal mechanical waves that propagate through air. Sound waves are caused by vibrations in objects and exist as differences in pressure. They can be thought of as vibrations in the medium the sound waves are traveling through. The vibrations in air cause our ear drums to vibrate, which our brain interprets as sound. The speed at which sound waves travel depends on the medium which they're passing through. Sound travels fastest through solids, and slowest through gases, but it varies with each substance. Below is a diagram of a sound wave, where the dots represent air molecules.



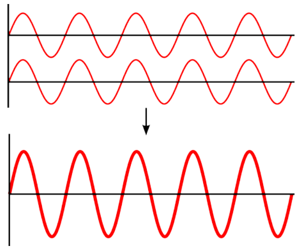
This guide was created by Christopher Addiego and Catherine Wu. To learn more about the student authors, visit http://www.ck12.org/about/about-us/team/interns.

Waves cont.

Wave Behavior

Two waves can interact by **interference**. Interference does not create any lasting change in either wave, but at the place where the waves meet, the amplitude of the two waves will merge.

Constructive Interference



Destructive Interference

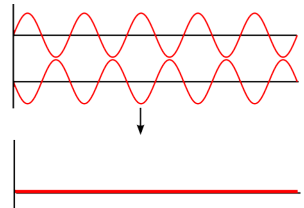
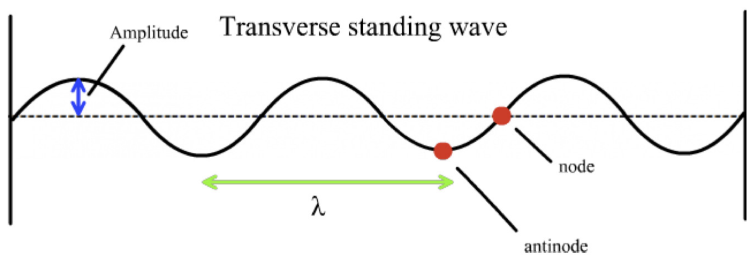


Image Credit: Wjh31, Quibik, CC-BY-SA 3.0

In physics, beats are the result of interference between sound waves. When two slightly different frequencies are emitted together, there will be both constructive and destructive interference which results in the sound alternating between loud and soft. The **beat frequency** is the difference of the two frequencies.
When a wave reaches a barrier, it is reflected and travels back the way it came.

- If the wave is not allowed to move at the barrier (a hard boundary), the wave will invert when it reflects back. Under the right conditions, a **standing wave** can be created. There are two important points in a standing wave: **nodes** and **antinodes**.



- If the wave is allowed to move at the barrier (a soft boundary), the wave will reflect back with the same orientation.

Resonance

All objects have a natural frequency at which they vibrate when struck. We can force an object to vibrate at a specific frequency by sending a sound wave at it. **Resonance** occurs when this forced frequency matches the natural frequency, causing the amplitude of vibration to increase. The idea that high notes can shatter glass comes from this idea - if a singer hits the right frequency, she can cause glass to resonate and shatter.

Doppler Effect

The **Doppler effect** occurs when either the wave or an observer is moving.

- If the observer and the source of the wave are moving toward each other, the wave will appear to have a higher frequency. In the case of a sound wave, the wave will seem to have a higher pitch.
- If they're moving away from each other, the wave will appear to have a lower frequency. In the case of a sound wave, the wave will seem to have a lower pitch.

The Doppler effect explains why a police siren sounds higher in pitch when the vehicle is moving towards you.

Important Equations

$v = f\lambda$	v - velocity	f - frequency	λ - wavelength
$T = \frac{1}{f}$	T - period		

Sample

At this level, problems with waves usually involve using the given information about a wave to find its other characteristics.

Sample

If a mechanical wave traveling down a slinky has a period of .5 s, and a wavelength of 1 m, what is the wave's frequency and speed?

Solution

First we'll find the frequency: $f = \frac{1}{T} = \frac{1}{.5\ \text{s}} = 2\ \text{Hz}$

Then, using the frequency, we can find the speed of the wave. $v = f\lambda = 2\ \text{Hz} \cdot 1\ \text{m} = 2\ \text{m/s}$

Wave Optics

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Physics

Big Picture

Visible light is one type of electromagnetic (EM) radiation and only represents a tiny portion of the whole EM spectrum. All electromagnetic radiation is transferred by waves that can interact with each other. Wave optics mainly has to do with the how these waves will interfere with each other. While we modeled light as rays in geometric optics, we will be dealing with how light behaves as a wave in this study guide.

Key Terms

Amplitude: In wave optics, the amplitude of an electromagnetic wave is the intensity of the wave.

Interference: When two waves of the same type meet, they combine to create a larger or smaller wave. For more information on interference, see the *Waves* study guide.

Interference Pattern: Pattern resulting from interference. Light interference patterns are alternating light (minima) and dark bands (maxima).

Diffraction: Occurs when a wave hits an obstacle or a barrier.

Fresnel Diffraction: An approximation that can be used to estimate propagation of waves when very close to an antenna emitting electromagnetic waves.

Fraunhoffer Diffraction: An approximation that can be used to estimate the propagation of waves when very far from an antenna emitting electromagnetic waves.

Resolving Power: The ability of a device to distinguish between different wavelengths in the electromagnetic spectrum.

Doppler Effect: Describes an apparent change in wavelength because of the relative motion of the source of a wave and an observer.

Light as a Wave

Light has a dual nature:

- In some situations, it is better to model light as particles called *photons*.
- In other situations, it is better to model light as a wave. Light waves are *electromagnetic waves*, which contain changing electric and magnetic fields that are oriented in directions perpendicular to the direction of travel.

Electromagnetic radiation is categorized by its wavelength, ranging from radio waves of wavelength 10^8 meters to gamma rays of wavelength 10^{-16} meters. Below is an illustration showing the divisions in the electromagnetic spectrum.

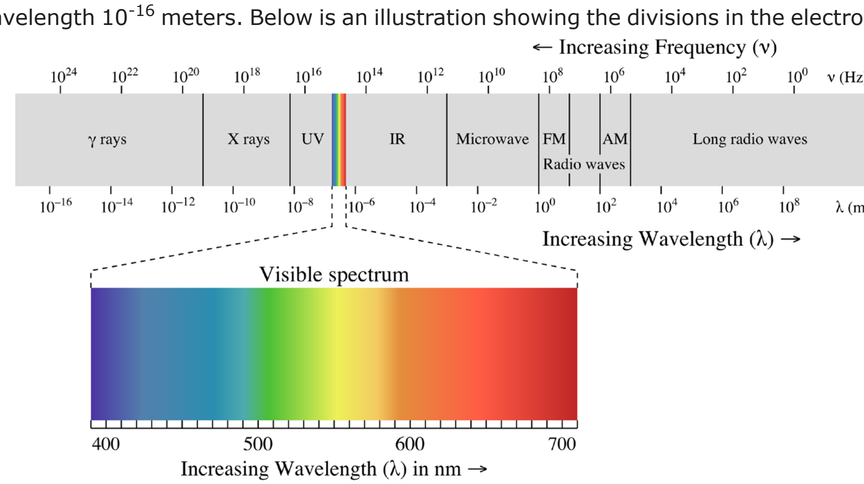


Image Credit: Philip Ronan, CC-BY-SA 2.5

In the case of visible light, the brightness of the light corresponds to the **amplitude** of the wave.

Interference

Electromagnetic waves can interfere constructively or destructively just like any other type of wave. However, it is possible to create easily visible **interference patterns** with light (unlike other types of waves such as sound).

Interference patterns are created any time that two light waves with the same wavelength reach the same point by traveling along slightly different paths. If we observe the light waves interacting on a surface such as a screen or a wall, there will be places where the interference pattern is bright (constructive interference) and places where the pattern is dark (destructive interferences). There are many situations where this is possible, including when light passes through a thin film or multiple separated slits.

Michelson Interferometer

The interferometer are used to split beams of light into different paths and then recombine them to create an interference pattern. Michelson used his design for an interferometer to demonstrate how the speed of light remained constant in an accelerating reference frame.

Physics

Wave Optics cont.

Interference (cont.)

Thin Film Interference

When there is a thin layer of a transparent substance on top of another material, some of the incident light will be reflected off the top of the surface and some will refract through the layer and then reflect off the bottom of the layer. The two rays of light traveled a different distance and will create an interference pattern when both rays emerge from the thin layer.

To the right is a diagram that shows the path of the light waves in the thin film.

The colors in soap bubbles are the result of thin film interference. The different colors reflect the thickness of the bubble.

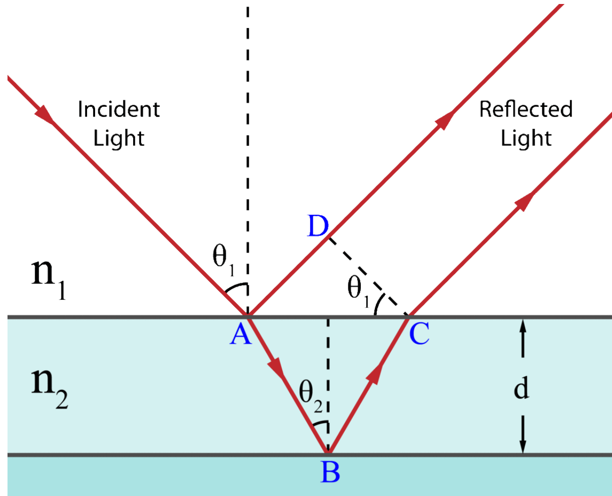


Image Credit: Chanli44, Public Domain

Diffraction

Light, like other waves, demonstrates **diffraction**. It is most noticeable when the wavelength of the wave is similar to the size of the obstacle.

Single Slit Diffraction

Passing light through a slit that is of comparable size to the wavelength will create an interference pattern. This can be explained by thinking of all the points in the opening as a point source for waves. The diffraction pattern will fall off in brightness very quickly from the central maxima. To the right is a diagram showing a single slit diffraction.

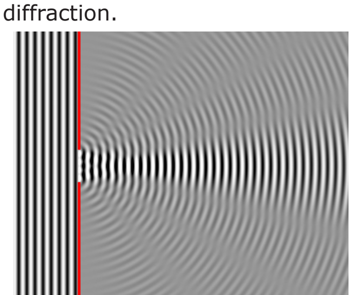
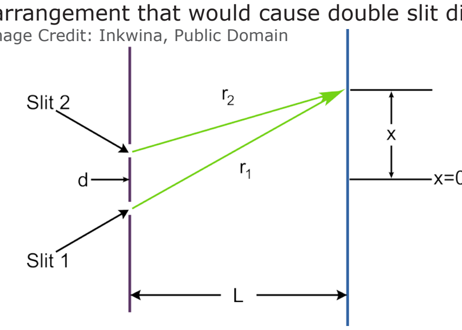


Image Credit: Dicklyon, Public Domain

Double Slit Diffraction

Double slit diffraction occurs when passing light through two slits separated by a small distance. The interference pattern is the result of the light waves from the two slits being slightly out of phase when they hit surface after traveling slightly different distances. The double slit experiment was used to prove that light is a wave. To the left is a diagram of an arrangement that would cause double slit diffraction.

Image Credit: Inkwina, Public Domain



Multiple Slit Diffraction

Multiple slit diffraction is very similar to double slit diffraction and creates a similar interference pattern, except the brightness will not decrease as fast from the central maxima.

Doppler Effect

Like sound waves, electromagnetic waves such as light experiences the Doppler effect. There are some special terms associated with the Doppler effect for light.

- *Red Shift:* Occurs when objects are moving away from each other, which increases the wavelength. It is called red shift because the emitted light shifts toward the red end of the visible spectrum.
- *Blue Shift:* Occurs when object's are moving toward each other, which decreases the wavelength.

Important Equations

Note: substitute $(n + \frac{1}{2})$ for n when trying to find a minima instead of a maxima. Also, the third equation uses the approximation that $\sin\theta \approx \tan\theta = \frac{x_m}{L}$ for small angles.

$c = \lambda f$	c - speed of light	λ - wavelength	f - frequency
$d\sin\theta = n\lambda$	d - separation of slits	θ - angular position of maxima/minima	n - number of maxima/minima from central maxima
$\frac{dx_m}{L} = n\lambda$	x_m - distance from central maxima	L - distance to screen	

Geometric Optics

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Physics

Big Picture

The principles of geometric optics describe the interactions of light on a macroscopic scale. Geometric optics is mainly used to determine how light will change direction and form images through reflection or refraction. The path of light is approximated by rays that travel in straight lines. All angles regarding reflection and refraction are measured from the normal, which is a line perpendicular to the surface the light is interacting with.

Key Terms

Reflection: When light travels away from an object at the same angle that it hits the surface.

Normal: Line perpendicular to the surface of interest.

Refraction: Occurs when light changes the medium it is moving through.

Snell's Law: Determines the angle at which a ray of light refracts.

Total Internal Reflection: Occurs when the light ray's incident angle is greater than the critical angle, causing all of the light to be reflected back into the original medium.

Image: An image forms where rays of light coming from an object seem to converge.

Real Image: Forms where the path of the light rays actually do converge.

Virtual Image: Forms whre the light rays actually converge.

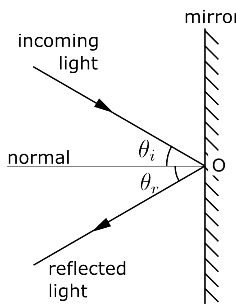
Focal Length (Focus): The point where originally parallel light rays converge when they reflect off a curved mirror or refract through a lens.

Lensmaker's Equation: Allows us to calculate the focal length of a lens based on the material and curvature.

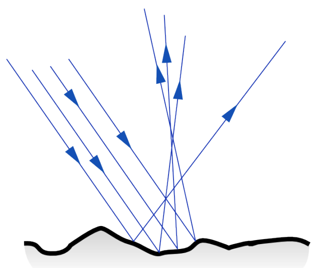
Reflection and Refraction

When light hits a surface, it always **reflects** away from the object at the same angle that it hits the surface. The angle is measured relative the **normal**. Depending on the surface, two types of reflection are possible:

- *Specular Reflection*: Rays of light reflect off a smooth surface parallel to each other. This means it is possible to see the original image that hit the surface. All mirrors employ specular reflection to display an **image**.
- *Diffuse Reflection*: When rays of light reflect off a rough surface, light reflects in all directions due to surface irregularities.



specular reflection
Image Credit: Johan Arvelius, CC-BY-SA 2.5



diffuse reflection
Image Credit: Marcelo Reis, CC-BY-SA 3.0

Light **refracts** when it travels from one medium to another (examples: from air to water, from glass to air). Light will change direction when entering a new medium because it will travel at a different speed in the new medium.

- How much light changes direction is determined by the medium's index of refraction, the ratio of the speed of light in a vacuum to the speed of light in the medium, and the incident angle of the light ray. This relationship is described by **Snell's law**.
- To a lesser degree, the angle of refraction also depends on the wavelength of light being refracted.
- All mirrors use specular reflection to create an image. Lenses are special shapes made of transparent material that refract light into an image.

Depending on the angle, an interesting phenomenon called **total internal reflection** can occur. Whenever light is moving from a medium of a higher index of refraction to a medium of lower index of refraction, some of the light is reflected back into the original medium. If the angle is greater than the critical angle, all the light is reflected.The critical angle is the angle at which light is refracted at 90° from the normal (along the interface between the two mediums).

Ray Diagrams

Ray diagrams illustrate the path of light when it interacts with mirrors and lenses.

- If the light is interacting with a mirror, it will reflect off the surface.
- If the light is interacting with a lens, it will refract through the lens.

Light from an object hitting a mirror or lens is approximated by two or three arrows that go towards the top, center, and bottom of the mirror or lens. Since ray diagrams are approximations, we will not need to calculate any specific angles of reflection or refraction.

Geometric Optics cont.

Ray Diagrams (cont.)

In a ray diagram, the solid lines indicate the actual path of light rays. **Real images** form where the rays converge. On the other hand, dotted lines indicate the path of virtual rays. They form **virtual images** when they converge.

- Real images can be projected onto a surface. You can see this by putting a white piece of paper on one side of a magnifying glass and then pointing it at a bright window. Move the paper back and forth relative to the magnifying glass until you see the window on the paper.
- Although the actual path of the light rays never intersect, when the diverging light rays hit your eyes, your brain will work backwards and construct an image at the point where the rays seem to intersect. Virtual images cannot be projected onto a surface. Flat mirrors always create virtual images.

There are three main characteristics of the image produced by the arrangements below that we need to keep track of:

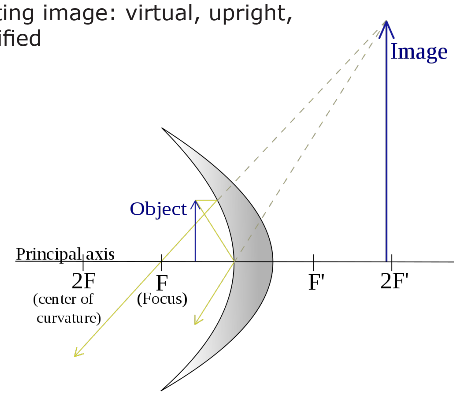
- type of image: real vs. virtual
- inverted vs. upright
- magnification: magnified vs. reduced

Concave Mirrors

Concave mirrors are shaped like a parabola and curve towards the object. One key factor of a concave mirror is that light rays coming in parallel (from an infinite distance) will all come together at the **focus**. There are a few different arrangements that are associated with concave mirrors. Each one produces a different sort of image.

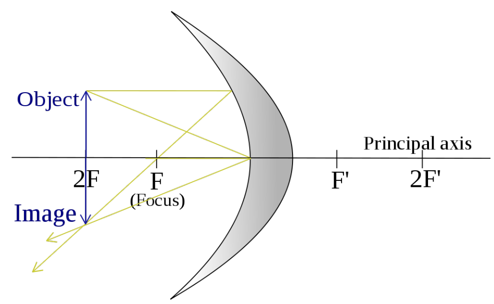
Object Between Focus and Mirror

Resulting image: virtual, upright, magnified



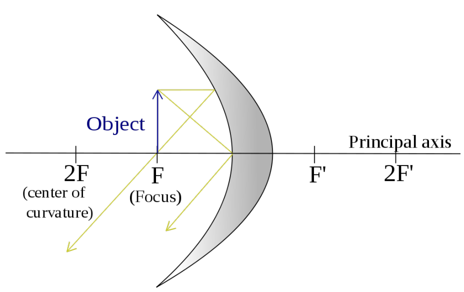
Object at 2F

Resulting image: real, inverted, same size



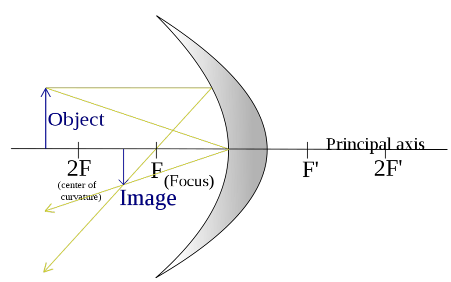
Object at Focus

Rays reflect parallely so an image never forms



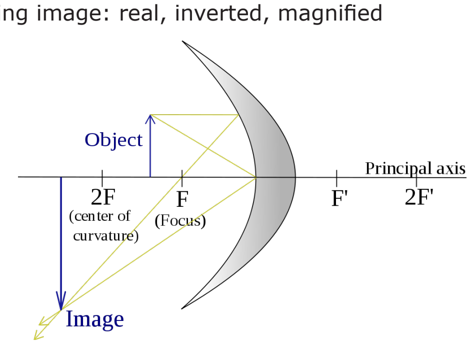
Object Beyond 2F

Resulting image: real, inverted, reduced



Object Between Focus and 2F

Resulting image: real, inverted, magnified



Geometric Optics cont.

Convex Mirror

Convex mirrors are mirrors that curve away from the object. Convex mirrors always produce the same kind of image.

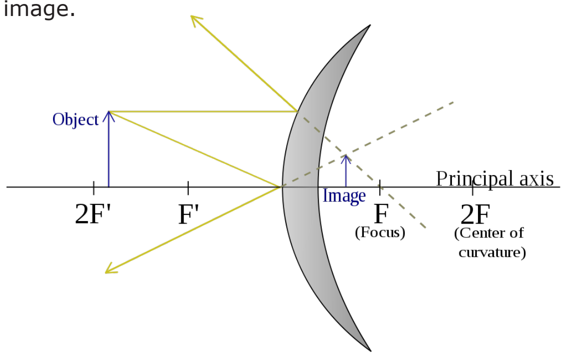


Image Credit: Cronholm144, CC-BY-SA 2.5

Resulting image: virtual, upright, reduced

Concave Lens

Also called a diverging lens because the light rays will diverge after passing through the lens. Diverging lenses always form a virtual, upright, diminished image.

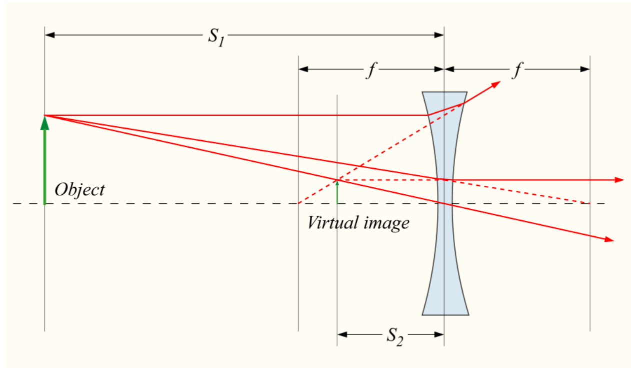


Image Credit: DrBob, GNU-FDL 1.2

Convex Lens

Also called converging lens because light rays will come together at the focus. There are a couple different configurations that are important.

Object Between Focus and Lens

Resulting image: real, inverted, reduced

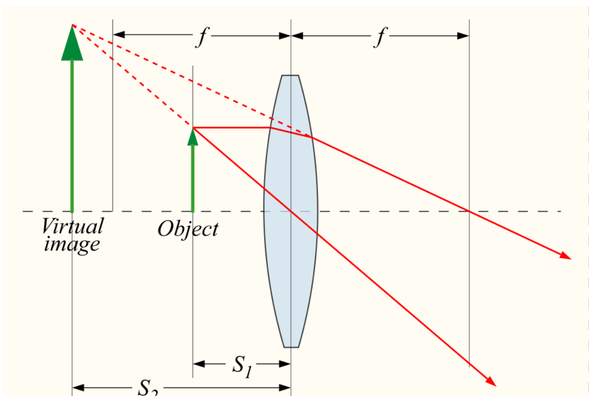


Image Credit: DrBob, GNU-FDL 1.2

Object Beyond Focus

Resulting image: virtual, upright, magnified

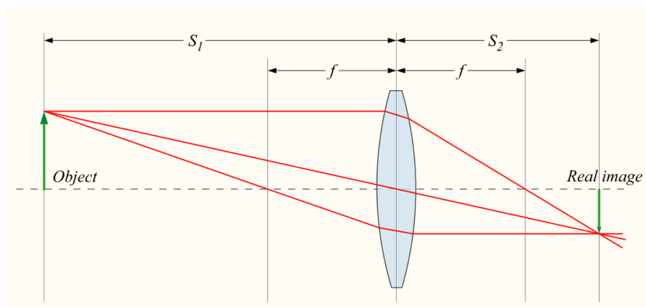


Image Credit: DrBob, GNU-FDL 1.2

Important Equations

Note: The Lensmaker's equation shown here is actually an approximation that assumes the lens is thin and is in air.

Snell's law:	$n_1 \sin\theta_2 = n_2 \sin\theta_2$	n - index of refraction	θ - angle from normal
Lensmaker's equation:	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$	f - focus/focal length	R - radius of lens curvature
	$\frac{1}{f} = \frac{1}{S_o} + \frac{1}{S_i}$	S_o - distance to object	S_i - distance to image

Notes

Momentum

Physics

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Big Picture

Momentum is the measurement of an object's inertia and is found using the mass and velocity. It is always conserved in a collision, allowing us to find the velocities of objects after a collision.

Key Terms

Momentum: Momentum is a vector quantity that measures of an object's inertia when it is in motion. SI units: kg•m/s

Law of Conservation of Momentum: Momentum is always conserved during collisions between objects.

Impulse: A force exerted over a period of time. SI units: N•s or (kg•m)/s

Collisions

Momentum is the product of mass and velocity ($p = mv$). We can think of momentum as how hard it is to move/stop a moving object. Thus, between a truck and a car moving at the same speed, the truck has more momentum.

- Newton's second law can be rewritten for momentum. $\mathbf{F}_{net} = m\mathbf{a} = m\frac{\Delta \mathbf{v}}{\Delta t} = \frac{\Delta \mathbf{p}}{\Delta t}$

Momentum can be transferred from one object to another. The **law of conservation of momentum** is used to determine the states of objects before or after collisions.

- When dealing with momentum in two dimensions, remember to break down all the vectors into the horizontal and vertical components. Momentum must be conserved in both dimensions.

- There are two main types of collisions:
    - *Perfectly elastic collisions* occur when energy is conserved in a collision and the colliding objects bounce back perfectly - the total momentum and energy is the same before and after the collision.
    - *Perfectly inelastic collisions* are when energy is not conserved in a collision and objects stick together. In this case, only momentum is conserved because some of the energy of the objects goes into producing sound waves and heat.

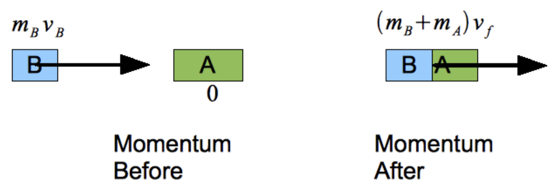

Figure: Example of inelastic collision. Initially, only block B is moving. After collision, blocks A and B are stuck together and moving to the right. If the collision is perfectly inelastic, the momentum before and after the collision is the same.

Impulse relates force to momentum. By exerting an external force over a certain time, the force is changing the momentum of the object. A large force exerted over a short time will produce the same change in momentum as a small force exerted over a long time.

- If an external force (a force outside the system) is applied to a system, the final momentum is not equal to the initial momentum. The change is equal to the impulse.

Notes

This guide was created by Christopher Addiego and Catherine Wu. To learn more about the student authors, visit http://www.ck12.org/about/about-us/team/interns.

Physics

Momentum Problem Guide

Important Equations

$p = mv$	p - momentum	m - mass	v - velocity
$J = \Delta p = F\Delta t$	J - impulse	F - force	t - time

Example Problems

Elastic Collision

An object (m_1) of mass 2 kg is moving at speed v_1 in the positive x-direction. It collides perfectly elastically with another object (m_2) of mass 3 kg moving at -3 m/s. If m_1 is moving at +1 m/s and m_2 is moving at +2 m/s after the collision, how fast was m_1 moving before the collision?

Solution

To solve this problem, we will use conservation of momentum. Since the collision was elastic, meaning that the objects did not stick together, we could also use conservation of kinetic energy.

$$p_i = p_f$$
start with the conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
sum the initial and final momentum of both objects

$$v_{1i} = \frac{m_1 v_{1f} + m_2 v_{2f} - m_2 v_{2i}}{m_1}$$
solve for v_{1i}

$$v_{1i} = \frac{2 \text{ kg} \cdot 1 \text{ m/s} + 3 \text{ kg} \cdot 2 \text{ m/s} - 3 \text{ kg} \cdot -3 \text{ m/s}}{2 \text{ kg}}$$
plug in known values

$$v_{1i} = 8.5 \text{ m/s}$$

Inelastic Collision

An object (m_1) of mass 2 kg is moving at 2 m/s in the positive x-direction. It collides perfectly inelastically with another object (m_2) of mass 3 kg moving at -1 m/s. How fast will the objects be moving after the collision?

Solution

This problem is very similar to the example above, except the objects collide inelastically. This means that the objects stick together after the collision. In this case, we would not be able to use conservation of kinetic energy.

$$p_i = p_f$$
start with the conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$$
there is only one term for the final momentum because the objects are stuck together

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$
solve for v_f

$$v_f = \frac{2 \text{ kg} \cdot 2 \text{ m/s} + 3 \text{ kg} \cdot -1 \text{ m/s}}{2 \text{ kg} + 3 \text{ kg}}$$
plug in known values

$$v_f = .2 \text{ m/s}$$

Notes

Circular Motion

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Physics

Big Picture

Rotating objects act differently than those in linear or projectile motion. All objects moving in a circular path have linear acceleration because they are constantly changing direction. All of Newton's laws of motion, which normally only apply to linear motion, have rotational equivalents.

Key Terms

Centripetal Force: A centripetal force is any constant force that causes an object to rotate in a circle. A centripetal force causes a centripetal acceleration, represented as a_c. SI units: N

Angular Displacement (θ): The angle that an object has moved around a central axis. SI units: rad

Angular Velocity (ω): The rate at which an object rotates around a central axis. SI units: rad/s

Tangential Speed: The linear speed of a rotating object. Called *tangent*ial speed because the direction of motion is always *tangent* to the circular path.

Angular Acceleration (α): The rate at which an object's angular velocity is changing. SI units: rad/s^2

Moment of Inertia (I): A measurement of an object's stationary inertia with respect to rotational motion. Moment of Inertia is the angular equivalent of mass. SI units: kg•m^2

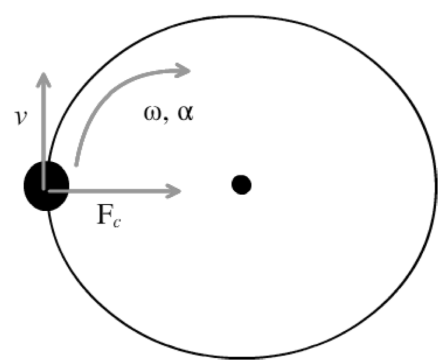
Torque (τ): A measurement of the ability for a force to rotate the object that it is acting on. Similar to how forces cause an acceleration, torques cause an angular acceleration. SI units: N•m

Angular Momentum (L): The measurement of a rotating object's inertia. Just like linear momentum, angular momentum is always conserved. SI units: kg•m^2•rad/s

Precession: The change in orientation of an object's rotational axis and is the result of conservation of momentum.

Centripetal Force

Centripetal forces are responsible for circular motion. The force vector will always be pointed toward the center of the circle and be perpendicular to the direction of the object's motion.



There are a few forces (called fictitious forces) that are associated with circular motion but do not actually exist when observing the rotating object from a stationary reference frame.

- Centrifugal force: An outward force that is wrongly believed to pull objects away from the center. When whirling a tennis ball on a string, many people believe that if the string were to break, the ball would fly away from the circle.

  In reality, it would go off in a tangential straight line because there is no force acting on it! The force felt is actually feeling the effects of Newton's third law. Since the person whirling the tennis ball exerts a centripetal force on the ball pointed towards the person, the object is also exerting an equal and opposite force on the person's hand.

Linear vs. Circular Motion

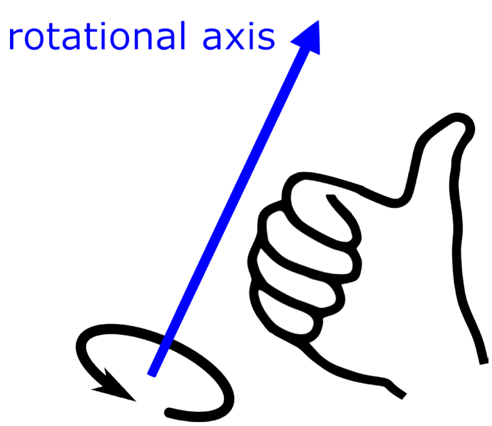


Image Credit: Wizard191, Public Domain

In circular motion, an object rotates in a circle around the rotational axis.

- To find the rotational axis, use the right-hand rule: use the fingers of your right hand to follow the direction of rotation, and your thumb will point along the axis.
- The distance between the rotational axis and the object is called the radius (r).
- To find the path length (distance an object travels along a circular path), use the geometric formula for finding arc length: $s = r\theta$, where s is the path length.
- The period T is the time it takes to complete one rotation. The frequency f is equal to $1/T$.

Circular Motion cont.

Linear vs. Circular Motion (cont.)

Circular motion is similar to linear motion in many ways.

Linear Quantity	Units	Angular Quantity	Units
displacement (**x**)	m	**angular displacement (θ)**	rad
velocity (**v**)	m/s	**angular velocity (α)**	rad/s
acceleration (**a**)	m/s^2	**angular acceleration (ω)**	rad/s^2
mass (*m*)	kg	**moment of inertia** (*I*)	kg•m^2
force (**F**)	N	**torque (τ)**	N•m
momentum (**p**)	kg•m/s	**angular momentum (L)**	kg•m^2/s

- Angular velocity and torque have two directions: clockwise and counterclockwise.
- Only the component of torque perpendicular to the radius of rotation will cause an object to accelerate.
- Angular acceleration is not the same as centripetal acceleration! Centripetal acceleration always points toward the rotation axis. Angular acceleration points in or against the direction of angular velocity.
- Angular velocity is related to period and frequency: $\omega = \frac{2\pi}{T} = 2\pi f$

Using angular quantities, we can rewrite the equations for linear motion.

- $KE_{rot} = \frac{1}{2}Iw^2$
- For constant α,
  - $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$
  - $\omega = \omega_o t + \alpha t$
  - $\omega^2 = \omega_o^2 + 2\alpha(\Delta\theta)$

Precession

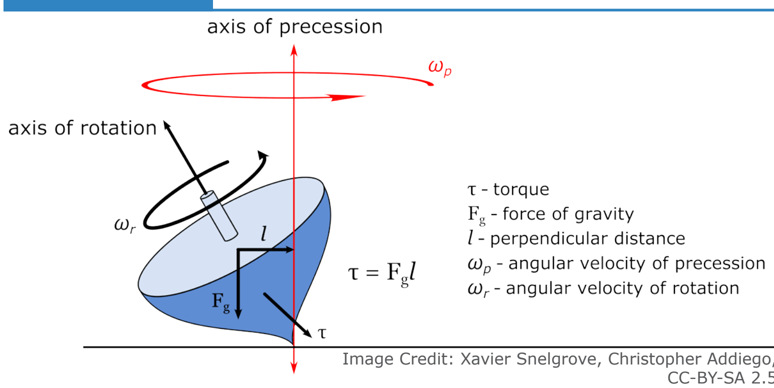


Image Credit: Xavier Snelgrove, Christopher Addiego, CC-BY-SA 2.5

Precession is most commonly viewed in the motion of a spinning top. When the top's center of gravity is not directly over the bottom tip of the top, gravity exerts a torque on the spinning top. Since the top's axis itself initially has zero angular momentum, the whole axis will rotate to counter the torque from gravity.

Notes

Circular Motion Problem Guide

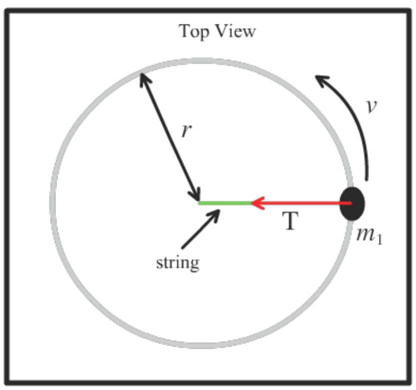
Important Equations

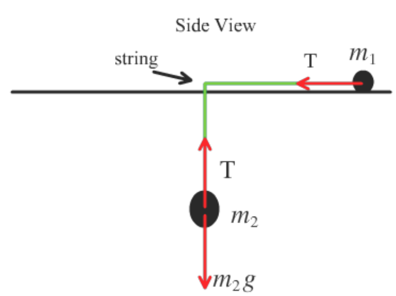
$F_c = m\dfrac{v^2}{r}$	F_c - centripetal force	m - mass	v - velocity	r - radius
$a_c = \dfrac{v^2}{r}$	a_c - centripetal acceleration			
$s = r\theta$	s - path length	θ - path length		
$\omega = \dfrac{v}{r}$	ω - angular velocity			
$\alpha = \dfrac{a}{r}$	α - angular acceleration			
$\tau = F_{\perp} r = Fr\sin\phi$	τ - torque	$F_{\perp}$ - force perpendicular to lever arm	Φ - angle between force vector and lever arm	
$\tau_{net} = I\alpha$	I - moment of inertia			
$L = Iw$	L - angular momentum			
$I = mr^2$	Note: this formula is for a particle of mass m at distance r from the axis of rotation			

To convert between degrees and radians, use the conversion factor: $1 \text{ rad} = \dfrac{180°}{\pi}$

Example Problem

A mass (m_1) is spinning horizontally on a frictionless table secured by a string of negligible mass. The string goes through a hole in the center of the table and is attached to another hanging object of unknown mass (we'll call it m_2). The radius of m_1's rotation is r, and m_1 is rotating with speed v. Find the mass of the object hanging under the table.





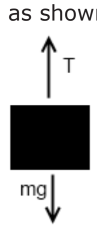
Solution

In this problem, the tension in the string is acting as a centripetal force to make the mass on top of the table rotate. Since the string is also attached to the hanging mass, the tension is also equal to the weight of the unknown mass. We can use these two pieces of information to find the mass of the hanging object.

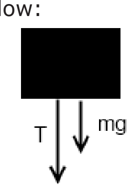
$m_2 g = T$	set the weight of the unknown object equal to the tension (T) in the string
$m_2 g = F_c$	since T is also providing the centripetal force, we can substitute F_c for T
$m_2 g = \dfrac{m_1 v^2}{r}$	substitute in the known values for F_c
$m_2 = \dfrac{m_1 v^2}{rg}$	solve for m_2

Mass on a String

At the bottom of its path, the free body diagram for a mass on a string is as shown below:



At the top of its path, the free body diagram for the mass is as shown below:



At the minimum speed needed for the mass to make it over, at the top of the path, the tension in the string will be 0.

Circular Motion Problem Guide cont.

Mass on a String (cont.)

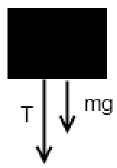
Example

A 5 g mass at rest is attached to the end of a 1 meter long string. At the top of the path, what speed will the mass be moving at if it is to just barely make it over?

Solution

Given: m = 5 g, l = 1 m

Find: v_{min}



$$F = \frac{mv_{min}^2}{r}$$

$F = T + mg$ because we're looking for the minimum speed, $T = 0$

$mg = \frac{mv_{min}^2}{l}$ set mg equal to the centripetal force

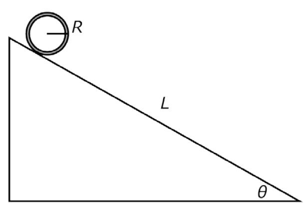
$v_{min} = \sqrt{lg} \approx 3.1$ m/s solve for v_{min}

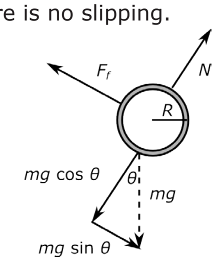
Rolling Bodies

If an object rolls down a ramp, there must be friction. Without friction, the object would slip down the ramp without rotating. Because friction causes the object to roll, friction is the force we plug in when we use the torque equation.

Example

A hoop of mass M, radius R, and moment of inertia of MR^2 is released from the top of a ramp of angle θ. What is the acceleration of the center of mass of the hoop? Assume there is no slipping.





Solution

Given: $m = M$, $r = R$, $I = MR^2$, let μ be the coefficient of static friction

Find: a

$\tau = I\alpha = MR^2(\frac{a}{R})$ and $\tau = F_\perp r = F_f R$ friction is the force providing torque

$MR^2(\frac{a}{R}) = F_f R$ set the two expressions for torque equal to each other

$$F_f = \mu N = \mu Mg\cos\theta$$

$$F_{net} = Ma = Mg\sin\theta - \mu Mg\cos\theta$$

$$a = g\sin\theta - \mu Mg\cos\theta$$

Pulleys

In a pulley system, tension causes the pulley to rotate, which means tension is the force we plug in for torque. If there are two blocks attached to a single string, we know that they will have the same acceleration because they are in the same system. Also, if they are of different mass, they will have different free body diagrams, so draw one for each!

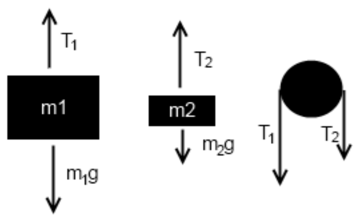
Example

A pulley of radius R is connected to a string with 2 blocks of different mass on both side. One block is 3 g and the other is 6 g. Find the rotational inertia of the pulley.

Solution

Given: m_1 = 6 g, m_2 = 3 g, $r = R$

Find: I



Since block 1 is 6 g and block 2 is 3 g, we can conclude that block 1 will fall in this system while block 2 rises.

$\tau = I\alpha = I\frac{a}{R}$ and $\tau = F_\perp R$

$$F_\perp = T_1 - T_2$$

$$I\frac{a}{R} = (T_1 - T_2)R$$

$$I = \frac{(T_1 - T_2)R^2}{a}$$

To find T_1 and T_2, use: $m_1 a = m_1 g - T_1$ because mass 1 is going downwards

$m_2 a = T_2 - m_2 g$ because mass 2 is going upwards

GRAVITATION

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Physics

Big Picture

Gravity is one of the four fundamental forces of our universe. Although it is the weakest of the forces, it is the one that people have the most experience with. The equations and concepts below explain gravity in a way that is consistent with classical mechanics, but the theory of general relativity is required for a complete explanation of our modern understanding of gravity.

Key Terms

Law of Universal Gravitation: Any two objects in the universe are attracted to each other by a force proportional to the masses of the two objects and inversely proportional to the square of the distance between the center of mass of each object. This law determines the force of gravity on all objects at any point in space. SI units: N

Gravitational Field: A force field that surrounds massive objects with acceleration vectors that pull smaller objects towards itself. The magnitude of the vector is equal to the acceleration of the object due to gravity at that distance from the object. SI units: m/s^2

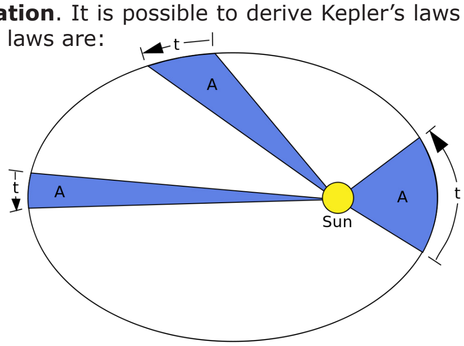
Field: Something that helps us keep track of forces.

Satellite: An object orbiting another object.

Kepler's Laws of Planetary Motion

Kepler came up with three laws of motion that describe the movements of the planets based on his observations of the solar system before Newton developed the **law of universal gravitation**. It is possible to derive Kepler's laws from the law of universal gravitation and Newton's laws of motion. Kepler's laws are:

1. Planets orbit in an ellipse with the sun at one of the foci.
2. For a given period of time, the angular displacement for a planet will always be equal. To the right is a diagram illustrating this law.
3. The square of the orbital period is proportional to the cube of half the length of the major axis of the planet's orbit.



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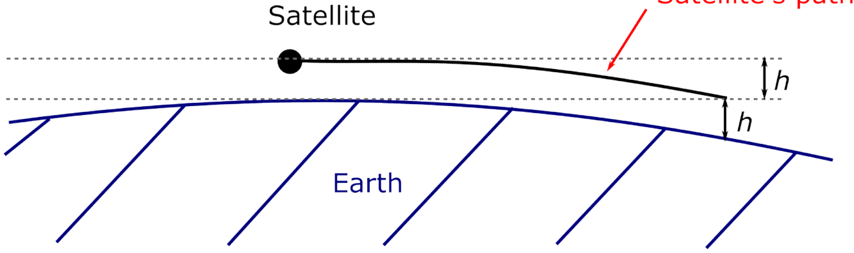
Satellites

In physics, anything orbiting another body is considered a **satellite**. For example, the Moon is considered to be Earth's satellite even though it is not referred to as a "satellite."

Satellites are able to stay in space despite being pulled down by gravity because of their horizontal velocity. Newton explains this concept by using an example: if we throw a ball horizontally, it will gradually fall to the ground. If we throw the ball much faster, it will travel for a longer time before hitting the ground because we threw it faster and because the Earth's surface curves away from the ball as it is pulled down by gravity. Hypothetically, if we throw the ball fast enough, the distance that the Earth's surface curves away from the ball will equal the distance the ball falls. Without air resistance, the ball would continue to fly around the earth forever.



The speeds necessary for satellite motion are extremely high. For example, the International Space Station has an average linear speed of about 17,200 mph (~7,700 m/s). This is why all satellites exist outside the Earth's atmosphere, where there is no air resistance to slow them down or to generate heat that would set the satellite on fire.

Important Equations

Law of Universal Gravitation: $F_g = \frac{Gm_1 m_2}{r^2}$

F_g - force of gravity

m - mass

G - gravitational constant $\approx 6.674 \times 10^{-11}$ N (m/kg)^2

r - distance between objects

Near Earth's surface, we can approximate: $g = \frac{Gm_{Earth}}{r^2_{Earth}} = 9.8$ m/s^2

This guide was created by Christopher Addiego and Catherine Wu. To learn more about the student authors, visit http://www.ck12.org/about/about-us/team/interns.

Energy

Physics

Big Picture

Energy is the measure of an object's ability to do work (exert a force over a distance) on another system or object. Energy can never be created or destroyed; it can only be transformed into a different type of energy. When somebody says energy has been lost, the person really means that the energy was turned into some form that is unusable. The conservation of energy is one of the fundamental laws of our universe.

Key Terms

Potential Energy: The potential energy of an object/ system is the energy that is stored and ready to be used in the object/ system. SI units: J

System: Any object or group of objects that we are studying. A system where energy is conserved is called a closed system.

Kinetic Energy: A measurement of the energy associated with an object in motion. Kinetic energy can also refer to energy associated with the vibration or motion of atoms or molecules. SI units: J

Work: If a force is exerted over a distance to move an object, the force is said to be doing work. SI unit: J

Conservative Forces: A force in which the work done to move a particle between two points is not affected by the path taken.

Non-Conservative Forces: A force in which the work done to move a particle between two points is affected by the path taken.

Power: The rate at which work is done by a system or object. SI units: J/s or W

Forms of Energy

There are many different types of **potential energy** that are differentiated by the way that the energy is stored.

- Gravitational potential energy is based on an object's position relative to some reference point, and chemical potential energy is stored in the chemical bonds between atoms.
- Gravitational potential energy is the *potential* for an object/**system** to move due to the force of gravity.
- Near the surface of the planet, gravitational potential energy $U_g = mgh$, m = mass, g = acceleration due to gravity, h = height. For Earth, $g = 9.8\ \text{m/s}^2$.

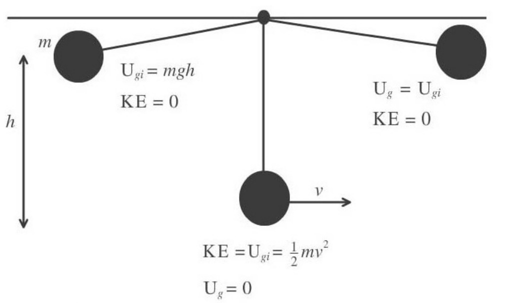
All moving objects have **kinetic energy**. $KE = \frac{1}{2}mv^2$

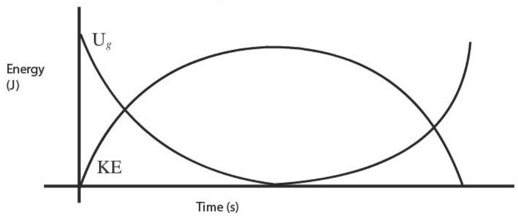
- Because the velocity v is squared in the formula for kinetic energy, faster objects have much more kinetic energy than slower ones.

Energy Diagrams

Energy diagrams allow us to visualize how energy is being transferred.

Below is a diagram for a swinging pendulum that shows how energy changes forms from gravitational potential energy to kinetic energy.





Work

In the work-energy theorem, the **work** done on an object is always equal to the change in kinetic energy of the object.

- No work is done if there is no motion or if the applied force is perpendicular to the direction of motion.
- If the force is a **conservative force**, the work done to move an object depends only on the displacement, not the total distance traveled. The work done on an object is equal to the displacement multiplied by the *component* of the force along the direction of motion.

For example, gravity is a conservative force. When an object's gravitational potential energy changes, only the object's displacement determines the work done. The diagram on the right shows how the change in energy is the same whether the red ball takes the green path or the blue path. Gravity does work *mgh* along either paths.

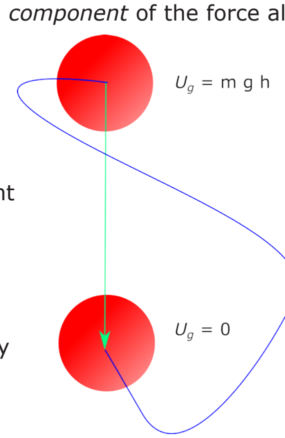


Image Credit: CompuChip, Public Domain

- Friction is an example of a **non-conservative force** because the work done actually depends on the total distance the object travels.
- **Power** is the rate work is done.

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Energy Problem Guide

Important Equations

$E_i = E_f$	E_i - initial energy	E_f - final energy		
$U_g = mgh$	U_g - gravitational potential energy	m - mass	g - acceleration due to gravity	h - height
$KE = \frac{1}{2}mv^2$	KE - kinetic energy	v - velocity		
$W = Fd = \Delta KE$	W - work	F - force	d - distance	
$P = \frac{\Delta W}{\Delta t}$	P - power	t - time		

Example

The law of conservation of energy tells us that energy is not created or destroyed. Instead, energy is transformed from one form to another. We can use the conservation of energy to help us solve many types of physics problems.

Tip: When solving problems with gravitational potential energy, pick a reference point that will make calculations easier. The reference point does not have to be the ground.

- In these types of problems, we usually ignore non-conservative forces such as friction and air resistance. If we cannot ignore non-conservative forces, some of the energy in a system will be "lost." (In fact, the energy is not really lost - it is transferred to another system, usually by heat or sound.)

Example

An object of mass m begins at rest and starts falling through the air. After falling a distance h, how fast is the object going? Air resistance is negligible.

Solution

It is possible to solve this using the equations for linear motion, but it is easier to solve by using the conservation of energy. We can set the point where the potential energy $PE = 0$ at distance h below where the ball started.

$PE_i + KE_i = PE_f + KE_f$	begin with an equation showing energy is conserved
$PE_i + 0 = 0 + KE_f$	substitute 0 for the initial kinetic energy and the final potential energy
$mgh = \frac{1}{2}mv^2$	substitute in the known values
$v = \sqrt{2gh}$	solve for v

Notes

ELECTROSTATICS

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Physics

Big Picture

All objects have positive and negative charges inside them. If the number of positive and negative charges are equal, as they most often are, then the object is neutral. Charged objects are objects with more positive charges than negative ones, or vice versa. Opposite charges attract, and similar charges repel. Electric fields are created by a net charge and point away from positive charges and towards negative charges. Many macroscopic forces can be attributed to the electrostatic forces between molecules and atoms.

Key Terms

Charge: Charge is carried by protons and electrons. Charge is always conserved in a closed system. SI unit: C

Coulomb's Law: Coulomb's law states that the force between two charges is proportional to the value of the two charges and inversely proportional to the square of the distance between them.

Electric Field: Electric fields surround electrically charged particles and affect other electrically charged particles. The field itself is a vector force field - it indicates the direction a positive charge at a given point would move. The electric field can also be thought of as the force per unit charge at a given point, similar to a gravitational field. SI units: N/C = V/m

Conductors: Materials where electricity can pass through easily, meaning that electrons can easily move inside the material.

Insulators: Materials where electricity cannot pass through easily.

Semiconductors: Materials with a conductivity between the conductivities of insulators and normal conductors.

Superconductors: Materials with exactly zero resistance below certain temperatures.

Plasma: A conductive state of matter, similar to gas, containing ions and/or free electrons.

Voltage: Potential energy measured as a difference in potential between two points in space. SI units: V

Equipotential Lines: Lines linking points of equal voltage around a point charge. Around point charges, equipotential lines just form circles around the charge. In more complex arrangements, equipotential lines can make all sorts of shapes. Equipotential surfaces are the same as equipotential lines, except in 3-dimensional space.

Coulomb's Law

There are two types of **charge**: positive and negative.

- Electrons have negative charge.
- Protons have positive charge.
- The magnitude of the charge is the same for electrons and protons: $e = 1.6 \times 10^{-19}$ C

Coulomb's law is used to calculate the force between two charged particles:

$F = \frac{kq_1q_2}{r^2}$, where k is a constant, q is the charge, and r is the distance between the charged particles.

The force can be attractive or repulsive depending on the charges:

- Like charges (charges with the same sign) repel
- Charges with opposite signs attract

Electric Field

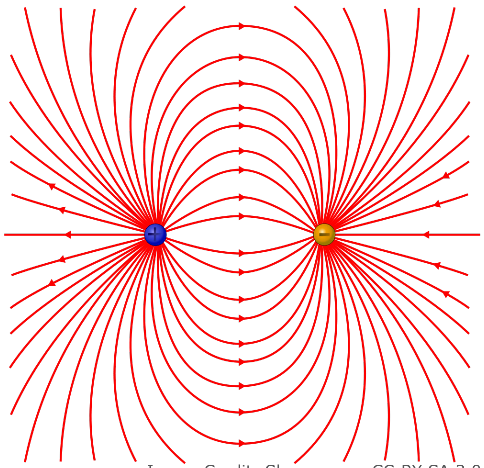
Electric force can be represented by an **electric field**.

The field E due to a single point charge q is: $E = \frac{kq}{r^2}$.

The field describes the force a charged object will feel if it enters the field. If another point charge q_0 is in the field, it will feel a force $F = Eq_0$.

A simple diagram can tell us a lot about an electric field - the density of the field lines in a region indicates the strength of the field, and parallel lines (like between two charged plates) mean that the field is constant.

Electric fields can interact with each other just like gravitational fields. To determine the direction and magnitude of the electric field at any point, just add the vectors from all of the different electric fields at that point. To the right is an illustration of the electric field between a positive charge and a negative charge. The lines show the direction a positive charge placed in the field will take.



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Physics

Electrostatics cont.

Gauss's Law

Gauss's law can be used to find the electric field at any point around a charge-carrying object. Coulomb's law can be derived by applying Gauss's Law to a point charge. When applied to relatively simple problems, Gauss's law essentially states that the electric field at a certain distance from a charged object is proportional to enclosed charge and inversely proportional to the area of the imaginary surface in 3-dimensional space that the electric field is passing through. This is an oversimplification of Gauss's law. To truly understand how powerful the Gauss's law is, you must understand a considerable amount of calculus. If you do know some calculus, the physics *Gauss's Law* study guide is devoted to basic Gauss's law problems.

Conductive Materials

Materials can be classified as **conductors** and **insulators**.

- Metals are generally good conductors because the electrons are loosely bound to the individual atoms. There are two other classes of conductors:
  - **Semiconductors**: Used in almost all modern electronics, semiconductors are the key component in transistors. Silicon is the most common semiconducting material.
  - **Superconductors**: In all normal conductors, there is some resistance to the flow of electrons present. However, when some materials are cooled to very low temperatures, they will exhibit exactly zero resistance. This is called superconductivity. Superconductivity is an phenomena that can only be understood through quantum mechanics.

Charging Methods

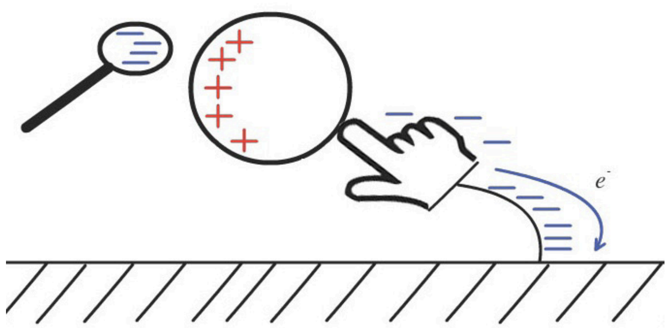
There are two ways to charge an object: by conduction or by induction.

Conduction is when we touch a charged object to a neutral object and the charges evenly distribute.

Charging by induction is when we charge an object without touching it. There are many methods for charging objects by induction, but here is one process for charging a single object by induction.

1. First touch one finger to the neutral object to ground the object.
2. Then, bring a charged object (we'll assume it's negatively charged, but it can be either) close to the neutral object. This causes negative charges in the neutral object to be repelled through your body to the ground.
3. When the finger is removed, the neutral object will be positively charged. When charging by induction, the originally neutral object will always end up with the opposite charge.

Below is a diagram illustrating this process. When charge moves, electrons are always the ones that move. Protons cannot move between atoms.



Notes

Electrostatics Problem Guide

Physics

Important Equations

Note: all equations involving a charge q are only valid when q is a point charge. Calculus is required to for problems involving continuous charge distribution. Also, the value $\frac{1}{4\pi\epsilon_o}$ is often written as a constant k. However, if you pursue physics beyond an introductory level, you will learn that $\frac{1}{4\pi\epsilon_o}$ is more than just a constant and carries greater significance.

$$F = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}$$

$$F = Eq = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}$$

$$V = Er = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$$

$$U_e = qV = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r}$$

F - force

E - electric field

V - voltage

U_e - electric potential energy

ε_0 - permittivity of free space

q - charge

d - distance

Example Problems

Aside from basic electrostatics problems that involve plugging in values and basic algebra, most problems encountered will combine electrostatics with some other area in physics. The overall strategy is to use your equations as a road map. Start with the known values and use different equations to make connections until you get to your answer.

Example 1

One positively charged particle of charge $+q$ and mass m_1 is orbiting around a fixed, negatively charged particle of charge $-q$ at a distance d. Determine the linear velocity of the moving particle.

Solution

In this problem, the electrostatic force (determined by Coulomb's law) acting between the particles is also the centripetal force keeping the positive charge in its orbit because the two particles are attracting each other. We can use this knowledge to solve for the linear velocity of the positive charge.

Tip: The negative sign on the second charge has been dropped because it is easier to do the calculations using only the absolute value of the charge. We can determine the direction of the force later on.

set centripetal force equal to Coulomb's law
$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_o}\frac{q_1 q_2}{r^2}$$

substitute in the given values
$$\frac{m_1 v^2}{d} = \frac{1}{4\pi\epsilon_o}\frac{q^2}{d^2}$$

solve for v
$$v = \sqrt{\frac{1}{4\pi\epsilon_o}\frac{q^2}{d}\frac{1}{m_1}}$$

Example 2

A charged particle with some initial velocity (v_0), charge ($+q$), and mass (m) is passing between two oppositely charged plates with a voltage difference (V) applied across them. The particle starts outside the plates, but let's assume that the plates extend infinity beyond where the particle starts. A diagram is provided below. If the particle starts halfway between the plates, how far will it travel before hitting one of the plates?

Solution

First we want to find the electric field between the plates. We know the field is constant between the plates, so we use:

$$V = Er \longrightarrow E = \frac{V}{r} = \frac{V}{d}$$

We can now apply this to finding the vertical acceleration of the particle by applying $F = Eq = ma$.

$$Eq = ma \longrightarrow \frac{V}{d}q = ma \longrightarrow a = \frac{Vq}{md}$$

Now we can use the equations of kinematics to find out how long it takes for the charge to hit the plate.

$$\Delta y = \frac{1}{2}at^2 \longrightarrow \frac{d}{2} = \frac{1}{2}at^2 \longrightarrow t = \sqrt{\frac{d}{a}} \longrightarrow t = \sqrt{\frac{md^2}{Vq}}$$

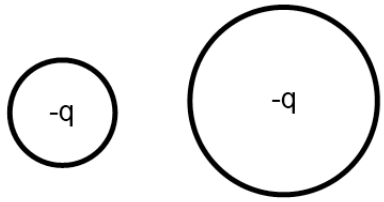
Now we can apply $\Delta x = vt$ to find how far the charge will travel.

$$\Delta x = vt \longrightarrow \Delta x = v_0\sqrt{\frac{md^2}{Vq}}$$

Electrostatics Problem Guide cont.

Charged Spheres

Whenever there are conducting metal spheres, remember that the charges are free to move within the object. Because like charges repel each other, charges tend to spread out - the charge of conducting objects is always at the surface.



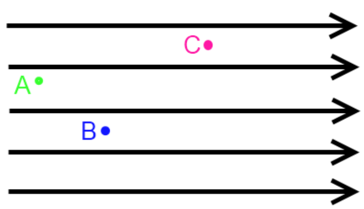
If the two conducting spheres of different radii (with the same charge -q) are connected by a metal wire, the charge will flow until the electric potential of both spheres is equal.

Equation to use: $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

Using this equation, we can see that the charge will flow from the small sphere to the large sphere - the negative charge will flow towards the higher potential (in this case, they're both negative, but the larger sphere's potential is less negative).

On the other hand, charges are not free to move in non-conducting objects. Therefore, there is an even distribution of the charge density throughout the object.

Electric Potential Energy



Equation to use: $U_e = qV$

If points A, B, and C represent positrons in an electric field, the field arrows represent the direction the positron will move in. Like in gravitational potential energy, where distance from the ground indicated how much potential energy an object had, the positron with the greatest distance to travel has the highest electric potential energy (in this case position A).

Keep in mind that electric potential energy (much like gravitational potential energy) does not depend on the path the object takes, rather the total distance moved!

Notes

Electrical Systems

CK-12 Study Guides

MADE BY STUDENTS!

Physics

Big Picture

Electrical systems involve the flow of electric charge through circuits of conducting material, usually a wire or metal cable. If there is a difference in voltage across a conductor, electrons will flow from the high voltage to low voltage creating a current. Current can be used to transfer energy between two points along a wire; this is the basis of all electronic devices. Unlike electrostatics which is mainly concerned with the activity of individual charges, electrical systems involve the movement of many electrons and the combined effects of the moving charges.

Key Terms

Voltage Source: A power source that provides a fixed voltage, usually a battery. While the voltage source has a certain amount of voltage available (also known as **emf**, or electromotive force), it tends to have some resistance, meaning the actual voltage (called the terminal voltage) it provides is less than the emf. SI unit: V

Voltage causes currents - think of voltage as how much force is pushing an electric current!

Voltage: Potential energy, measured as a difference between potential at two points in space. SI units: V

Voltmeter: Measures the potential difference (voltage) between two points in a circuit.

Current: The flow of electric charges through a wire. SI units: A

Ammeter: Measures current.

Resistance: The amount a device opposes the flow of a current. SI units: Ω

Capacitance: A device's capacity to store charge. SI units: F

Ohm's Law: The current between two points through a conductor is directly proportional to the voltage between those two points and inversely proportional to the resistance between those two points.

Power: The concept of power in electrical systems is exactly the same as in mechanics: the rate of energy transfer. However, power in electrical systems is calculated with different formulas. SI units: J

DC Circuits: Also known as direct current circuits, electrons in DC circuits are constantly moving along a wire towards the positive charge.

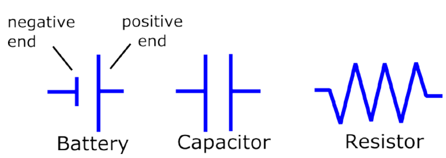
RC Circuits: Circuit comprised of resistors and capacitors connected to a voltage source.

Can you guess what the R and C in RC circuit stand for now?

Circuits

Circuits include a **voltage source** (usually a battery) and a conducting wire connecting opposite ends of the voltage source, providing a closed loop for charge to flow (**DC circuits**). Some other elements that may be included in a circuit are resistors and capacitors.

In circuit diagrams, we use these symbols:



When working with circuit diagrams, we are usually calculating one of the following:

- **voltage**
- **current**
- **resistance**
- **capacitance** (if capacitors are included)

Ohm's law relates current (I), voltage (V), and resistance (R): $V = IR$.

Two other important rules are Kirchoff's laws:

- *Kirchoff's Law for Voltage* (also known as the Loop Rule): The sum of all the potential differences in a given loop is equal to 0.
- *Kirchoff's Law for Current* (also known as the Junction Rule): At any junction, the sum of the current flowing in equals the sum of the current flowing out.

Resistors

Resistors are devices that resist the flow of current and are used to control the current flowing through a circuit. The **power** that a battery provides can be calculated by $P = IV$. This power flows through the circuit until it hits a resistor, which dissipates some energy. This energy goes into heating the resistor.

Resistance is low in a conductor and high in an insulator.

This guide was created by Christopher Addiego and Catherine Wu. To learn more about the student authors, visit http://www.ck12.org/about/about-us/team/interns.

Electrical Systems cont.

Resistors (cont.)

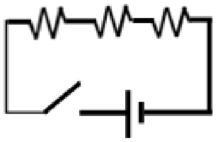
Resistors in Series

The current passes through each resistor and the total voltage drop across the resistors is equal to their sum. The total resistance is equal to the sum of the resistance of each resistor.

$I_{total} = I_1 = I_2 = I_3 = \ldots$

$V_{total} = V_1 + V_2 + V_3 + \ldots$

$R_{total} = R_1 + R_2 + R_3 + \ldots$



If the current splits up (and can't go through both resistors), then the resistors are in parallel.

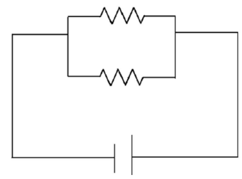
Resistors in Parallel

The current is split up between the resistors, so the current is equal to the sum of the current through each resistor. The voltage drop across each resistor is the same. The reciprocal of the total resistance is equal to the sum of the reciprocal of each resistor's resistance.

$I_{total} = I_1 + I_2 + I_3 + \ldots$

$V_{total} = V_1 = V_2 = V_3 = \ldots$

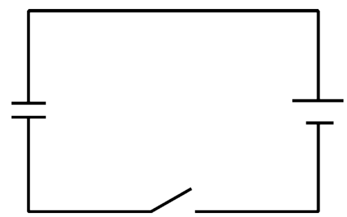
$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$



Capacitors

Capacitors are devices that store energy. When the switch is closed, the voltage source forces electrons from one plate to the other, creating equal and opposite charges on the two plates. This creates a potential difference that increases until the potential difference is equal to the emf of the voltage source. At this point, the voltage source can no longer add charges to the plates. Thus, the flow of current stops and it is as if the switch had been opened again!

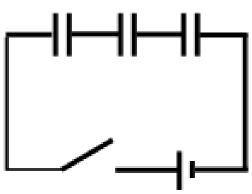
A large capacitance means more charge can be stored. Capacitance depends on the area of the plates, the distance between plates, and the dielectric constant (varies for different materials between the plates).



Capacitors in Series

The total capacitance is equal to the sum of the reciprocal of each capacitor's capacitance.

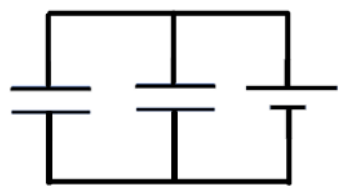
$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$



Capacitors in Parallel

The total capacitance is equal to the sum of the capacitance of each capacitor.

$C_{total} = C_1 + C_2 + C_3 + \ldots$



*Since resistor has the letter "s", R_{total} of resistors in a series equals the **s**um of all the resistances.*

*Since capacitor has the letter "p", C_{total} of capacitors in **p**arallel equals the sum of the capacitance.*

Transistors

Transistors are semiconductors that are used to amplify and switch electrical signals. They have three or more terminals that can connect to outside circuits. When a voltage or current passes through one terminal, the current flowing through the other terminals changes. Oftentimes, the output power is significantly greater than the input making transistors very effective signal amplifiers.

RC Circuits

An **RC circuit** is a circuit comprised of resistors, capacitors, and a voltage source. Initially, the capacitor is uncharged and the potential difference across the resistor is the emf. When the switch is closed, the current and the potential difference across the resistor start to decrease as charge accumulates on the capacitor.

At time = 0, the capacitor is uncharged so it acts like a wire.

$I = \frac{emf}{R}$ $\quad$ I - initial/maximum current

As time $\rightarrow \infty$, the capacitor is fully charged and there is no current running through the capacitor, making the system resemble an open circuit.

Electrical System Problem Guide

Important Equations

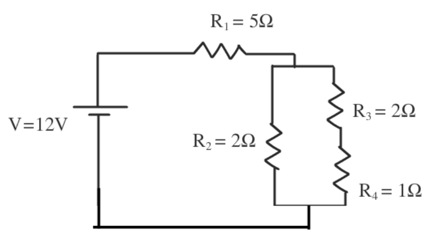
$V = IR$	V - voltage	I - current	R - resistance
$P = IV = I^2R = \frac{V^2}{R}$	P - power		
$C = \frac{Q}{V}$	C - capacitance	Q - charge	
$C = \epsilon_0 \frac{A}{d}$	ε_0 - dielectric constant	A - area of plate	d - distance between plates
$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = Pt$	E - energy		
$I = \frac{\Delta Q}{\Delta t}$			

Example Problems

The general strategy for any circuit problem is to first find the total equivalent resistance or capacitance, depending on what kind of circuit you are dealing with. Using that, we can find out whatever information on the circuit as a whole that has been left out. Then, work your way back down using Ohm's law in in order to find the characteristics of smaller sections of the circuit until you get to your answer. When finding total resistance or capacitance, it is usually easier to work starting with the most buried components. The problem below will illustrate this concept.

Example 1

In the circuit below, find the voltage drop and the power dissipated by the resistor labeled R_2.



Solution

Before we can even think about the individual resistors, we need to find out how much current is running through the whole circuit. So, to do that, we will first need to find the total resistance. We'll start by summing the resistance of resistors 3 and 4.

$$R_{3,4} = R_3 + R_4 = 2\Omega + 1\Omega = 3\Omega$$

Next we'll combine $R_{3,4}$ with R_2.

$$\frac{1}{R_{2,3,4}} = \frac{1}{R_2} + \frac{1}{R_{3,4}} = \frac{1}{2\Omega} + \frac{1}{3\Omega}$$

$$\frac{1}{R_{2,3,4}} = \frac{5}{6}\Omega \longrightarrow R_{2,3,4} = \frac{6}{5}\Omega$$

Finally we can find R_t by combining $R_{2,3,4}$ with R_1.

$$R_t = R_1 + R_{2,3,4} = 5\Omega + \frac{6}{5}\Omega = 6.2\Omega$$

Note that the three steps above can be combined into one expression: $R_t = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3+R_4}}$

Now, we can find the amount of current that runs through the whole circuit using Ohm's Law.

$$V = IR_t \longrightarrow I = \frac{V}{R_t} = \frac{12V}{6.2\Omega} = 1.94A$$

Now, since R_1 and the group $R_{2,3,5}$ are in series, we know that the same amount of current is going through both of these resistors. So, we can use Ohm's Law to find the voltage drop across $R_{2,3,4}$.

$$V_{2,3,4} = IR_{2,3,4} = 1.94A \cdot 1.2\Omega = 2.33V$$

Since, R_2 and the group $R_{3,4}$ are in parallel, we know that the voltage applied across them is equal. So, know the answer to the first part of the problem is $V_2 = 2.31$ V. We can use this information to find the current through the resistor and then the power it dissipates.

$$V_2 = I_2R_2 \longrightarrow I_2 = \frac{V_2}{R_2} = \frac{2.33V}{2\Omega} = 1.17A$$

Now we can find the power the resistor dissipates.

$$P = I_2^2R_2 = (1.17A)^2 \cdot 2\Omega = 2.7W$$