Total Physics

Total Physics

Peter MacDonald

AUTHOR
Peter MacDonald

To access a customizable version of this book, as well as other interactive content, visit www.ck12.org

CK-12 Foundation is a non-profit organization with a mission to reduce the cost of textbook materials for the K-12 market both in the U.S. and worldwide. Using an open-content, web-based collaborative model termed the **FlexBook®**, CK-12 intends to pioneer the generation and distribution of high-quality educational content that will serve both as core text as well as provide an adaptive environment for learning, powered through the **FlexBook Platform®**.

Printed: August 30, 2012

Contents

CHAPTER 1

Introduction to Physics

Chapter Outline

Adobe Flash

MEDIA

Click image to the left for more content.

Physics happens every day and at every moment. This is a 240 fps super slow motion video taken for a grade 10 science class.

1.1 Problem-Solving Models

Here you'll be exposed to many different methods that you can use to solve a problem and the way that these methods should fit into your overall problem-solving plan.

Suppose you're taking a standardized test to get into college and you encounter a type of problem that you've never seen before. What tools could you use to help solve the problem? Is there anything you should do before trying to solve the problem? Is there anything you should do afterwards? In this Concept, you'll be presented with a step-by-step guide to problem solving and some strategies that you can use to solve any problem.

Guidance

A Problem-Solving Plan

Much of mathematics applies to real-world situations. To think critically and to problem solve are mathematical abilities. Although these capabilities may be the most challenging, they are also the most rewarding.

To be successful in applying mathematics in real-life situations, you must have a "toolbox" of strategies to assist you. Many algebra lessons are devoted to filling this toolbox so you become a better problem solver and can tackle mathematics in the real world.

Step #1: Read and Understand the Given Problem

Every problem you encounter gives you clues needed to solve it successfully. Here is a checklist you can use to help you understand the problem.

$\surd$ Read the problem carefully. Make sure you read all the sentences. Many mistakes have been made by failing to fully read the situation.

$\surd$ Underline or highlight key words. These include mathematical operations such as *sum, difference*, and *product,* and mathematical verbs such as *equal, more than, less than*, and *is.* Key words also include the nouns the situation is describing, such as *time, distance, people,* etc.

$\surd$ Ask yourself if you have seen a problem like this before. Even though the nouns and verbs may be different, the general situation may be similar to something else you've seen.

$\surd$ What are you being asked to do? What is the question you are supposed to answer?

$\surd$ What facts are you given? These typically include numbers or other pieces of information.

Once you have discovered what the problem is about, the next step is to declare what variables will represent the nouns in the problem. Remember to use letters that make sense!

Step #2: Make a Plan to Solve the Problem

The next step in problem-solving is to **make a plan** or **develop a strategy.** How can the information you know assist you in figuring out the unknown quantities?

Here are some common strategies that you will learn.

- Drawing a diagram
- Making a table
- Looking for a pattern
- Using guess and check
- Working backwards
- Using a formula
- Reading and making graphs
- Writing equations
- Using linear models
- Using dimensional analysis
- Using the right type of function for the situation

In most problems, you will use a combination of strategies. For example, drawing a diagram and looking for patterns are good strategies for most problems. Also, making a table and drawing a graph are often used together. The "writing an equation" strategy is the one you will work with the most frequently in your study of algebra.

Step #3: Solve the Problem and Check the Results

Once you develop a plan, you can use it to **solve the problem.**

The last step in solving any problem should always be to **check and interpret** the answer. Here are some questions to help you to do that.

- Does the answer make sense?
- If you substitute the solution into the original problem, does it make the sentence true?
- Can you use another method to arrive at the same answer?

Step #4: Compare Alternative Approaches

Sometimes a certain problem is best solved by using a specific method. Most of the time, however, it can be solved by using several different strategies. When you are familiar with all of the problem-solving strategies, it is up to you to choose the methods that you are most comfortable with and that make sense to you. In this book, we will often use more than one method to solve a problem. This way we can demonstrate the strengths and weaknesses of different strategies when applied to different types of problems.

Regardless of the strategy you are using, you should always implement the problem-solving plan when you are solving word problems. Here is a summary of the problem-solving plan.

Step 1: Understand the problem.

Step 2: Devise a plan – Translate. Come up with a way to solve the problem. Set up an equation, draw a diagram, make a chart, or construct a table as a start to begin your problem-solving plan.

Step 3: Carry out the plan – Solve.

Step 4: Check and Interpret: Check to see if you have used all your information. Then look to see if the answer makes sense.

Solve Real-World Problems Using a Plan

Example A

Jeff is 10 years old. His younger brother, Ben, is 4 years old. How old will Jeff be when he is twice as old as Ben?

Solution: Begin by understanding the problem. Highlight the key words.

Jeff is **10** years old. His younger brother, **Ben**, is **4** years old. **How old** will Jeff be **when he is twice as old as Ben**?

The question we need to answer is. "What is Jeff's age when he is twice as old as Ben?"

You could guess and check, use a formula, make a table, or look for a pattern.

The key is "twice as old." This clue means two times, or double Ben's age. Begin by doubling possible ages. Let's look for a pattern.

$4 \times 2 = 8$. Jeff is already older than 8.

$5 \times 2 = 10$. This doesn't make sense because Jeff is already 10.

$6 \times 2 = 12$. In two years, Jeff will be 12 and Ben will be 6. Jeff will be twice as old.

Jeff will be 12 years old when he is twice as old as Ben.

Example B

Another way to solve the problem above is to write an algebraic equation.

Solution:

Let x be the age of Ben. We want to know when Jeff will be twice as old as Ben, which can be expressed as $2x$. We also know that since Jeff is 10 and Ben is 4, that Jeff is 6 years older than Ben. Jeff's age can be expressed as $x+6$. We want to know when Jeff's age will be twice Ben's age, so putting these together, we get

$2x = x+6$.

What value of x would satisfy this equation? Solving this equation, we can find that $x = 6$. But $x = 6$ is Ben's age, and Jeff is 6 years older so $x+6 = 6+6 = 12$.

When Jeff is 12, he will be twice Ben's age, since 12 is twice the age of 6.

Example C

Matthew is planning to harvest his corn crop this fall. The field has 660 rows of corn with 300 ears per row. Matthew estimates his crew will have the crop harvested in 20 hours. How many ears of corn will his crew harvest per hour?

Solution: Begin by highlighting the key information.

Matthew is planning to harvest his corn crop this fall. The field has **660 rows** of corn with **300 ears per row**. Matthew estimates his crew will have the **crop harvested in 20 hours. How many ears of corn** will his crew **harvest per hour**?

You could draw a picture (it may take a while), write an equation, look for a pattern, or make a table. Let's try to use reasoning.

We need to figure out how many ears of corn are in the field: $660(300) = 198,000$. There are 198,000 ears in the field. It will take 20 hours to harvest the entire field, so we need to divide 198,000 by 20 to get the number of ears picked per hour.

$$\frac{198,000}{20} = 9,900$$

The crew can harvest 9,900 ears per hour.

Vocabulary

Algebraic equation: An **algebraic equation** is a mathematical sentence connecting an expression to a value, a variable, or another expression with an equal sign (=).

Guided Practice

The sum of angles in a triangle is 180 degrees. If the second angle is twice the size of the first angle and the third angle is three times the size of the first angle, what are the measures of the angles in the triangle?

Solution:

Step 1 is to read and determine what the problem is asking us. After reading, we can see that we need to determine the measure of each angle in the triangle. We will use the information given to figure this out.

Step 2 tells us to devise a plan. Since we are given a lot of information about how the different pieces are related, it looks like we can write some algebraic expressions and equations in order to solve this problem. Let a be the measure of the first angle. The second angle is twice the first, so think about how you can express that algebraically.

The correct expression is $2a$. Also, the third angle is three times the size of the first so that would be $3a$. Now the other piece of information given to us is that all three angles must add up to 180 degrees. From this we will write an equation, adding together the expressions of the three angles and setting them equal to 180.

$a+2a+3a=180$

Step 3 is to solve the problem. Simplifying this we get

$6a=180$

$a=30$

Now we know that the first angle is 30 degrees, which means that the second angle is 60 degrees and the third is 90 degrees. Let's check whether these three angles add up to 180 degrees.

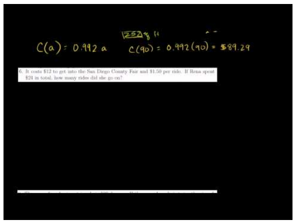
$30+60+90=180$

The three angles do add up to 180 degrees.

Step 4 is to consider other possible methods. We could have used guess and check and possibly found the correct answer. However, there are many choices we could have made. What would have been our first guess? There are so many possibilities for where to start with guess and check that solving this problem algebraically was the simplest way.

Practice

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra:Word Problem-Solving Plan 1 (10:12)



6. It costs $12 to get into the San Diego County Fair and $1.50 per ride. If Rena spent $24 in total, how many rides did she go on?

MEDIA

Click image to the left for more content.

1. What are the four steps to solving a problem?
2. Name three strategies you can use to help make a plan. Which one(s) are you most familiar with already?
3. Which types of strategies work well together? Why?
4. Suppose Matthew's crew takes 36 hours to harvest the field. How many ears per hour will they harvest?
5. Why is it difficult to solve Ben and Jeff's age problem by drawing a diagram?
6. How do you check a solution to a problem? What is the purpose of checking the solution?
7. There were 12 people on a jury, with four more women than men. How many women were there?
8. A rope 14 feet long is cut into two pieces. One piece is 2.25 feet longer than the other. What are the lengths of the two pieces?
9. A sweatshirt costs $35. Find the total cost if the sales tax is 7.75%.
10. This year you got a 5% raise. If your new salary is $45,000, what was your salary before the raise?
11. It costs $250 to carpet a room that is $14\ ft \times 18\ ft$. How much does it cost to carpet a room that is $9\ ft \times 10\ ft$?
12. A department store has a 15% discount for employees. Suppose an employee has a coupon worth $10 off any item and she wants to buy a $65 purse. What is the final cost of the purse if the employee discount is applied before the coupon is subtracted?
13. To host a dance at a hotel, you must pay $250 plus $20 per guest. How much money would you have to pay for 25 guests?

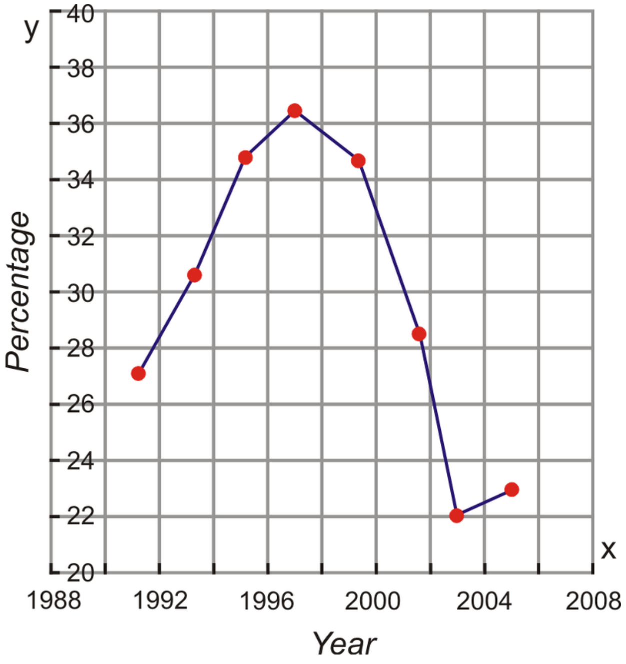
14. It costs \$12 to get into the San Diego County Fair and \$1.50 per ride. If Rena spent \$24 in total, how many rides did she go on?
15. An ice cream shop sells a small cone for \$2.92, a medium cone for \$3.50, and a large cone for \$4.25. Last Saturday, the shop sold 22 small cones, 26 medium cones, and 15 large cones. How much money did the store take in?

Mixed Review

1. Choose an appropriate variable for the following situation: *It takes Lily 45 minutes to bathe and groom a dog. How many dogs can she groom in an 9-hour day?*
2. Translate the following into an algebraic inequality: *Fourteen less than twice a number is greater than or equal to 16.*
3. Write the pattern of the table below in words and using an algebraic equation.

x	-2	-1	0	1
y	-8	-4	0	4

1. Check that $m = 4$ is a solution to $3y - 11 \geq -3$.
2. What is the domain and range of the graph shown?



1.2 Mathematical Review: Solving Equations

Here you'll learn about equations that have the variable on both sides and how to solve them.

Bill and Kate are both reading a 500-page novel. So far, Bill has read 70 pages and Kate has read 50 pages, but from this point forward, Bill plans to read 25 pages per day, while Kate plans to read 29 pages per day. After how many days will they have read the same number of pages? Do you know how to set up and solve an equation to answer a question like this? Such an equation would have the variable on both sides, and in this Concept, you'll learn how to solve this type of equation.

Guidance

As you may now notice, equations come in all sizes and styles. There are single-step, double-step, and multi-step equations. In this Concept, you will learn how to solve equations with a variable appearing on each side of the equation. The process you need to solve this type of equation is similar to solving a multi-step equation. The procedure is repeated here.

Procedure to Solve Equations:

1. Remove any parentheses by using the Distributive Property or the Multiplication Property of Equality.

2. Simplify each side of the equation by combining like terms.

3. Isolate the ax term. Use the Addition Property of Equality to get the variable on one side of the equal sign and the numerical values on the other.

4. Isolate the variable. Use the Multiplication Property of Equality to get the variable alone on one side of the equation.

5. Check your solution.

Karen and Sarah have bank accounts. Karen has a starting balance of $125.00 and is depositing $20 each week. Sarah has a starting balance of $43 and is depositing $37 each week. When will the girls have the same amount of money?

To solve this problem, you could use the "guess and check" method. You are looking for a particular week in which the bank accounts are equal. This could take a long time! You could also translate the sentence into an equation. The number of weeks is unknown so this is our variable. Call it w. Now translate this situation into an algebraic equation:

$$125+20w=43+37w$$

This is a situation in which the variable w appears on both sides of the equation. To begin to solve for the unknown, we must use the Addition Property of Equality to gather the variables on one side of the equation.

Example A

Determine when Sarah and Karen will have the same amount of money.

Solution: Using the Addition Property of Equality, move the variables to one side of the equation:

$$125+20w-20w=43+37w-20w$$

Simplify: $125=43+17w$

Solve using the steps from a previous Concept:

$$\begin{aligned}125-43&=43-43+17w\\82&=17w\\82\div17&=17w\div17\\w&\approx4.82\end{aligned}$$

It will take about 4.8 weeks for Sarah and Karen to have equal amounts of money.

Example B

Solve for y when $2y-5=3y+10$.

Solution:

To solve this equation, we need to get both terms with a variable onto the same side. The easiest way to do this is to subtract the variable with the smaller coefficient from each side:

$$\begin{aligned}&2y-5=3y+10\\&-2y+2y-5=-2y+3y+10\\&-5=y+10\\&-5-10=y+10-10\\&-15=y\end{aligned}$$

Checking our answer:

$$\begin{aligned}&2(-15)-5=3(-15)+10\\&-30-5=-45+10\\&-35=-35\end{aligned}$$

Example C

Solve for $h: 3(h+1) = 11h - 23.$

Solution: First you must remove the parentheses by using the Distributive Property:

$$3h+3 = 11h-23$$

Gather the variables on one side:

$$3h-3h+3 = 11h-3h-23$$

Simplify:

$$3 = 8h-23$$

Solve using the steps from a previous Concept:

$$\begin{aligned} 3+23 &= 8h-23+23 \\ 26 &= 8h \\ 26 \div 8 &= 8h \div 8 \\ h &= \frac{13}{4} = 3.25 \end{aligned}$$

Vocabulary

Distributive Property: For any real numbers M, N, and K:

$$\begin{aligned} M(N+K) &= MN+MK \\ M(N-K) &= MN-MK \end{aligned}$$

The Addition Property of Equality: For all real numbers a, b, and c:

If $a = b$, then $a+c = b+c$.

The Multiplication Property of Equality: For all real numbers a, b, and c:

If $a = b$, *then* $a(c) = b(c)$.

Guided Practice

Solve for g when $-5g+3 = -8g+9$.

Solution:

In this case, our two terms with the variable are negative. The same suggestion in Example B works, and since $-8 < -5$, we will subtract -8. However, subtracting -8 can be simplified to adding 8 (Opposite-Opposite Property). This will leave us with a positive coefficient in front of our variable.

$$\begin{aligned}
&-5g+3=-8g+9\\
&+8g-5g+3=+8g-8g+9\\
&3g+3=9\\
&3g+3-3=9-3\\
&3g=6\\
&\frac{1}{3}\cdot 3g=\frac{1}{3}\cdot 6\\
&g=2
\end{aligned}$$

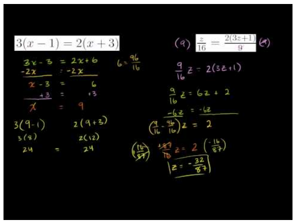
Checking the answer:

$$\begin{aligned}
&-5(2)+3=-8(2)+9\\
&-10+3=-16+9\\
&-7=-7
\end{aligned}$$

Therefore, $g=2$.

Practice

Sample explanations for some of the practice exercises below are available by viewing the following video. Note that there is not always a match between the number of the practice exercise in the video and the number of the practice exercise listed in the following exercise set. However, the practice exercise is the same in both. CK-12 Basic Algebra:Equations with Variables onBoth Sides (9:28)



MEDIA

Click image to the left for more content.

In 1 – 13, solve the equation.

1. $3(x-1)=2(x+3)$
2. $7(x+20)=x+5$
3. $9(x-2)=3x+3$
4. $2\left(a-\frac{1}{3}\right)=\frac{2}{5}\left(a+\frac{2}{3}\right)$
5. $\frac{2}{7}\left(t+\frac{2}{3}\right)=\frac{1}{5}\left(t-\frac{2}{3}\right)$
6. $\frac{1}{7}\left(v+\frac{1}{4}\right)=2\left(\frac{3v}{2}-\frac{5}{2}\right)$
7. $\frac{y-4}{11}=\frac{2}{5}\cdot\frac{2y+1}{3}$
8. $\frac{z}{16}=\frac{2(3z+1)}{9}$
9. $\frac{q}{16}+\frac{q}{6}=\frac{(3q+1)}{9}+\frac{3}{2}$
10. $21+3b=6-6(1-4b)$
11. $-2x+8=8(1-4x)$

12. $3(-5v-4) = -6v-39$
13. $-5(5k+7) = 25+5k$
14. Manoj and Tamar are arguing about how a number trick they heard goes. Tamar tells Andrew to think of a number, multiply it by five, and subtract three from the result. Then Manoj tells Andrew to think of a number, add five, and multiply the result by three. Andrew says that whichever way he does the trick he gets the same answer. What was Andrew's number?
15. I have enough money to buy five regular priced CDs and have $6 left over. However, all CDs are on sale today for $4 less than usual. If I borrow $2, I can afford nine of them. How much are CDs on sale for today?
16. Jaime has a bank account with a balance of $412 and is saving $18 each week. George has a bank account with a balance of $874 and is spending $44 dollars each week. When will the two have the same amount of money?
17. Cell phone plan A charges $75.00 each month and $0.05 per text. Cell phone plan B charges $109 dollars and $0.00 per text.
    a. At how many texts will the two plans charge the same?
    b. Suppose you plan to text 3,000 times per month. Which plan should you choose? Why?
18. To rent a dunk tank, Modern Rental charges $150 per day. To rent the same tank, Budgetwise charges $7.75 per hour.
    a. When will the two companies charge the same?
    b. You will need the tank for a 24-hour fund raise-a-thon. Which company should you choose?

Mixed Review

1. Solve for $t: -12+t = -20$.
2. Solve for $r: 3r-7r = 32$.
3. Solve for $e: 35 = 5(e+2)$.
4. 25 more than four times a number is 13. What is the number?
5. Find the opposite of $9\frac{1}{5}$. Write your answer as an improper fraction.
6. Evaluate $(|b|-a)-(|d|-a)$. Let $a = 4,\ b = -6$, and $d = 5$.
7. Give an example of an integer that is not a counting number.

Quick Quiz

1. Determine the inverse of addition.
2. Solve for $w: -4w = 16$.
3. Write an equation to represent the following situation and solve. *Shauna ran the 400 meter dash in 56.7 seconds, 0.98 seconds less than her previous time. What was her previous time?*
4. Solve for $b: \frac{1}{2}b+5 = 9$.
5. Solve for $q: 3q+5-4q = 19$.

1.3 Metric Units

Students will learn about the metric system and how to convert between metric units.

Frequently Used Measurements, Greek Letters, and Prefixes

Key Applications

The late, great physicist Enrico Fermi used to solve problems by making educated guesses. For instance, say you want to *guess*timate the number of cans of soda drank by everybody in San Francisco in one year. You'll come pretty close if you guess that there are about 800,000 people in S.F., and that each person drinks on average about 100 cans per year. So, 80,000,000 cans are consumed every year. Sure, this answer is wrong, but it is likely not off by more than a factor of 10 (i.e., an "order of magnitude"). That is, even if we guess, we're going to be in the *ballpark*of the right answer. That is always the first step in working out a physics problem.

Measurements

TABLE 1.1: **Types of Measurements**

Type of measurement	*Commonly used symbols*	*Fundamental units*
length or position	d, x, L	meters (m)
time	t	seconds (s)
velocity or speed	v, u	meters per second (m/s)
mass	m	kilograms (kg)
force	$\mathbf{F}$	Newtons (N)
energy	E, K, U, Q	Joules (J)
power	P	Watts (W)
electric charge	q, e	Coulombs (C)
temperature	T	Kelvin (K)
electric current	I	Amperes (A)
electric field	$\mathbf{E}$	Newtons per Coulomb (N/C)
magnetic field	$\mathbf{B}$	Tesla (T)

Prefixes

TABLE 1.2: **Prefix Table**

SI prefix	In Words	Factor
nano (n)	billionth	$1 * 10^{-9}$
micro (μ)	millionth	$1 * 10^{-6}$
milli (m)	thousandth	$1 * 10^{-3}$
centi (c)	hundreth	$1 * 10^{-2}$
deci (d)	tenth	$1 * 10^{-1}$
deca (da)	ten	$1 * 10^{1}$
hecto (h)	hundred	$1 * 10^{2}$
kilo (k)	thousand	$1 * 10^{3}$
mega (M)	million	$1 * 10^{6}$

TABLE 1.2: (continued)

SI prefix	In Words	Factor
giga (G)	billion	$1 * 10^9$

Greek Letters

TABLE 1.3: Frequently used Greek letters.

μ "mu"	τ "tau"	Φ "Phi"*	ω "omega"	ρ "rho"
θ "theta"	π "pi "	Ω "Omega"*	λ "lambda"	Σ "Sigma"*
α "alpha"	β "beta"	γ "gamma"	Δ "Delta"*	ε "epsilon"

Two very common Greek letters are Δ and Σ . Δ is used to indicate that we should use the change or difference between the final and initial values of that specific variable. Σ denotes the sum or net value of a variable.

Guidance

- Every answer to a physics problem must include units. Even if a problem explicitly asks for a speed in meters per second (m/s), the answer is 5 m/s, not 5.
- If a unit is named after a person, it is capitalized. So you write "10 Newtons," or "10 N," but "10 meters," or "10 m."
- Metric units use a base numbering system of 10. Thus a centimeter is ten times larger than a millimeter. A decimeter is 10 times larger than a centimeter and a meter is 10 times larger than a decimeter. Thus a meter is 100 times larger than a centimeter and 1000 times larger than a millimeter. Going the other way, one can say that there are 100 cm contained in a meter.
- **Rounding:** Round your answers to a reasonable number of digits. If the answer from the calculator is 1.369872654389, but the numbers in the problem are like 21 m/s and 12s, you should round off to 1.4. To report the full number would be misleading (i.e. you are telling me you know something to very high accuracy, when in fact you don't). Do not round your numbers while doing the calculation, otherwise you'll probably be off a bit due to the dreaded 'round off error'.
- **Does your answer make sense:** Remember you are smarter than the calculator. Check your answer to make sure it is reasonable. For example, if you are finding the height of a cliff and your answer is 0.00034 m. That can't be right because the cliff is definitely higher than 0.34 mm. Another example, the speed of light is $3 \times 10^8\ m/s$ (i.e. light travels 300 million meters in one second). Nothing can go faster than the speed of light. If you calculate the speed of a car to be $2.4 \times 10^{10}\ m/s$, you know this is wrong and possibly a calculator typ-o.

Example 1

Question: Convert 2500 m/s into km/s

Solution: A km (kilometer) is 1000 times bigger than a meter. Thus, one simply divides by 1000 and arrives at 2.5 km/s

Example 2

Question: The lengths of the sides of a cube are doubling each second. At what rate is the volume increasing?

Solution:The cube side length, x, is doubling every second. Therefore after 1 second it becomes $2x$. The volume of the first cube of side x is $x \times x \times x = x^3$. The volume of the second cube of side $2x$ is $2x \times 2x \times 2x = 8x^3$. The ratio of the second volume to the first volume is $8x^3/x^3 = 8$. Thus the volume is increasing by a factor of 8 every second.

Watch this Explanation

MEDIA

Click image to the left for more content.

Time for Practice

1. A tortoise travels 15 meters (m) west, then another 13 centimeters (cm) west. How many meters total has she walked?

2. A tortoise, Bernard, starting at point A travels 12 m west and then 150 millimeters (mm) east. How far west of point A is Bernard after completing these two motions?
3. $80 \text{ m} + 145 \text{ cm} + 7850 \text{ mm} = X$ mm. What is$X$?
4. A square has sides of length 45 mm. What is the area of the square in mm^2?
5. A square with area 49 cm^2 is stretched so that each side is now twice as long. What is the area of the square now? Include a sketch.
6. A rectangular solid has a square face with sides 5 cm in length, and a length of 10 cm. What is the volume of the solid in cm^3? Sketch the object, including the dimensions in your sketch.
7. As you know, a cube with each side 4 m in length has a volume of 64 m^3. Each side of the cube is now doubled in length. What is the *ratio* of the new volume to the old volume? Why is this ratio **not** simply 2? Include a sketch with dimensions.
8. What is the ratio of the mass of the Earth to the mass of a single proton? (See equation sheet.)
9. A spacecraft can travel 20 km/s. How many km can this spacecraft travel in 1 hour (h)?

Answers

1. 15.13 m
2. 11.85 m
3. 89,300 mm
4. 2025 mm^2
5. 196 cm^2
6. 250 cm^3
7. 8 : 1, each side goes up by 2 cm, so it will change by 2^3
8. $3.5 \times 10^{51} : 1$
9. 72,000 km/h

1.4 Unit Conversions

Students will learn how to convert units from metric to english system and vice verse using dimensional analysis.

Key Equations

$$\begin{aligned} 1 \text{ meter} &= 3.28 \text{ feet} \\ 1 \text{ mile} &= 1.61 \text{ kilometers} \\ 1 \text{ lb. (1 pound)} &= 4.45 \text{ Newtons} \end{aligned}$$

Guidance

- The key to converting units is to multiply by a clever factor of one. You can always multiply by 1, because it does not change the number. Since 1 in. is equal to 2.54 cm, then

$1 = \frac{2.54 \text{ cm}}{1 \text{ in}} = \frac{1 \text{ in}}{2.54 \text{ cm}}$ thus, one can multiply by this form of 1 in order to cancel units (see video below).

- Write out every step and show all your units cancelling as you go.
- When converting speeds from metric to American units, remember the following rule of thumb: a speed measured in mi/hr is about double the value measured in m/s (*i.e.,* 10 {m/s} is equal to about 20 MPH). Remember that the speed itself hasn't changed, just our representation of the speed in a certain set of units.
- When you're not sure how to approach a problem, you can often get insight by considering how to obtain the units of the desired result by combining the units of the given variables. For instance, if you are given a distance (in meters) and a time (in hours), the only way to obtain units of speed (meters/hour) is to divide the distance by the time. This is a simple example of a method called *dimensional analysis*, which can be used to find equations that govern various physical situations without any knowledge of the phenomena themselves.

Example 1

Question: 20 m/s = ? mi/hr

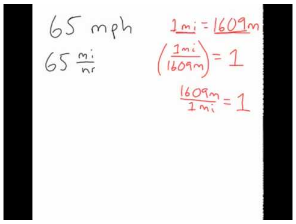
Solution:

20 m/s (1 mi/1600 m) = .0125 mi/s

.0125 mi/s (60 s/min) = .75 mi/min

.75 mi/min (60 min/hr) = 45 mi/hr

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. Estimate or measure your height.
   1. Convert your height from feet and inches to meters.
   2. Convert your height from feet and inches to centimeters $(100 \text{ cm} = 1 \text{ m})$
2. Estimate or measure the amount of time that passes between breaths when you are sitting at rest.
   1. Convert the time from seconds into hours
   2. Convert the time from seconds into milliseconds (ms)
3. Convert the French speed limit of 140 km/hr into mi/hr.
4. Estimate or measure your weight.
   1. Convert your weight in pounds into a mass in kg
   2. Convert your mass from kg into μg
   3. Convert your weight into Newtons
5. Find the SI unit for pressure.
6. An English lord says he weighs 12stone.
   1. Convert his weight into pounds (you may have to do some research online)
   2. Convert his weight in stones into a mass in kilograms
7. If the speed of your car increases by 10 mi/hr every 2 seconds, how many mi/hr is the speed increasing every second? State your answer with the units mi/hr/s.

Answers

1. a. A person of height 5 ft. 11 in. is 1.80 m tall b. The same person is 180 cm
2. a. 3 seconds $= 1/1200$ hours b. $3x10^3$ ms
3. 87.5 mi/hr
4. If the person weighs 150 lb then a. 67.9 kg (on Earth) b. 67.9 billion μg c. this is equivalent to 668 N
5. Pascals (Pa), which equals N/m^2
6. a. 168 lb.,b. 76.2 kg
7. 5 mi/hr/s

CHAPTER 2 Kinematics

Chapter Outline

MEDIA
Click image to the left for more content.

The video was originall shot in slow motion using 120 fps. This is a brief demonstration of the types of analyses the Tracker Video Analysis (http://www.cabrillo.edu/ dbrown/tracker/) program is capable of computing. The Tracker program is open source so download it for yourself and begin analyzing the kinematics of objects.

2.1 Position and Displacement

Students will learn the meaning of an object's position, the difference between distance and displacement and some basic graphing of position vs. time.

The Big Idea

Speed represents how quickly an object is moving through space. Velocity is speed with a direction, making it a *vector* quantity. If an object's velocity changes with time, the object is said to be accelerating. As we'll see in the next chapters, understanding the acceleration of an object is the key to understanding its motion. We will assume constant acceleration throughout this chapter.

Key Equations

$$\text{Symbols}\begin{cases}\Delta(\text{anything}) & \text{Final value - initial value}\\ \text{anything}_o & \text{Initial value}\end{cases}$$

$$\text{Scalors}\begin{cases}t & \text{Time in seconds, s}\\ d = |\Delta x_1| + |\Delta x_2| & \text{Distance (in meters, m)}\\ v = |v| & \text{Speed (in meters per second, m/s)}\end{cases}$$

$$\text{Vectors}\begin{cases}x = x(t) & \text{Position at a specific time, } t\\ \Delta x = x_f - x_o & \text{Displacement}\end{cases}$$

When beginning a one dimensional problem, define a positive direction. The other direction is then taken to be negative. Traditionally, "positive" is taken to mean "to the right"; however, any definition of direction used consistently throughout the problem will yield the right answer.

Key Concepts

- When you begin a problem, define a coordinate system. For positions, this is like a number line; for example, positive $(+x)$ positions can be to the right of the origin and negative $(-x)$ positions to the left of the origin.
- For velocity v you might define positive as *moving to the right* and negative as *moving to the left*. What would it mean to have a **positive position** and a **negative velocity?**

Guidance

Position is the location of the object (whether it's a person, a ball or a particle) at a given moment in time. Displacement is the difference in the object's position from one time to another. Distance is the total amount the object has traveled in a certain period of time. Displacement is a vector quantity (direction matters), where as distance is a scalor (only the amount matters). Distance and displacement are the same in the case where the object travels in a straight line and always moving in the same direction.

Example 1

Problem: An indecisive car goes 120 m North, then 30 m south then 60m North. What is the car's distance and displacement?

Solution:

Distance is the total amount traveled. Thus distance = 120 + 30 + 60 m = 210 m

Displacement is the amount displaced from the starting position. Thus displacement = 120 - 30 + 60 m = 150 m.

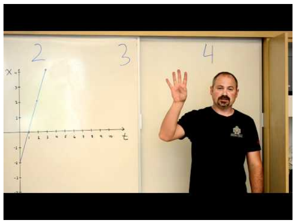
Example 2

Problem: An 8th grader is timed to run 24 feet in 12 seconds, what is her speed in meters per second?

Solution:

$$\begin{aligned} \Delta x &= vt \\ 24\ ft &= v(12\ s) \\ v &= 24\ ft/12\ s = 2\ ft/s \\ v &= 2\ ft/s * (1m/3.28\ ft) = 0.61\ m/s \end{aligned}$$

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. What is the difference between distance d and displacement $\triangle x$? Write a few sentences explaining this.
2. Does the odometer reading in a car measure distance or displacement?
3. Imagine a fox darting around in the woods for several hours. Can the displacement $\triangle x$ of the fox from his initial position ever be larger than the total distance d he traveled? Explain.
4. Your brother borrowed the scissors from your room and now you want to use them. Do you care about the distance the scissors have traveled or their displacement? Explain your answer.
5. You're trying to predict how long it's going to take to get to Los Angeles for the long weekend. Do you care about the distance you'll travel or your displacement? Explain your answer.

2.2 Average Velocity

Students will learn the meaning of speed, velocity and average velocity.

Key Equations

Speed = distance/time

Average Velocity

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

Guidance

Speed is the distance traveled divided by the time it took to travel that distance. Velocity is the instantaneous speed and direction. Average velocity is the displacement divided by the time.

Example 1

Pacific loggerhead sea turtles migrate over 7,500 miles (12,000 km) between nesting beaches in Japan and feeding grounds off the coast of Mexico. If the average speed of a loggerhead is about 45 km/day, how long does it take for it to complete the distance of a one-way migration?

Question: $t = ?$ [days]

Given: $\Delta x = 12,000\ km$

$v_{avg} = 45\ km/day$

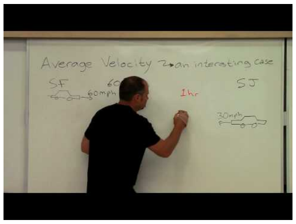
Equation: $v_{avg} = \frac{\Delta x}{t}$ therefore $t = \frac{\Delta x}{v_{avg}}$

Plug n' Chug: $t = \frac{\Delta x}{v_{avg}} = \frac{12{,}000\ km}{45\ km/day} = 267\ days$

Answer:

$$\boxed{\textbf{267 days}}$$

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. Two cars are heading right towards each other, but are 12 km apart. One car is going 70 km/hr and the other is going 50 km/hr. How much time do they have before they collide head on?
2. You drive the 10 miles to work at an average speed of 40 mph. On the way home you hit severe traffic and drive at an average speed of 10 mph. What is your average speed for the trip?
3. The following data represent the first 30 seconds of actor Crispin Glover's drive to work.

TABLE 2.1:

Time (s)	Position (m)	Distance (m)
0	0	0
5	10	10
10	30	30
15	30	30
20	20	40
25	50	70
30	80	120

1. a. Sketch the graphs of position vs. time and distance vs. time. Label your x and y axes appropriately.
   b. Why is there a discrepancy between the distance covered and the change in position during the time period between $t = 25$ s and $t = 30$ s?
   c. What do you think is going on between $t = 10$ s and $t = 15$ s?
   d. What is the displacement between $t = 10$ s and $t = 25$ s?
   e. What is the distance covered between $t = 10$ s and $t = 25$ s?
   f. What is the average velocity during the first 30 seconds of the trip?
   g. What is the average velocity between the times $t = 20$ s and $t = 30$ s?
   h. During which time interval(s) was the velocity negative?
   i. Sketch the velocity vs. time and speed vs. time graphs. Label your x and y axes appropriately.

Answers

1. 0.1 hours = 6 minutes

2. 16 mph

2.3 Velocity and Acceleration

Students will learn the meaning of acceleration, how it is different than velocity and how to calculate average acceleration.

Key Equations

$v =$ velocity (m/s)

$v_o =$ initial velocity

$v_f =$ final velocity

$\Delta v =$ change in velocity $= v_f - v_o$

$v_{avg} = \frac{\Delta x}{\Delta t}$

$a =$ acceleration (m/s^2)

$a_{avg} = \frac{\Delta v}{\Delta t}$

Guidance

- Acceleration is the rate of change of velocity. So in other words, acceleration tells you how quickly the velocity is increasing or decreasing. An acceleration of 5 m/s^2 indicates that the velocity is increasing by $5m/s$ in the positive direction every second.
- Gravity near the Earth pulls an object downwards toward the surface of the Earth with an acceleration of 9.8 $m/s^2 (\approx 10\ m/s^2)$. In the absence of air resistance, all objects will fall with the same acceleration. The letter $g$is used as the symbol for the acceleration of gravity.
  - When talking about an object's acceleration, whether it is due to gravity or not, the acceleration of gravity is sometimes used as a unit of measurement where $1g = 9.8m/s^2$. So an object accelerating at 2g's is accelerating at $2 * 9.8m/s^2$ or $19.6m/s^2$
- *Deceleration* is the term used when an object's *speed* (i.e. magnitude of its velocity) is decreasing due to acceleration in the opposite direction of its velocity.

Example 1

A Top Fuel dragster can accelerate from 0 to 100 mph (160 km/hr) in 0.8 seconds. What is the average acceleration in m/s^2?

Question: $a_{avg} = ?\ [m/s^2]$

Given: $v_o = 0\ m/s$

$v_f = 160\ km/hr$

$t = 0.8\ s$

Equation: $a_{avg} = \frac{\Delta v}{t}$

Plug n' Chug: Step 1: Convert km/hr to m/s

$$v_f = \left(160 \frac{km}{hr}\right) \left(\frac{1{,}000\ m}{1\ km}\right) \left(\frac{1\ hr}{3{,}600\ s}\right) = 44.4\ m/s$$

Step 2: Solve for average acceleration:

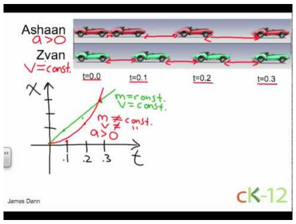
$$a_{avg} = \frac{\Delta v}{t} = \frac{v_f - v_o}{t} = \frac{44.4\ m/s - 0\ m/s}{0.8\ s} = 56\ m/s^2$$

Answer:

$$\boxed{\mathbf{56\ m/s^2}}$$

Note that this is over $5\frac{1}{2}$ *g's!*

Watch this Explanation



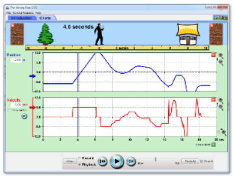
MEDIA

Click image to the left for more content.

Simulation

MEDIA

Click image to the left for more content.

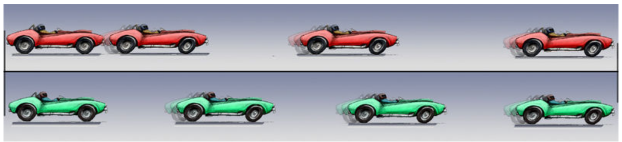


The Moving Man (PhETSimulation)

Time for Practice

1. Ms. Reitman's scooter starts from rest and accelerates at $2.0 m/s^2$.
   a. Where will the scooter be relative to its starting point after 7.0 seconds?
   b. What is the scooter's velocity after 1s? after 2s? after 7s?
2. A horse is galloping forward with an acceleration of $3\ m/s^2$. Which of the following statements is not necessarily true? You may choose more than one.
   a. The horse is increasing its speed by 3 m/s every second, from 0 m/s to 3 m/s to 6 m/s to 9 m/s.
   b. The speed of the horse will triple every second, from 0 m/s to 3 m/s to 9 m/s to 27 m/s.
   c. Starting from rest, the horse will cover 3 m of ground in the first second.
   d. Starting from rest, the horse will cover 1.5 m of ground in the first second.

3. Below are images from a race between Ashaan (above) and Zyan (below), two daring racecar drivers. High speed cameras took four pictures in rapid succession. The first picture shows the positions of the cars at $t = 0.0$. Each car image to the right represents times 0.1, 0.2, and 0.3 seconds later.



   a. Who is ahead at $t = 0.2$ s ? Explain.
   b. Who is accelerating? Explain.
   c. Who is going fastest at $t = 0.3$ s? Explain.
   d. Which car has a constant velocity throughout? Explain.
   e. Graph x vs. t and v vs. t. Put both cars on same graph; label which line is which car.
   f. Which car is going faster at $t = 0.2$ s (Hint: Assume they travel the same distance between 0.1 and 0.2 seconds)?

Answers

1. a. 49 m b. 2 m/s, 4 m/s, 14 m/s

3. See Video above

2.4 Graphical Analysis

Students will learn how to graph motion vs time. Specifically students will learn how to take the slope of a graph and relate that to the instantaneous velocity or acceleration for position or velocity graphs, respectively. Finally students will learn how to take the area of a velocity vs time graph in order to calculate the displacement.

Key Equations

For a graph of position vs. time. The slope is the rise over the run, where the rise is the displacement and the run is the time. thus,

Slope = $v_{avg} = \frac{\Delta x}{\Delta t}$

Note: Slope of the tangent line for a particular point in time = the instantaneous velocity

For a graph of velocity vs. time. The slope is the rise over the run, where the rise is the change in velocity and the run is the time. thus,

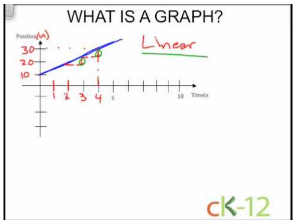
Slope = $a_{avg} = \frac{\Delta v}{\Delta t}$

Note: Slope of the tangent line for a particular point in time = the instantaneous acceleration

Guidance

- One must first read a graph correctly. For example on a position vs. time graph (thus the position is graphed on the y-axis and the time on the x-axis) for a given a data point, go straight down from it to get the time and straight across to get the position.
- If there is constant acceleration the graph x vs. t produces a parabola. The slope of the x vs. t graph equals the instantaneous velocity. The slope of a v vs. t graph equals the acceleration.
- The **slope** of the graph v vs. t can be used to find **acceleration;** the **area** of the graph v vs. t can be used to find **displacement.** Welcome to calculus!

What is a Graph



MEDIA

Click image to the left for more content.

Watch this Explanation

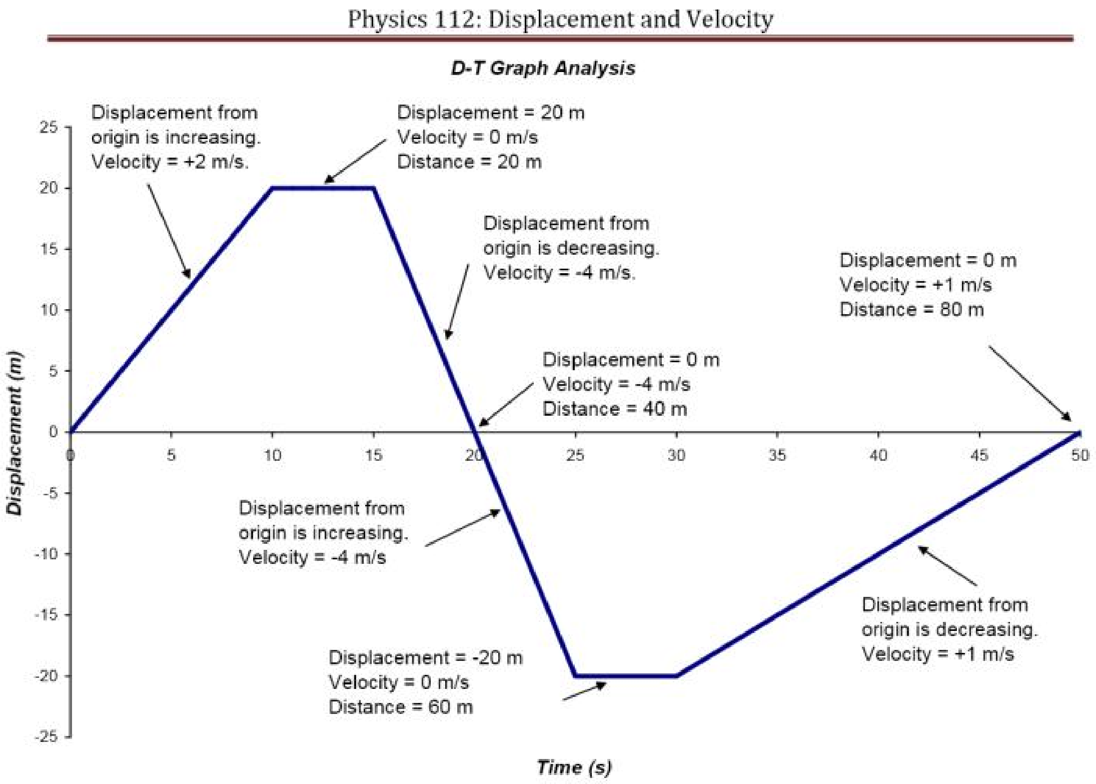
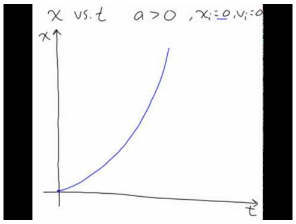


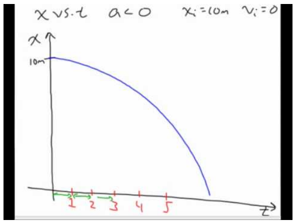
FIGURE 2.1

Analysis of a displacement - time graph.



MEDIA

Click image to the left for more content.



MEDIA

Click image to the left for more content.

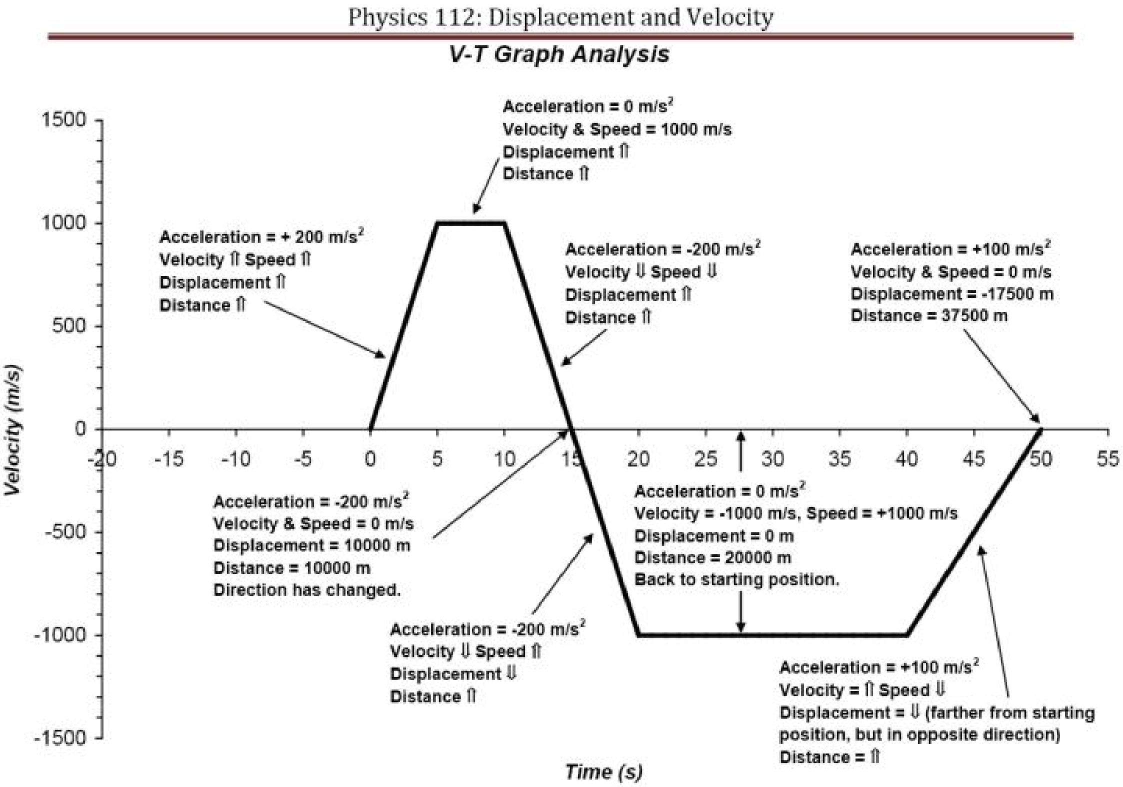
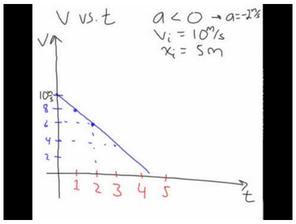


FIGURE 2.2

Analysis of a velocity - time graph.



MEDIA

Click image to the left for more content.

Word Problems

Example 1

A car accelerates from zero to 35 m/s in 7.3 seconds.

a) What is the average acceleration?

b) What distance was covered during the acceleration?

Example 2

An object is moving at 55 m/s [E] and undergoes an acceleration of 2.5 m/s^2 [W] for 10s.

a) What is the final velocity of the car?

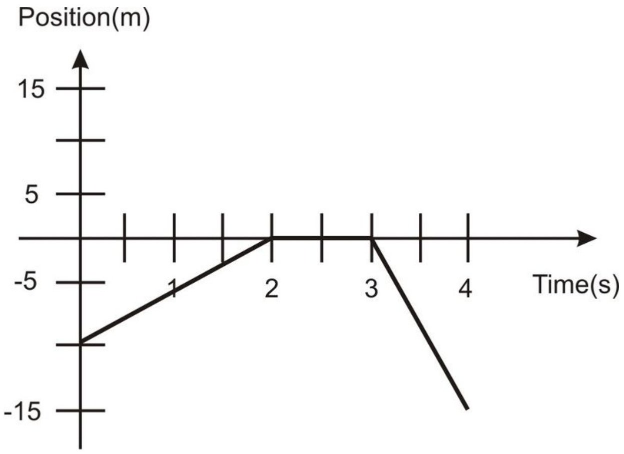
b) What was the final displacement of the car?

Example 3

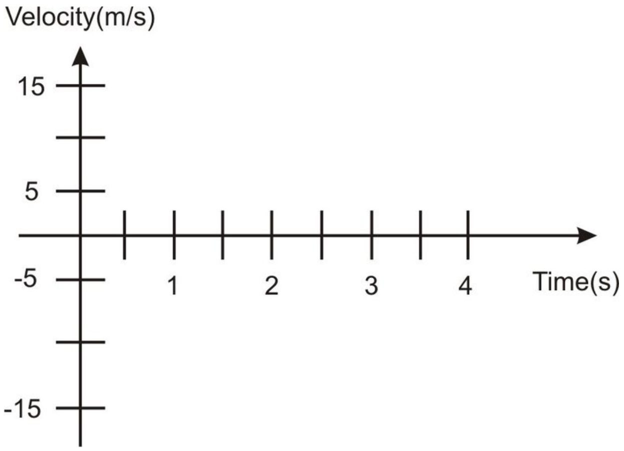
Standing near the edge of a cliff a baseball is launched straight up with a velocity of 15 m/s. The ball is in the air for a total of 4.5 s before it hits the ground at the bottom of the cliff. Find the height of the cliff.

Time for Practice

1. The position graph below is of the movement of a fast turtle who can turn on a dime.

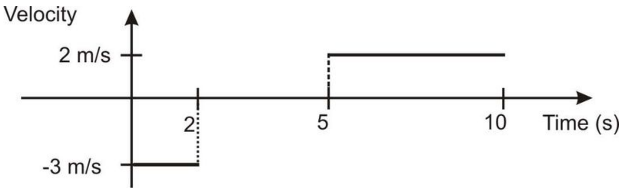


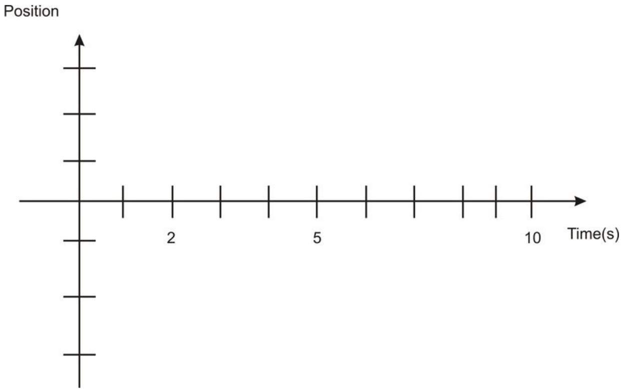
a. Sketch the velocity vs. time graph of the turtle below.



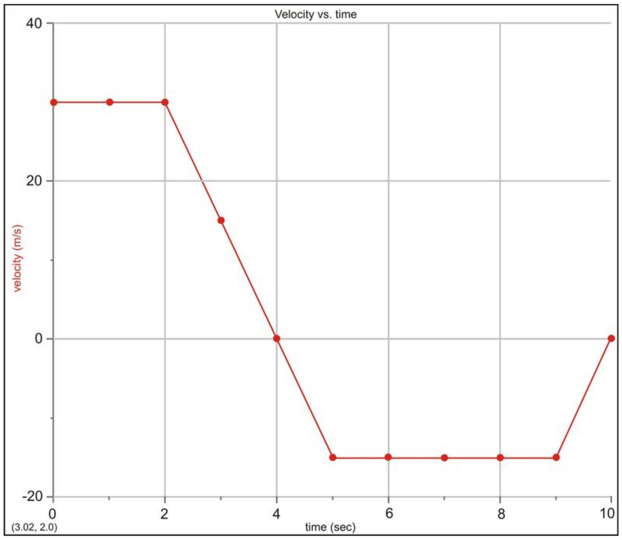
b. Explain what the turtle is doing (including both *speed* and *direction*) from: i) 0-2s. ii) 2-3s. iii) 3-4s. c. How much distance has the turtle covered after 4s? d. What is the turtle's displacement after 4s?

2. Draw the position vs. time graph that corresponds to the velocity vs. time graph below. You may assume a starting position $x_0 = 0$. Label the $y-$axis with appropriate values.





3. The following velocity-time graph represents 10 seconds of actress Halle Berry's drive to work (it's a rough morning).

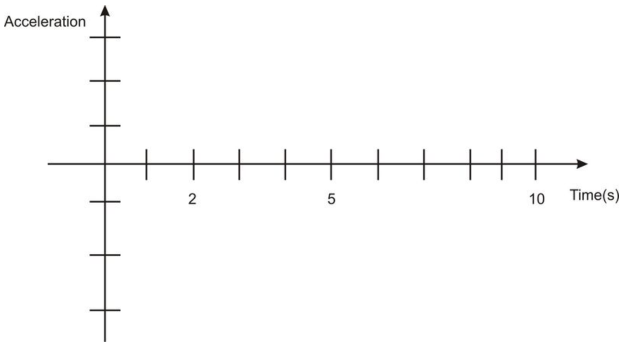


a. Fill in the tables below – remember that *displacement* and *position* are not the same thing!

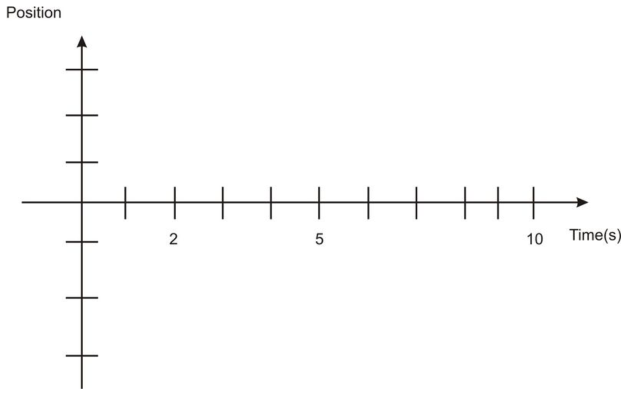
TABLE 2.2:

Interval (s)	Displacement (m)	Acceleration(m/s^2)	**Instantaneous Time (s)**	**Position (m)**
			0 sec	0 m
0-2 sec				
			2 sec	
2-4 sec				
			4 sec	
4-5 sec				
			5 sec	
5-9 sec				
			9 sec	
9-10 sec				
			10 sec	

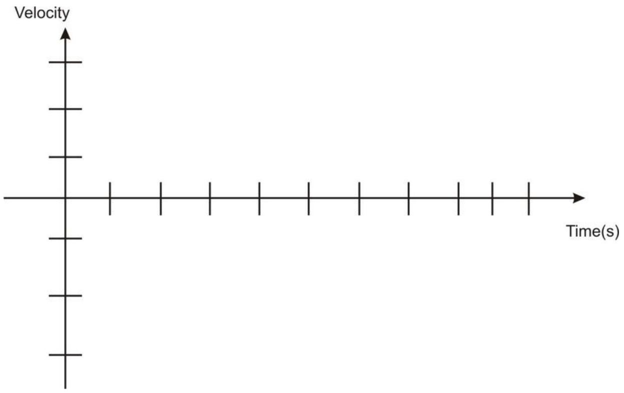
b. On the axes below, draw an *acceleration-time* graph for the car trip. Include numbers on your acceleration axis.



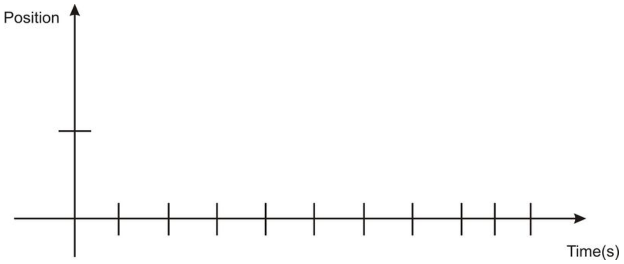
c. On the axes below, draw a *position-time* graph for the car trip. Include numbers on your position axis. Be sure to note that some sections of this graph are linear and some curve – why?



1. Two cars are drag racing down El Camino. At time $t = 0$, the yellow Maserati starts from rest and accelerates at 10 m/s^2. As it starts to move it's passed by a '63 Chevy Nova (cherry red) traveling at a constant velocity of 30 m/s. a. On the axes below, show a line for each car representing its speed as a function of time. Label each line.



b. At what time will the two cars have the same speed (use your graph)? c. On the axes below, draw a line (or curve) for each car representing its *position* as a function of time. Label each curve.



d. At what time would the two cars meet (other than at the start)?

Answers:

1c. 25 m

1d. -5 m

2. discuss in class

3. discuss in class

4b. 3 sec

4d. 6 sec

2.5 Motion Analysis Review

Students will learn how to solve all types of problems using the kinematic equations.

In this Concept, you will learn how to solve all types of problems using the kinematic equations.

Key Equations

Averages

$$v_{avg} = \frac{\Delta x}{\Delta t}$$
$$a_{avg} = \frac{\Delta v}{\Delta t}$$

The Big Three

^2 = v_0^2 + 2a Δx

Guidance

- When beginning a one dimensional problem, define a positive direction. The other direction is then taken to be negative. Traditionally, "positive" is taken to mean "up" for vertical problems and "to the right" for horizontal problems; however, any definition of direction used consistency throughout the problem will yield the right answer.

- Gravity near the Earth pulls an object toward the surface of the Earth with an acceleration of $9.8 \text{ m/s}^2 (\approx 10 \text{ m/s}^2)$. In the absence of air resistance, all objects will fall with the same acceleration. Air resistance can cause low-mass, large area objects to accelerate more slowly.

- The Big Three equations define the graphs of position and velocity as a function of time. When there is no acceleration (constant velocity), position increases linearly with time – distance equals rate times time. Under constant acceleration, velocity increases linearly with time but distance does so at a quadratic rate. The slopes of the position and velocity graphs will give instantaneous velocity and acceleration, respectively.

Example Problem 1

While driving through Napa you observe a hot air balloon in the sky with tourists on board. One of the passengers accidentally drops a wine bottle and you note that it takes 2.3 seconds for it to reach the ground. (a) How high is the balloon? (b) What was the wine bottle's velocity just before it hit the ground?

*Question **a**:* $h = ?\,[m]$

Given: $t = 2.3\ s$

$g = 10\ m/s^2$

$v_i = 0\ m/s$

Equation: $\Delta x = v_i t + \frac{1}{2}at^2$ or $h = v_i t + \frac{1}{2}gt^2$

Plug n' Chug: $h = 0 + \frac{1}{2}(10\ m/s^2)(2.3\ s)^2 = 26.5\ m$

Answer:

$$\boxed{\mathbf{26.5\ m}}$$

Question ***b****:* $v_f = ?\,[m/s]$

Given: (same as above)

Equation: $v_f = v_i + at$

Plug n' Chug: $v_f = v_i + at = 0 + (10\ m/s^2)(2.3\ s) = 23\ m/s$

Answer:

$$\boxed{\mathbf{23\ m/s}}$$

Example Problem 2

The second tallest building in the world is the Petronas Tower in Malaysia. If you were to drop a penny from the roof which is 378.6 m (1242 ft) high, how long would it take to reach the ground? You may neglect air friction.

Question: $t = ?\,[s]$

Given: $h = 378.6\ m$

$g = 10\ m/s^2$

$v_i = 0\ m/s$

Equation: $\Delta x = v_i t + \frac{1}{2}at^2$ or $h = v_i t + \frac{1}{2}gt^2$

Plug n' Chug: since $v_i = 0$, the equation simplifies to $h = \frac{1}{2}gt^2$ rearranging for the unknown variable, t, yields

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(378.6\ m)}{10.0\ m/s^2}} = 8.70\ s$$

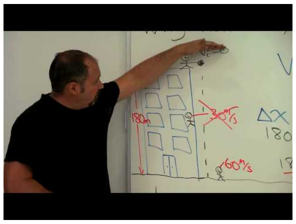
Answer:

$$\boxed{\mathbf{8.70\ s}}$$

Watch this Explanation

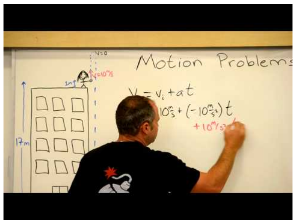
MEDIA

Click image to the left for more content.



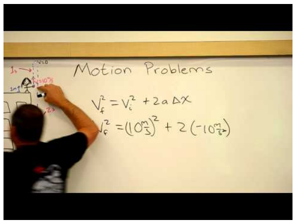
MEDIA

Click image to the left for more content.



MEDIA

Click image to the left for more content.



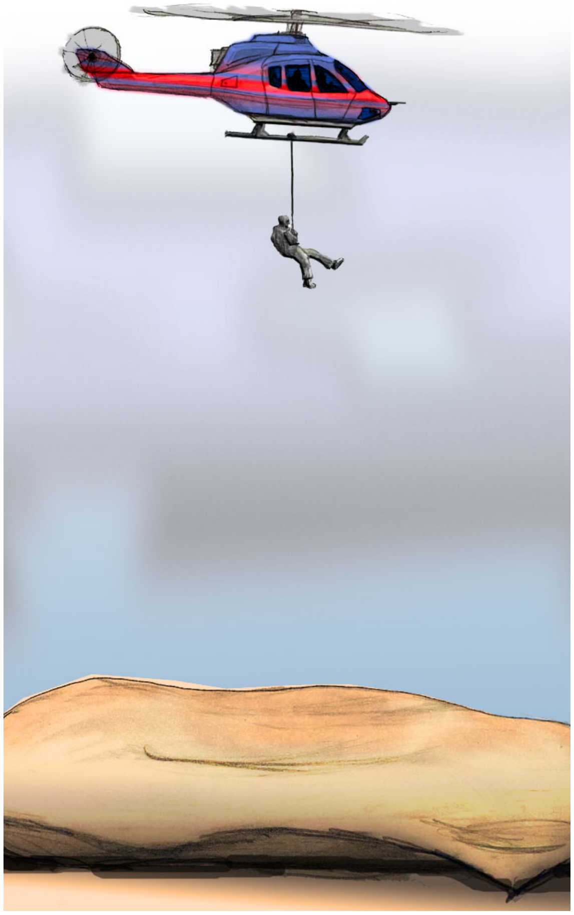
MEDIA

Click image to the left for more content.

Time for Practice

1. Sketchy LeBaron, a used car salesman, claims his car is able to go from 0 to 60 mi/hr in 3.5 seconds.
   a. What is the average acceleration of this car? Give your answer in $\mathrm{m/s^2}$. (Hint: you will have to perform a conversion.)
   b. How much distance does this car cover in these 3.5 seconds? Express your answer twice: in meters and in feet.
   c. What is the speed of the car in mi/hr after 2 seconds?
2. Michael Jordan had a vertical jump of about 48 inches.
   a. Convert this height into meters.
   b. Assuming no air resistance, at what speed did he leave the ground?
   c. What is his speed 3/4 of the way up?
   d. What is his speed just before he hits the ground on the way down?
3. You are sitting on your bike at rest. Your brother comes running at you from behind at a speed of 2 m/s. At the exact moment he passes you, you start up on your bike with an acceleration of 2 $\mathrm{m/s^2}$.
   a. Draw a picture of the situation, defining the starting positions, speeds, etc.
   b. At what time *t* do you have the same speed as your brother?
   c. At what time *t* do you pass your brother?
   d. Draw another picture of the exact moment you catch your brother. Label the drawing with the positions and speeds at that moment.
   e. Sketch a position vs. time graph for both you and your brother, labeling the important points (*i.e.,* starting point, when you catch him, etc.)
   f. Sketch a speed vs. time graph for both you and your brother, labeling the important points (*i.e.,* starting point, when you catch him, etc.)

4. You are standing at the foot of the Bank of America building in San Francisco, which is 52 floors (237 m) high. You launch a ball straight up in the air from the edge of the foot of the building. The initial vertical speed is 70 m/s. (For this problem, you may ignore your own height, which is very small compared to the height of the building.)
   a. How high up does the ball go?
   b. How fast is the ball going right before it hits the top of the building?
   c. For how many seconds total is the ball in the air?
5. Measure how high you can jump vertically on Earth. Then, figure out how high you would be able to jump on the Moon, where acceleration due to gravity is $1/6^{th}$ that of Earth. Assume you launch upwards with the same speed on the Moon as you do on the Earth.
6. A car is smashed into a wall during Weaverville's July 4^{th} Destruction Derby. The car is going 25 m/s just before it strikes the wall. It comes to a stop 0.8 seconds later. What is the average acceleration of the car during the collision?



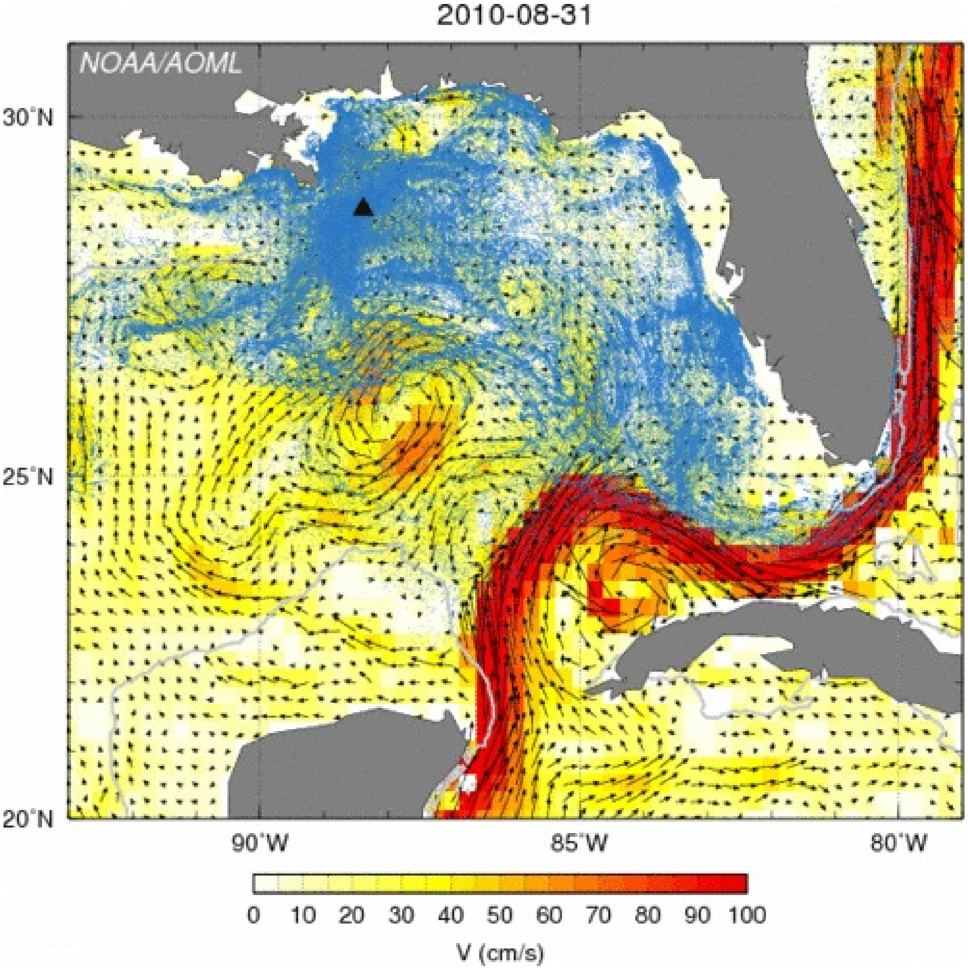
7. A helicopter is traveling with a velocity of 12 m/s directly upward. Directly below the helicopter is a very large and very soft pillow. As it turns out, this is a good thing, because the helicopter is lifting a large man. When the man is 20 m above the pillow, he lets go of the rope.
    a. What is the speed of the man just before he lands on the pillow?
    b. How long is he in the air after he lets go?
    c. What is the greatest height reached by the man above the ground? (Hint: this should be greater than 20 m. Why?
    d. What is the distance between the helicopter and the man three seconds after he lets go of the rope?
8. You are speeding towards a brick wall at a speed of 55 MPH. The brick wall is only 100 feet away.
    a. What is your speed in m/s?
    b. What is the distance to the wall in meters?
    c. What is the minimum acceleration you should use to avoid hitting the wall?
9. What acceleration should you use to increase your speed from 10 m/s to 18 m/s over a distance of 55 m?
10. You drop a rock from the top of a cliff. The rock takes 3.5 seconds to reach the bottom.
    a. What is the initial speed of the rock?
    b. What is the magnitude (i.e., *numerical value*) of the acceleration of the rock at the moment it is dropped?
    c. What is the magnitude of the acceleration of the rock when it is half-way down the cliff?
    d. What is the height of the cliff?

Answers to Selected Problems

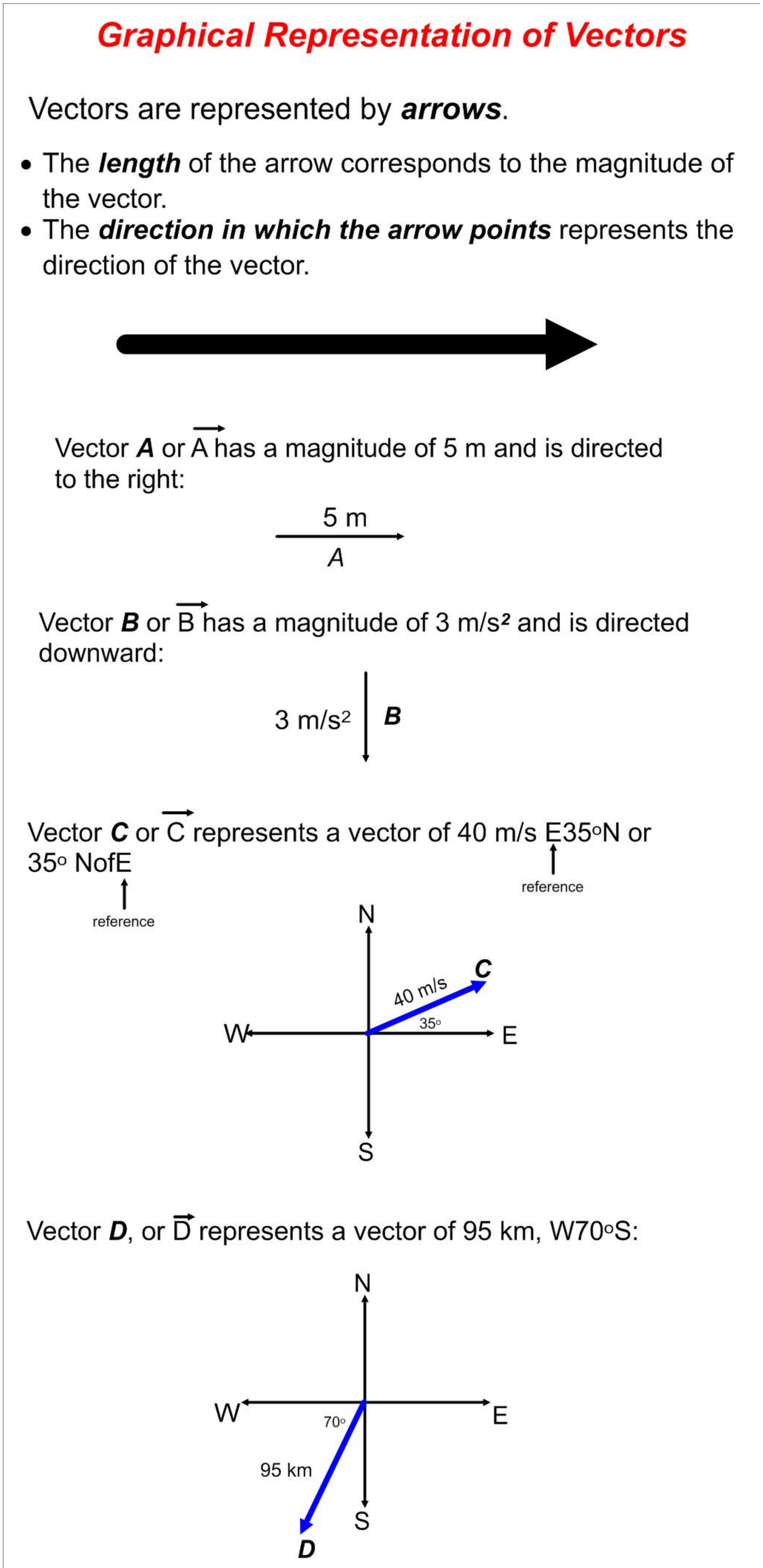
1. a. 7.7 $\mathrm{m/s^2}$ b. 47 m, 150 feet c. 34 m/s
2. a. 1.22 m b. 4.9 m/s c. 2.46 m/s d. -4.9 m/s

3. b. 1 second c. at 2 seconds d. 4 m
4. a. 250 m b. 13 m/s, -13 m/s c. 14 s for round trip
5. Let's say we can jump 20 feet (6.1 m) in the air. ☺ Then, on the moon, we can jump 36.5 m straight up.
6. -31m/s^2
7. 23 m/s b. 3.6 seconds c. 28 m d. 45 m
8. 25 m/s b. 30 m c. 2.5 m/s^2
9. 2.04 m/s^2
10. a. $v_0 = 0$ b. 10 m/s^2 c. $-10\ \text{m/s}^2$ d. 60 m

2.6 Vector Addition



The above image shows water currents for the Gulf of Mexico area near Texas and Florida of the United States. Students will learn how to read vector maps such as this and what information can be learned from it.



Graphical Representation of Vectors

Vectors are represented by ***arrows***.

- The ***length*** of the arrow corresponds to the magnitude of the vector.
- The ***direction in which the arrow points*** represents the direction of the vector.

Vector ***A*** or $\overrightarrow{A}$ has a magnitude of 5 m and is directed to the right:

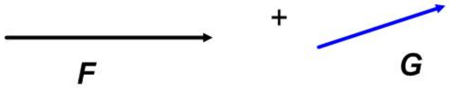
Vector ***B*** or $\overrightarrow{B}$ has a magnitude of 3 m/s² and is directed downward:

Vector ***C*** or $\overrightarrow{C}$ represents a vector of 40 m/s E35°N or 35° NofE

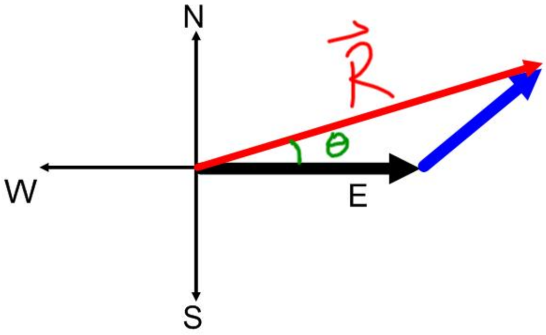
Vector ***D***, or $\overrightarrow{D}$ represents a vector of 95 km, W70°S:

Adding Vectors Graphically

Method #1: Tip-To-Tail Method



To add vectors graphically, they must first be ***lined up tip-to-tail.***



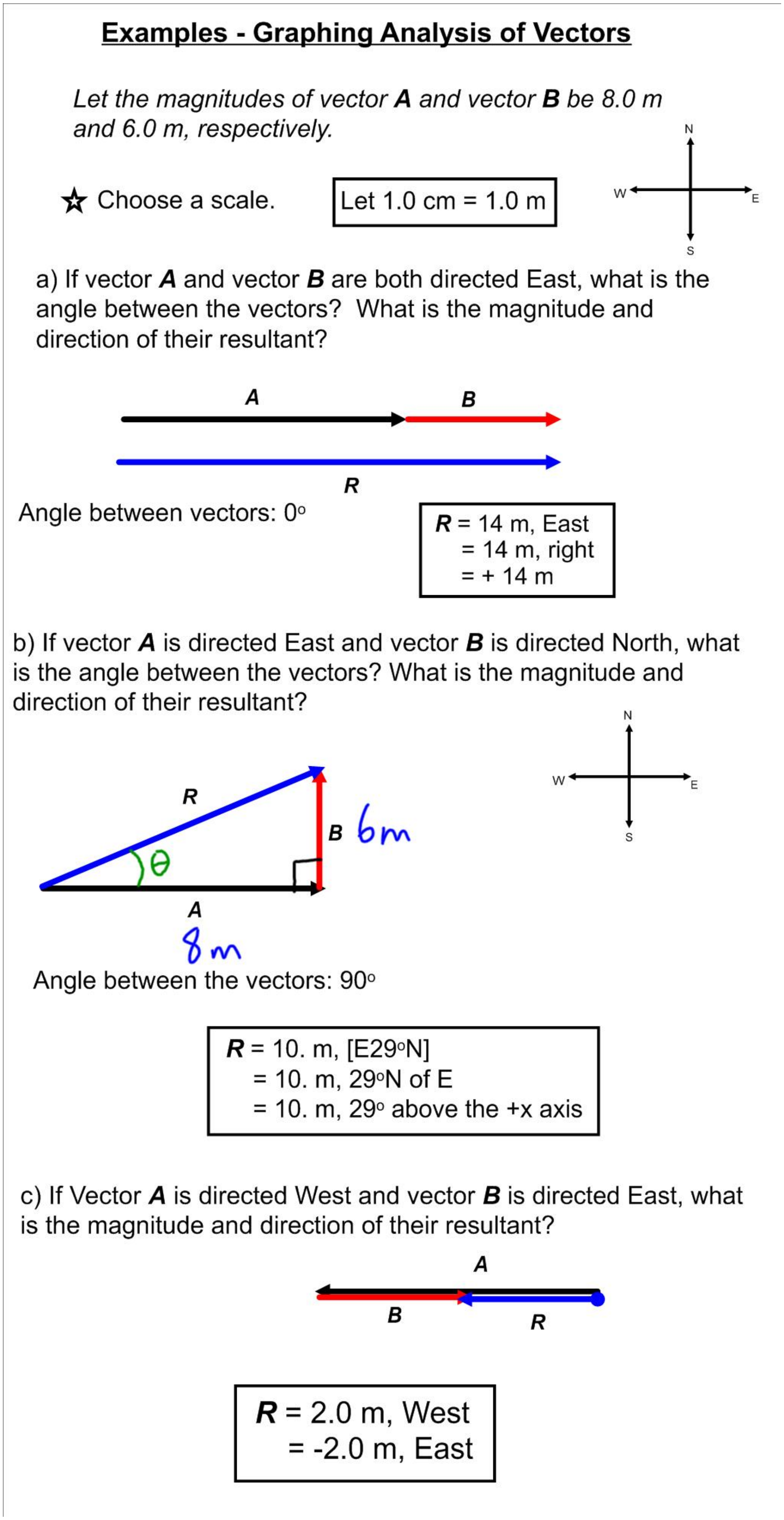
The vector sum of ***F*** and ***G*** is the vector, ***R***. It connects the tail of the first arrow to the tip of the last arrow.

Why is the letter ***R*** used for the vector sum?

Physicists call the vector sum the ***resultant vector*** *or the* ***resultant****.*

Why is the graphical method not considered the best method to use when adding vectors?

If the vectors are not drawn precisely, your final answer will not be accurate.



Examples - Graphing Analysis of Vectors

*Let the magnitudes of vector **A** and vector **B** be 8.0 m and 6.0 m, respectively.*

☆ Choose a scale.

Let 1.0 cm = 1.0 m

a) If vector **A** and vector **B** are both directed East, what is the angle between the vectors? What is the magnitude and direction of their resultant?

Angle between vectors: 0°

R = 14 m, East
= 14 m, right
= + 14 m

b) If vector **A** is directed East and vector **B** is directed North, what is the angle between the vectors? What is the magnitude and direction of their resultant?

Angle between the vectors: 90°

R = 10. m, [E29°N]
= 10. m, 29°N of E
= 10. m, 29° above the +x axis

c) If Vector **A** is directed West and vector **B** is directed East, what is the magnitude and direction of their resultant?

R = 2.0 m, West
= -2.0 m, East

Find the resultant, graphically, for each of the following:

1. From home a car drives 16 km [E], and then 24 km [S].
2. A person runs 2.0 m/s [N] then 4.0 m/s [E30.^{o}N].
3. A ball is kicked 25 m [W20.0^{o}S] then kicked again 35 m [W60.0^{o}N].
4. A basketball is passed 15 m due West, then 20. m due North, and finally 8.0 m due East.
5. A police car drives 70. km due North, then 80. km [E40.^{o}N], and finally 50. km [E50.0^{o}S].
6. A laser beam travels 1500 km [W30.^{o}S], 2100 km [E20.^{o}S], and finally 2700 km [W10.^{o}S].

2.7 References

1. CK-12 Foundation. . CCSA
2. CK-12 Foundation. . CCSA

CHAPTER 3 Introduction to Forces

Chapter Outline

MEDIA

Click image to the left for more content.

Super slow motion video of a hockey shot demonstrating the concept of impulse and momentum.

3.1 Types of Forces and FBDs

Students will learn how to draw a free-body diagram and apply it to the real world.

The Big Idea

Acceleration is caused by force. The more force applied, the greater the acceleration that is produced. Objects with high masses have more inertia and thus resist more strongly changes to its current velocity. In the absence of applied forces, objects simply keep moving at whatever speed they are already going. All forces come in pairs because they arise in the interaction of two objects — you can't hit without being hit back! In formal language[1]:

Newton's 1^{st} *Law: Every body continues in its state of rest, or of uniform motion in a right* (straight) *line, unless it is compelled to change that state by forces impressed upon it.*

Newton's 2^{nd} *Law: The change of motion is proportional to the motive force impressed; and is made in the direction of the right* (straight) *line in which that force is impressed.*

Newton's 3^{rd} *Law: To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*

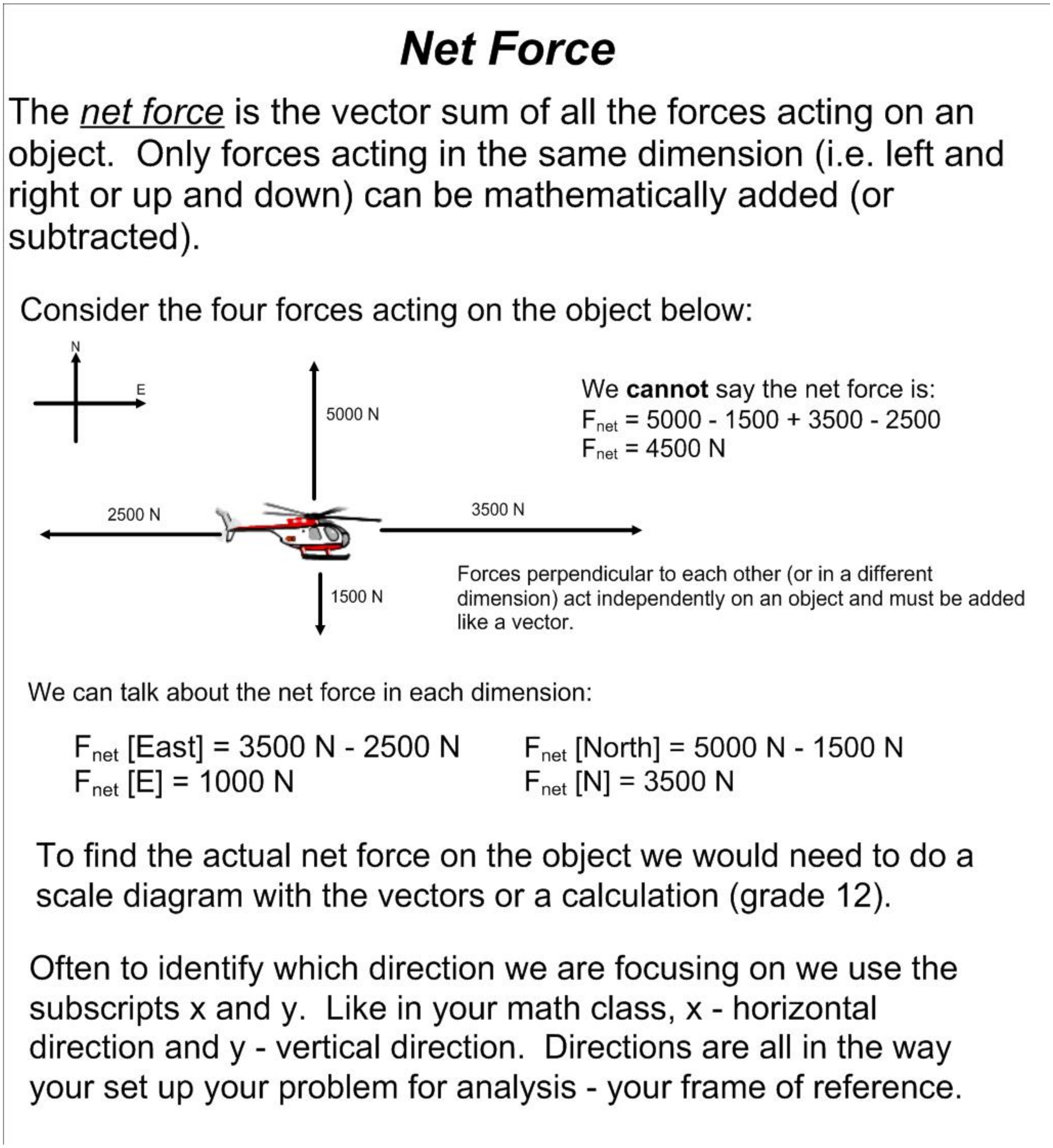
[1] Principia in modern English, Isaac Newton, University of California Press, 1934

Key Concepts

- An object will not change its state of motion (i.e., accelerate) unless an unbalanced force acts on it. Equal and oppositely directed forces do not produce acceleration.
- If no unbalanced force acts on an object the object remains at constant velocity or at rest.
- The force of gravity is called weight and equals m**g**, where g is the acceleration due to gravity of the planet ($g = 9.8\ m/s^2 \sim 10\ m/s^2$, downward, on Earth).
- Your mass does not change when you move to other planets, because mass is a measure of how much *matter* your body contains, and not how much gravitational force you feel.
- Newton's 3^{rd} Law states for every force there is an equal but opposite reaction force. To distinguish a third law pair from merely oppositely directed pairs is difficult but very important. Third law pairs must obey three rules: they must be of the same type of force, they are exerted on two different objects and they are equal in magnitude and oppositely directed. *Example:* A block sits on a table. The Earth's gravity on the block and the force of the table on the block are equal and opposite. **But these are not third law pairs,** because they are both on the same object and the forces are of different types. The proper third law pairs are: (1) earth's gravity on block/block's gravity on earth and (2) table pushes on block/ block pushes on table.
- **Pressure** is often confused with force. Pressure is force spread out over an area; a small force exerted on a very small area can create a very large pressure; i.e., poke a pin into your arm!
- The force of **friction** can actually be described in terms of the *coefficient of friction, μ.* It is determined experimentally and varies depending upon the two surfaces in contact.
- **Static friction** acts between two surfaces that are in contact but not in motion with respect to each other. This force prevents objects from sliding. It always opposes potential motion, and it rises in magnitude to a maximum value given by the formula below.[2]

- **Kinetic friction** acts between two surfaces that are in contact and in motion with respect to each other. This force reduces the acceleration and it always opposes the direction of motion.[2]

[2] Ultimately many of these "contact" forces are due to attractive and repulsive electromagnetic forces between atoms in materials.



Net Force

The *net force* is the vector sum of all the forces acting on an object. Only forces acting in the same dimension (i.e. left and right or up and down) can be mathematically added (or subtracted).

Consider the four forces acting on the object below:

We **cannot** say the net force is:
$F_{net} = 5000 - 1500 + 3500 - 2500$
$F_{net} = 4500$ N

Forces perpendicular to each other (or in a different dimension) act independently on an object and must be added like a vector.

We can talk about the net force in each dimension:

F_{net} [East] = 3500 N - 2500 N
F_{net} [E] = 1000 N

F_{net} [North] = 5000 N - 1500 N
F_{net} [N] = 3500 N

To find the actual net force on the object we would need to do a scale diagram with the vectors or a calculation (grade 12).

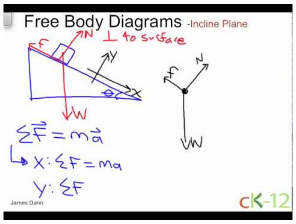
Often to identify which direction we are focusing on we use the subscripts x and y. Like in your math class, x - horizontal direction and y - vertical direction. Directions are all in the way your set up your problem for analysis - your frame of reference.

FIGURE 3.1

Guidance

For every problem involving forces it is essential to draw a free body diagram (FBD) before proceeding to the problem solving stage. The FBD allows one to visualize the situation and also to make sure all the forces are accounted. In addition, a very solid understanding of the physics is gleaned and many questions can be answered solely from the FBD.

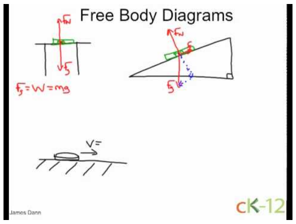
Example 1



MEDIA

Click image to the left for more content.

Watch this Explanation

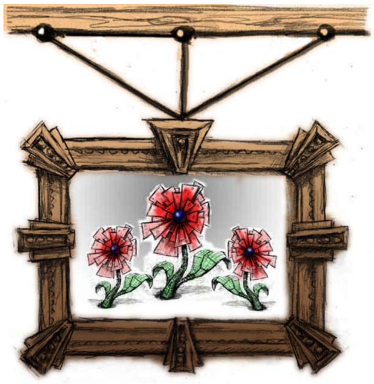


MEDIA

Click image to the left for more content.

Time for Practice

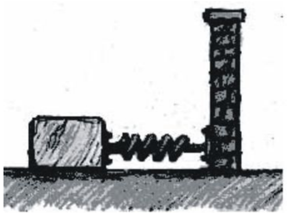
1. Draw free body diagrams (FBDs) for all of the following objects involved (in **bold**) and label all the forces appropriately. Make sure the lengths of the vectors in your FBDs are proportional to the strength of the force: smaller forces get shorter arrows!
   a. A **man** stands in an elevator that is accelerating upward at 2 m/s^2.
   a. A boy is dragging a **sled** at a constant speed. The boy is pulling the sled with a rope at a 30° angle.
   a. The **picture** shown here is attached to the ceiling by three wires.

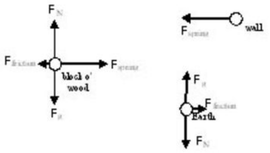


   a. A **bowling ball** rolls down a lane at a constant velocity.

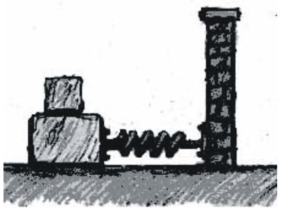
a. A **car** accelerates down the road. There is friction f between the tires and the road.

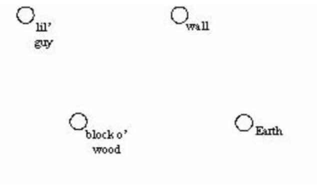
2. For the following situation, identify the 3^{rd} law force pairs on the associated free body diagrams. Label each member of one pair `[U+0080][U+009C]`A, `[U+0080][U+009D]` each member of the next pair `[U+0080][U+009C]`B, `[U+00` and so on. The spring is stretched so that it is pulling the block of wood to the right.





Draw free body diagrams for the situation below. Notice that we are pulling the bottom block *out from beneath* the top block. There is friction between the blocks! After you have drawn your FBDs, identify the 3^{rd} law force pairs, as above.





Answers

Discuss in class

3.2 Newton's First Law

The **First Law is about inertia; objects at rest stay at rest unless acted upon and objects in motion continue that motion in a straight line unless acted upon**. Prior to Newton and Galileo, the prevailing view on motion was still Aristotle's. According to his theory the natural state of things is at rest; force is required to keep something moving at a constant rate. This made sense to people throughout history because on earth, friction and air resistance slow moving objects. When there is no air resistance (or other sources of friction), a situation approximated in space, Newton's first law is much more evident.

The amount of inertia an object has is simply related to the mass of the object. Mass and Weight are two different things. Mass (typically in units of kg or grams) is basically a measure of what comprises an object. Weight is the measure of how much the force of gravity is pulling on you. In fact, instead of saying 'I weigh 80 lb.', one could say that 'the force of gravity is pulling on me with a force of 80 lb.' The metric unit for weight (and force) is the Newton.

Key Equations

$$F_g = mg$$

;

The force of gravity (i.e. your weight) is equal to the mass of the object multiplied by the acceleration of gravity for that planet.

$$1\text{lb.} = 4.45\text{N}$$

Guidance

- An object will not change its state of motion (i.e., accelerate) unless a net force acts on it. Equal and oppositely directed forces do not produce acceleration.
- If no net force acts on an object the object remains at constant velocity or at rest.

Example 1

Question: What is the weight of a 90 kg person on Earth? What about the moon?

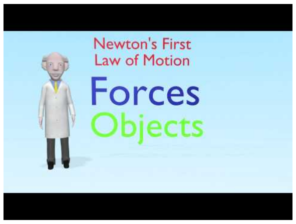
Answers: On Earth,

$$F_g = mg = (90\ kg)(9.8\ m/s^2) = 882\ N$$

On moon,

$$F_g = mg = (90\ kg)(1.6\ m/s^2) = 144\ N$$

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. When hit from behind in a car crash, a passenger can suffer a neck injury called whiplash . Explain in terms of inertia how this occurs, and how headrests can prevent the injury.
2. A cheetah can outrun a gazelle in a short straight race, but the gazelle can escape with its life by zigzagging. The cheetah is more massive than the gazelle – explain how this strategy works.
3. If your hammer develops a loose head, you can tighten it by banging it on the ground. A little physics secret though – it's better to bang the hammer *head up* rather than *head down*. Explain, using inertia.
4. If a man weighs 140 lb. on Earth, what is his weight in Newtons and his mass in kg?

Answers

1. The passenger's head will remain at rest for the split second when the seat exerts a big force on the passenger's back causing a "whiplash" on your neck. This is an example of newton's first law because your head is not acted on by an unbalanced force while the rest of your body is. A head rest causes your head to accelerate with the rest of your body.
2. The cheetah must exert a bigger force to change directions than the gazelle because the cheetah has more inertia. This extra force needed for the cheetah to change directions allows the gazelle to get away.
3. The head of the hammer has more inertia than the tail. So when you bang the hammer "head up" the head exerts a large force on the rest of the hammer in order to come to a stop.
4. 620 N, 62 kg (using 10 m/s/s for acceleration of gravity

3.3 Friction

In this lesson students will learn about friction, the difference between static and kinetic friction and how to solve problems involving friction.

Key Equations

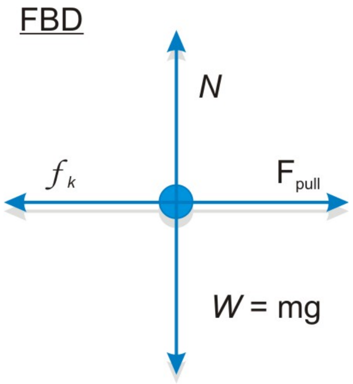
$$\text{Static and Kinetic Friction} \begin{cases} f_s \leq \mu_s |F_N| & \text{Opposes potential motion of surfaces in contact} \\ f_k = \mu_k |F_N| & \text{Opposes motion of surfaces in contact} \end{cases}$$

Guidance

- The force of **friction** can actually be described in terms of the *coefficient of friction,* μ. It is determined experimentally and varies depending upon the two surfaces in contact.
- **Static friction** (μ_s) acts between two surfaces that are in contact but not in motion with respect to each other. This force prevents objects from sliding. It always opposes potential motion, and it rises in magnitude to a maximum value given by the formula below.[2]
- **Kinetic friction** (μ_k) acts between two surfaces that are in contact and in motion with respect to each other. This force reduces the acceleration and it always opposes the direction of motion.[2]

Example 1

Calculate the force necessary to slide a 4.7-kg chair across a room at a constant speed if the coefficient of kinetic friction between the chair and the floor is 0.68.



Question: $F = ?\,[N]$

Given: $m = 4.7\ kg$

$\mu_k = 0.68$

$g = 9.81\ m/s^2$

Equations: $\sum F = ma$

$\sum F_y = N - mg = 0, \text{so } N = mg$

$$\sum F_x = ma_x$$
$$F_{\text{pull}} - f_k = 0 \text{ (because the chair is moving at } constant\ speed, \text{ so } a = 0)$$
$$F_{\text{pull}} = \mu_k N$$

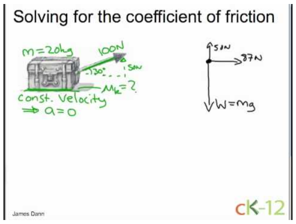
Plug n' Chug: The force necessary to move the chair at a constant speed is equal to the frictional force between the chair and the floor. However in order to calculate the frictional force you must first determine the normal force which is (in this case) equal to the weight (i.e. F_g) of the chair.

$$N = mg = (4.7\ kg)(9.81\ m/s^2) = 47\ N$$
$$F_{\text{pull}} = \mu_k\ N = (0.68)(47\ N) = 32\ N$$

Answer:

$$\boxed{\textbf{32 N}}$$

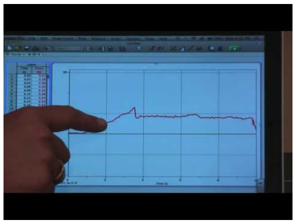
Example 2



MEDIA

Click image to the left for more content.

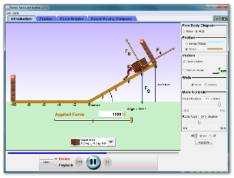
Watch this Explanation



MEDIA

Click image to the left for more content.

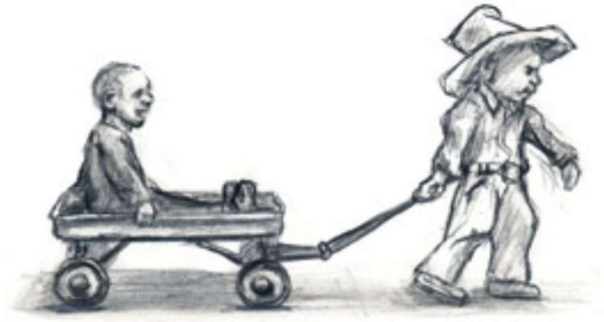
Simulation



RampForces and Motion (PhETSimulation)

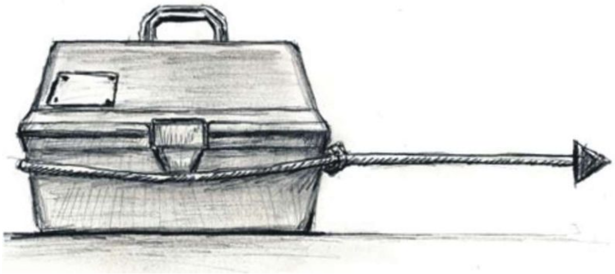
Time for Practice

1. Nathan pulls his little brother in a wagon, using a rope inclined at 30° above the horizontal. The wagon and brother have a total mass of 80 kg, the average coefficient of friction between the wagon wheels and the floor is 0.08, and Nathan pulls on the rope with a force of 100 N.

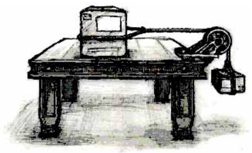


   a. Draw a force diagram for the wagon, labeling all forces.
   b. Calculate the horizontal and vertical components of Nathan's pull. Label them on your diagram (use dotted lines for components so as not to confuse them with other forces).
   c. Calculate the normal force acting on the wagon. (HINT: It is NOT equal to the weight! Use your FBD above).
   d. Calculate the force of friction on the wagon.
   e. Calculate the horizontal acceleration of the wagon.

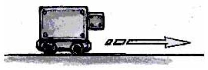
2. When the 20 kg box to the right is pulled with a force of 100 N, it just starts to move (i.e. the maximum value of static friction is overcome with a force of 100 N). What is the value of the coefficient of static friction, μ_S?



3. A different box, this time 5 kg in mass, is being pulled with a force of 20 N and is sliding with an acceleration of 2 m/s^2. Find the coefficient of kinetic friction, μ_K.
4. The large box on the table is 30 kg and is connected via a rope and pulley to a smaller 10 kg box, which is hanging. The 10 kg mass is the highest mass you can hang without moving the box on the table. Find the coefficient of static friction μ_S.



5. A block has a little block hanging out to its side, as shown:



As you know, if the situation is left like this, the little block will just fall. But if we accelerate the leftmost block to the right, this will create a normal force between the little block and the big block, and if there is a coefficient of friction between them, then the little block won't slide down! Clever, eh?

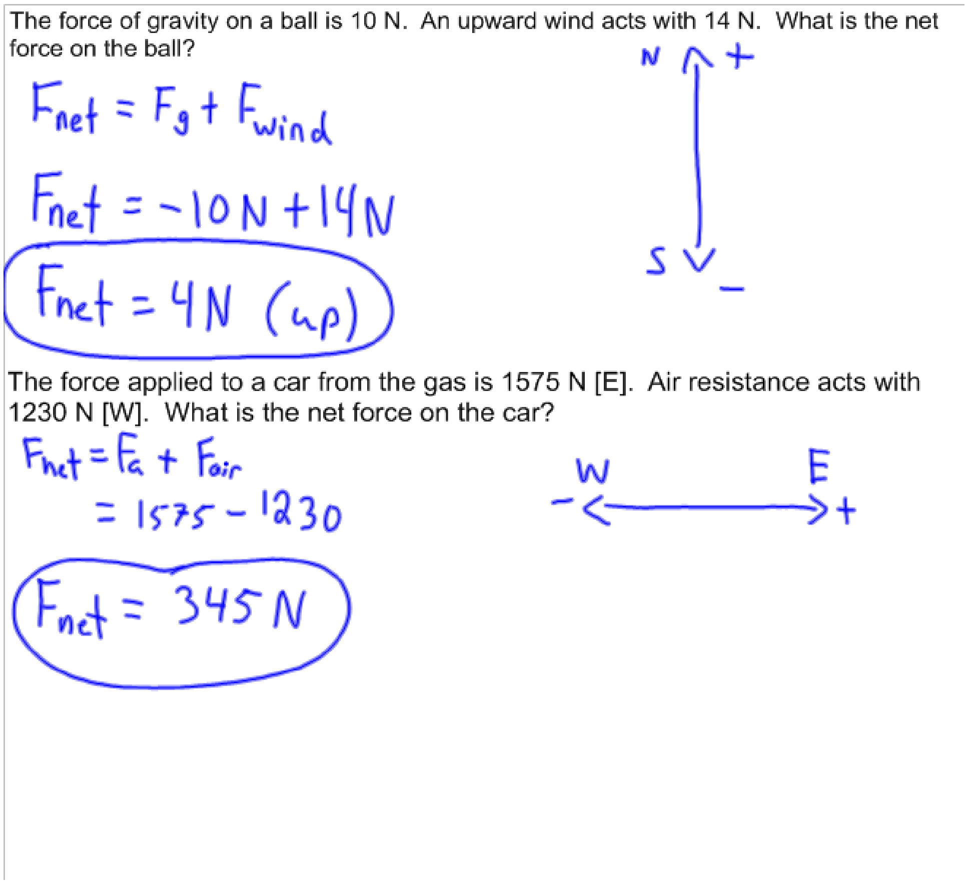
a. The mass of the little block is 0.15 kg. What frictional force is required to keep it from falling? (State a magnitude and direction.)
b. If both blocks are accelerating to the right with an acceleration $a = 14.0\ \text{m}/\text{s}^2$, what is the normal force on the little block provided by the big block?
c. What is the minimum coefficient of static friction required?

Answers

1. b. $F_x = 87\ N, F_y = 50\ N$ c. $N = 750\ N$ d. $f = 60\ N$ e. $0.34\ m/s^2$
2. 0.5
3. 0.2
4. 0.33
5. a. 1.5 N; 2.1 N; 0.71

3.4 Friction and Net Force Examples

Friction & Net Force Examples



The force of gravity on a ball is 10 N. An upward wind acts with 14 N. What is the net force on the ball?

$F_{net} = F_g + F_{wind}$

$F_{net} = -10\,N + 14\,N$

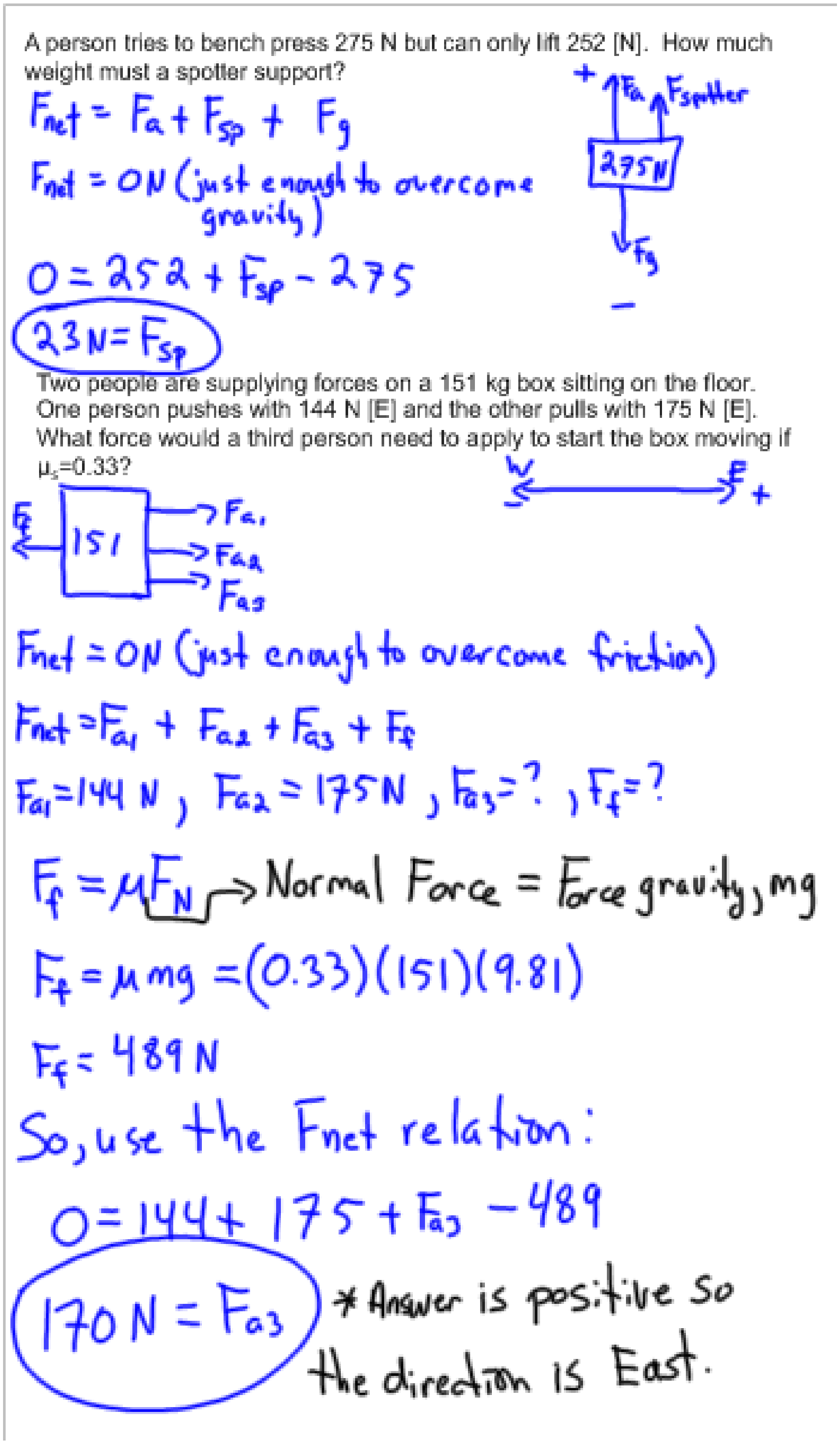
$\boxed{F_{net} = 4N \text{ (up)}}$

The force applied to a car from the gas is 1575 N [E]. Air resistance acts with 1230 N [W]. What is the net force on the car?

$F_{net} = F_a + F_{air}$

$= 1575 - 1230$

$\boxed{F_{net} = 345\,N}$



A person tries to bench press 275 N but can only lift 252 [N]. How much weight must a spotter support?

$F_{net} = F_a + F_{sp} + F_g$

$F_{net} = 0N$ (just enough to overcome gravity)

$0 = 252 + F_{sp} - 275$

$\boxed{23N = F_{sp}}$

Two people are supplying forces on a 151 kg box sitting on the floor. One person pushes with 144 N [E] and the other pulls with 175 N [E]. What force would a third person need to apply to start the box moving if μ_s=0.33?

$F_{net} = 0N$ (just enough to overcome friction)

$F_{net} = F_{a1} + F_{a2} + F_{a3} + F_f$

$F_{a1} = 144\ N$, $F_{a2} = 175N$, $F_{a3} = ?$, $F_f = ?$

$F_f = \mu \underbrace{F_N} \rightarrow$ Normal Force = Force gravity, mg

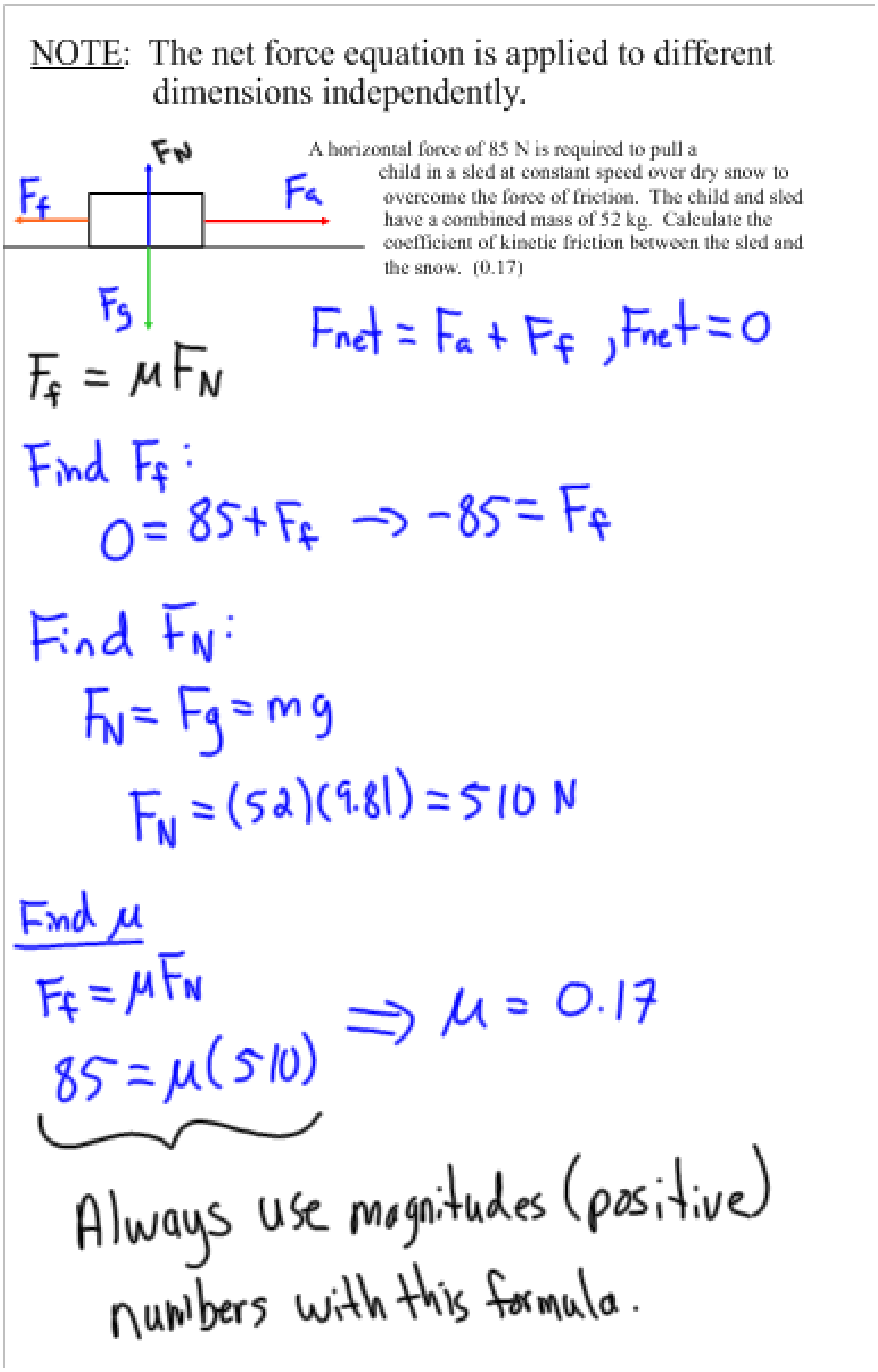
$F_f = \mu mg = (0.33)(151)(9.81)$

$F_f = 489\ N$

So, use the Fnet relation:

$0 = 144 + 175 + F_{a3} - 489$

$\boxed{170\ N = F_{a3}}$ * Answer is positive so the direction is East.



NOTE: The net force equation is applied to different dimensions independently.

A horizontal force of 85 N is required to pull a child in a sled at constant speed over dry snow to overcome the force of friction. The child and sled have a combined mass of 52 kg. Calculate the coefficient of kinetic friction between the sled and the snow. (0.17)

$F_{net} = F_a + F_f$, $F_{net} = 0$

$F_f = \mu F_N$

Find F_f:

$0 = 85 + F_f \rightarrow -85 = F_f$

Find F_N:

$F_N = F_g = mg$

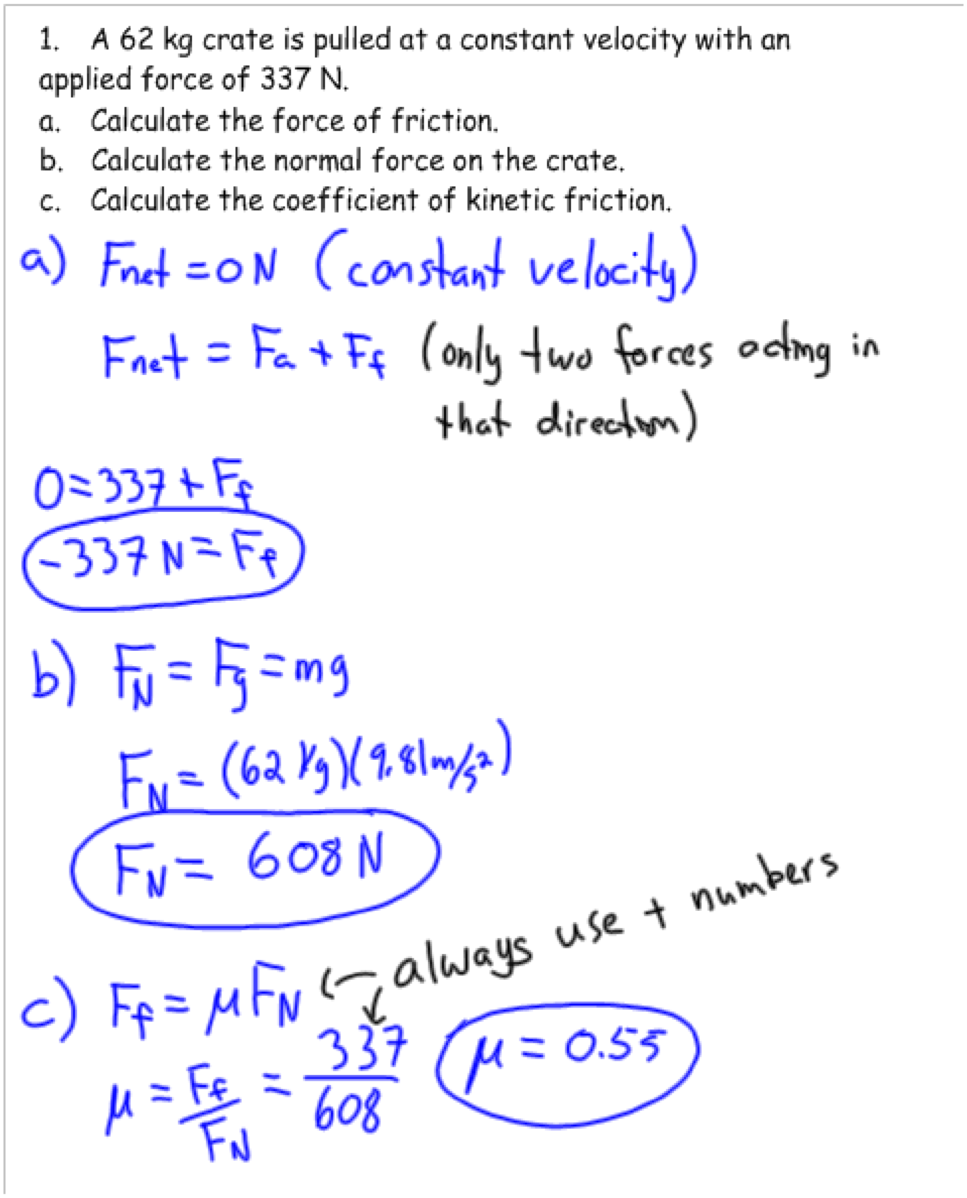
$F_N = (52)(9.81) = 510$ N

Find μ

$F_f = \mu F_N$

$\underbrace{85 = \mu(510)} \Rightarrow \mu = 0.17$

Always use magnitudes (positive) numbers with this formula.



1. A 62 kg crate is pulled at a constant velocity with an applied force of 337 N.

a. Calculate the force of friction.

b. Calculate the normal force on the crate.

c. Calculate the coefficient of kinetic friction.

a) $F_{net} = 0\text{ N}$ (constant velocity)

$F_{net} = F_a + F_f$ (only two forces acting in that direction)

$0 = 337 + F_f$

$\boxed{-337\text{ N} = F_f}$

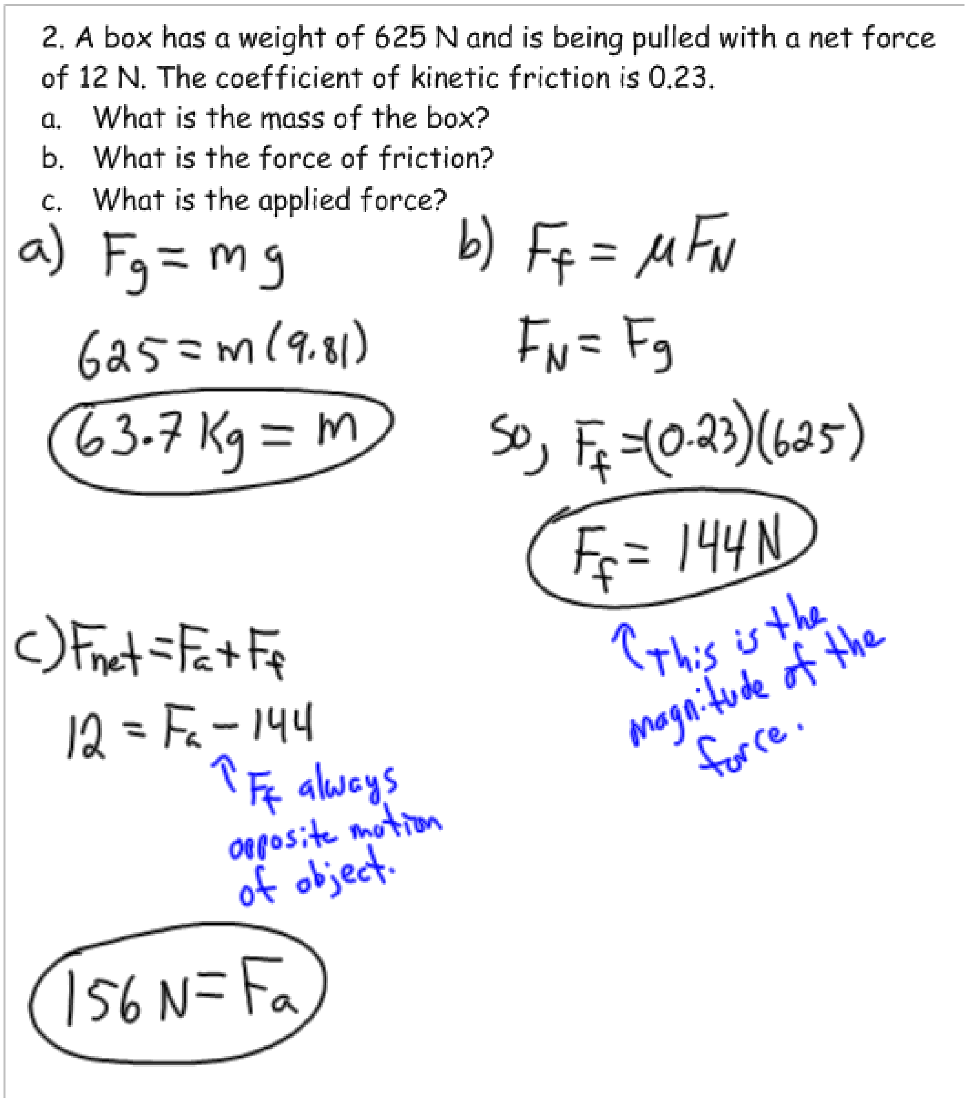
b) $F_N = F_g = mg$

$F_N = (62\text{ kg})(9.81\text{ m/s}^2)$

$\boxed{F_N = 608\text{ N}}$

c) $F_f = \mu F_N$ ← always use + numbers

$\mu = \frac{F_f}{F_N} = \frac{337}{608}$ $\boxed{\mu = 0.55}$



2. A box has a weight of 625 N and is being pulled with a net force of 12 N. The coefficient of kinetic friction is 0.23.

a. What is the mass of the box?

b. What is the force of friction?

c. What is the applied force?

a) $F_g = mg$

$625 = m(9.81)$

$\boxed{63.7\text{ Kg} = m}$

b) $F_f = \mu F_N$

$F_N = F_g$

So, $F_f = (0.23)(625)$

$\boxed{F_f = 144\text{ N}}$

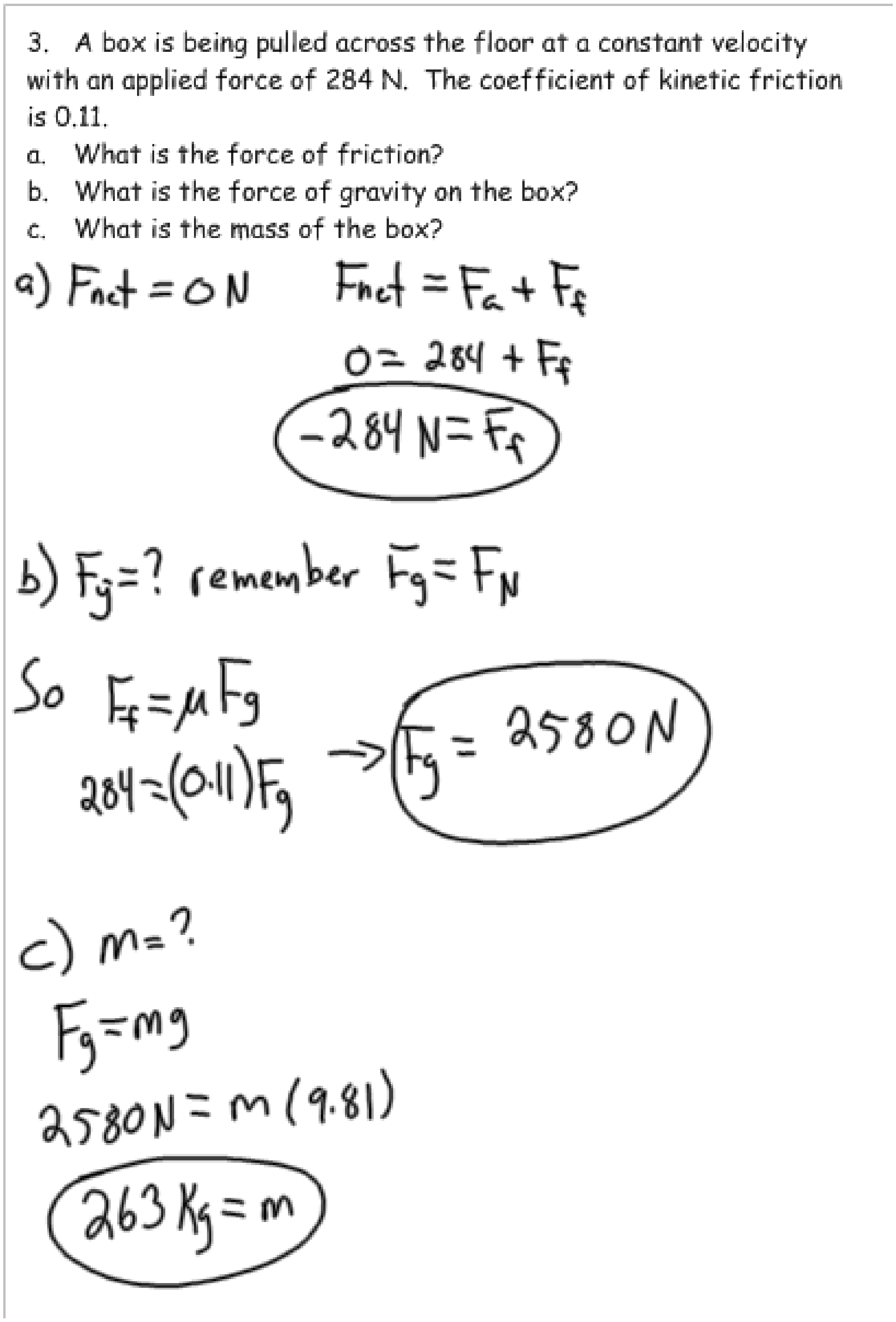
↑ this is the magnitude of the force.

c) $F_{net} = F_a + F_f$

$12 = F_a - 144$

↑ F_f always opposite motion of object.

$\boxed{156\text{ N} = F_a}$



3. A box is being pulled across the floor at a constant velocity with an applied force of 284 N. The coefficient of kinetic friction is 0.11.

a. What is the force of friction?

b. What is the force of gravity on the box?

c. What is the mass of the box?

a) $F_{net} = 0\text{ N}$ $\quad F_{net} = F_a + F_f$

$0 = 284 + F_f$

$\boxed{-284\text{ N} = F_f}$

b) $F_g = ?$ remember $F_g = F_N$

So $F_f = \mu F_g$

$284 = (0.11) F_g \rightarrow \boxed{F_g = 2580\text{ N}}$

c) $m = ?$

$F_g = mg$

$2580\text{ N} = m(9.81)$

$\boxed{263\text{ Kg} = m}$

3.5 Newton's Second Law

The acceleration experienced by an object will be proportional to the applied force and inversely proportional to its mass. If there are multiple forces, they can be added as vectors and it is the *net* force that matters.

Key Equations

Newton's Second Law describes his famous equation for the motion of an object

> *The change of motion is proportional to the motive force impressed; and is made in the direction of the right* (straight) *line in which that force is impressed.*

The "motion" Newton mentions in the Second Law is, in his language, the product of the mass and velocity of an object — we call this quantity momentum — so **the Second Law is actually the** famous equation**:**

$$\vec{F} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = m\vec{a} \qquad [1]$$

$$\text{Force Sums} \begin{cases} \vec{F_{\text{net}}} = \sum_i F_i = m\vec{a} & \text{Net force is the vector sum of all the forces} \\ F_{\text{net},x} = \sum_i F_{ix} = ma_x & \text{Horizontal components add also} \\ F_{\text{net},y} = \sum_i F_{iy} = ma_y & \text{As do vertical ones} \end{cases}$$

Guidance

To calculate the net force on an object, you need to calculate all the individual forces acting on the object and then add them as vectors. This requires some mathematical skill.

Example 1

A 175-g bluebird slams into a window with a force of 19.0 N. What is the bird's acceleration?

FBD

Question: $a = ?\,[m/s^2]$

Given: $m = 175\ grams = 0.175\ kg$

$F = 19.0\ N$

Equation: $a = \frac{F_{net}}{m}$

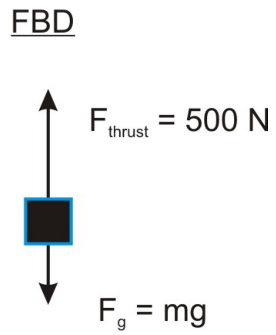
Plug n' Chug: $a = \frac{F_{net}}{m} = \frac{19.0\ N}{0.175\ kg} = \frac{19.0 \frac{kg \cdot m}{s^2}}{0.175\ kg} = 109 \frac{m}{s^2}$

Answer:

$$\boxed{\mathbf{109\ m/s^2}}$$

Example 2

Calculate the acceleration of a rocket that has 500N of thrust force and a mass of 10kg.



Question: $a = ?\,[m/s^2]$

Given: $m = 10\ kg$

$F_{\text{thrust}} = 500\ N$

$g = 9.81\ m/s^2$

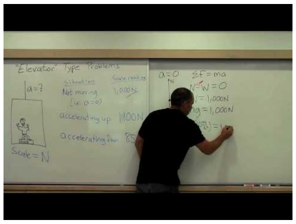
Equations: $\sum F_{\text{individual forces}} = ma$

or, in this case, $\sum F_{y-\text{direction forces}} = ma_y$

Plug n' Chug: Use FBD to "fill in" Newton's second law equation:

$$\begin{aligned}\sum F_{y-\text{direction forces}} &= ma_y\\ F - Mg &= Ma\\ 500N - 10\ kg(9.81\ m/s^2) &= 10kg(a)\\ a &= 40\ m/s^2\end{aligned}$$

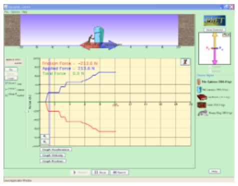
Watch this Explanation



MEDIA

Click image to the left for more content.

Simulation



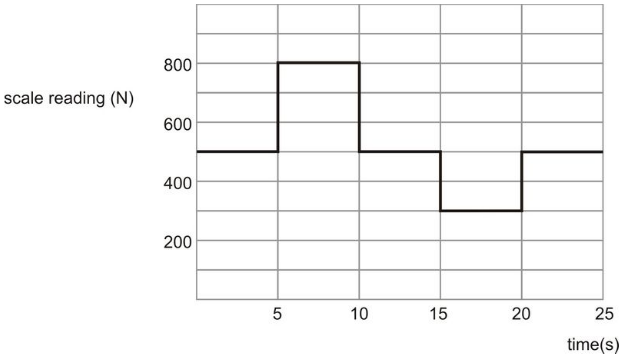
Forces in 1 Dimension (PhET Simulation)

Time for Practice

1. During a rocket launch, the rocket's acceleration increases greatly over time. Explain, using Newton's Second Law. (Hint: most of the mass of a rocket on the launch pad is fuel).
2. When pulling a paper towel from a paper towel roll, why is a quick jerk more effective than a slow pull?
3. You pull a wagon with a force of 20 N. The wagon has a mas of 10 kg. What is the wagon's acceleration?
4. The man is hanging from a rope wrapped around a pulley and attached to both of his shoulders. The pulley is fixed to the wall. The rope is designed to hold 500 N of weight; at higher tension, it will break. Let's say he has a mass of 80 kg. Draw a free body diagram and explain (using Newton's Laws) whether or not the rope will break.

5. Now the man ties one end of the rope to the ground and is held up by the other. Does the rope break in this situation? What precisely is the difference between this problem and the one before?

6. A crane is lowering a box of mass 50 kg with an acceleration of 2.0 m/s^2.
   a. Find the tension F_T in the cable.
   b. If the crane lowers the box at a constant speed, what is the tension F_T in the cable?
7. A physics student weighing 500 N stands on a scale in an elevator and records the scale reading over time. The data are shown in the graph below. At time $t = 0$, the elevator is at rest on the ground floor.

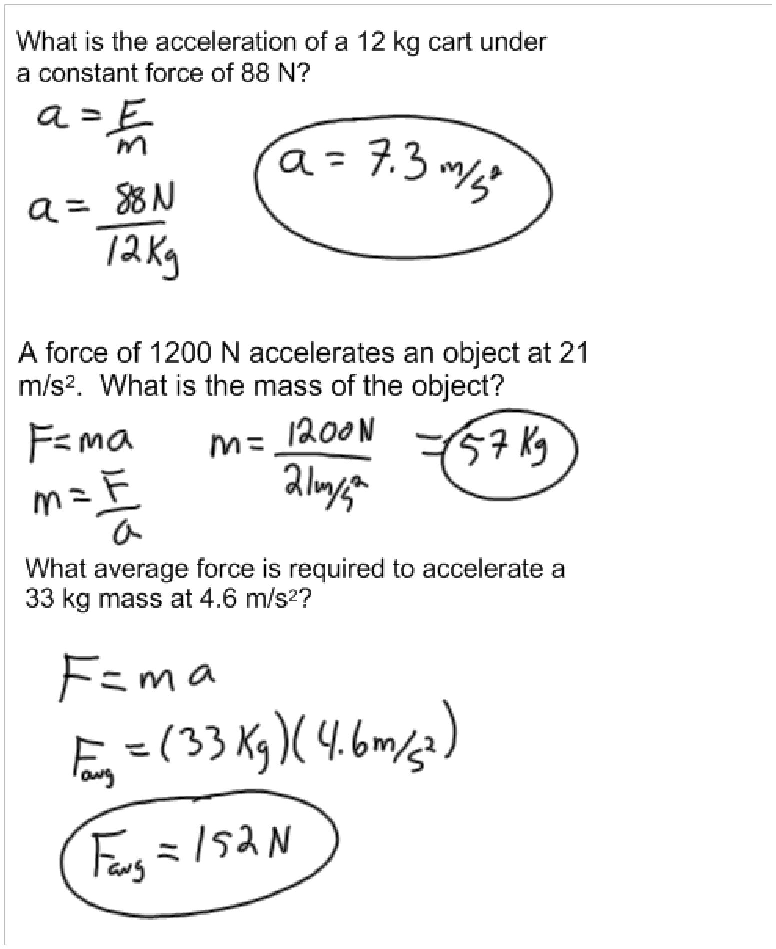


a. Draw a FBD for the person, labeling all forces.
b. What does the scale read when the elevator is at rest?
c. Calculate the acceleration of the person from 5-10 sec.
d. Calculate the acceleration of the person from 10-15 sec. Is the passenger at rest?
e. Calculate the acceleration of the person from 15-20 sec. In what direction is the passenger moving?
f. Is the elevator at rest at $t = 25\ s$. Justify your answer.

Answers (using $g = 10\ m/s^2$):

1. According to Newton's second law: the acceleration of an object is inversely proportional to it's mass, so if you decrease it's mass while keeping the net force the same, the acceleration will increase.
2. When you jerk the paper towel, the paper towel that you are holding onto will accelerate much more quickly than the entire roll causing it to rip. Again, acceleration is inversely proportional to the mass of the object.
3. $2m/s^2$
4. The rope will not break because his weight of 800 N is distributed between the two ropes.
5. Yes, because his weight of 800 N is greater than what the rope can hold.
6. a. 400 N b. 500 N
7. b. 500 N c. 6 m/s^2 d. 0 e. $-4\ m/s^2$

3.6 Newton's 2nd Law Examples



What is the acceleration of a 12 kg cart under a constant force of 88 N?

$a = \frac{F}{m}$

$a = \frac{88 N}{12 Kg}$

$a = 7.3\ m/s^2$

A force of 1200 N accelerates an object at 21 m/s². What is the mass of the object?

$F = ma$

$m = \frac{F}{a}$

$m = \frac{1200 N}{21 m/s^2} = 57\ Kg$

What average force is required to accelerate a 33 kg mass at 4.6 m/s²?

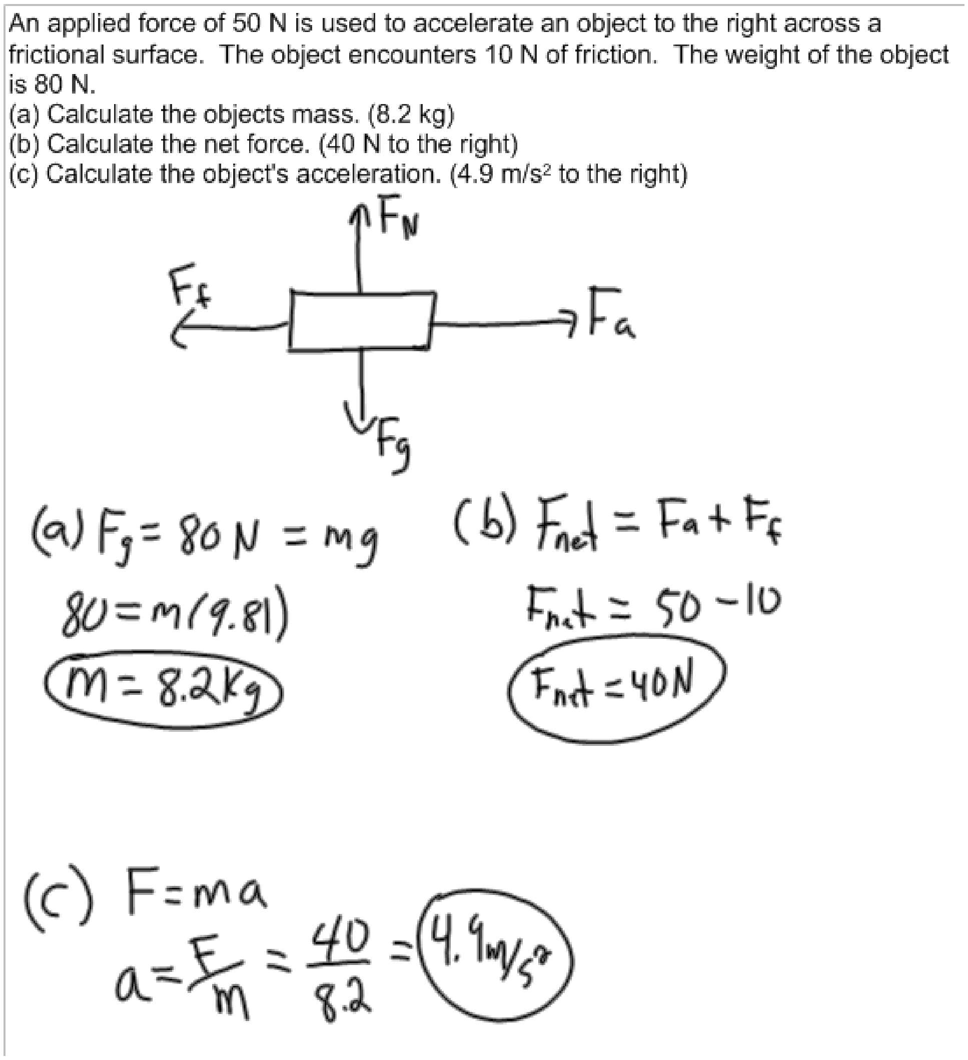
$F = ma$

$F_{avg} = (33\ Kg)(4.6\ m/s^2)$

$F_{avg} = 152\ N$

FIGURE 3.2

Force Examples Using Newton's Second Law



An applied force of 50 N is used to accelerate an object to the right across a frictional surface. The object encounters 10 N of friction. The weight of the object is 80 N.

(a) Calculate the objects mass. (8.2 kg)

(b) Calculate the net force. (40 N to the right)

(c) Calculate the object's acceleration. (4.9 m/s² to the right)

(a) $F_g = 80\text{ N} = mg$

$80 = m(9.81)$

$\boxed{m = 8.2\text{ Kg}}$

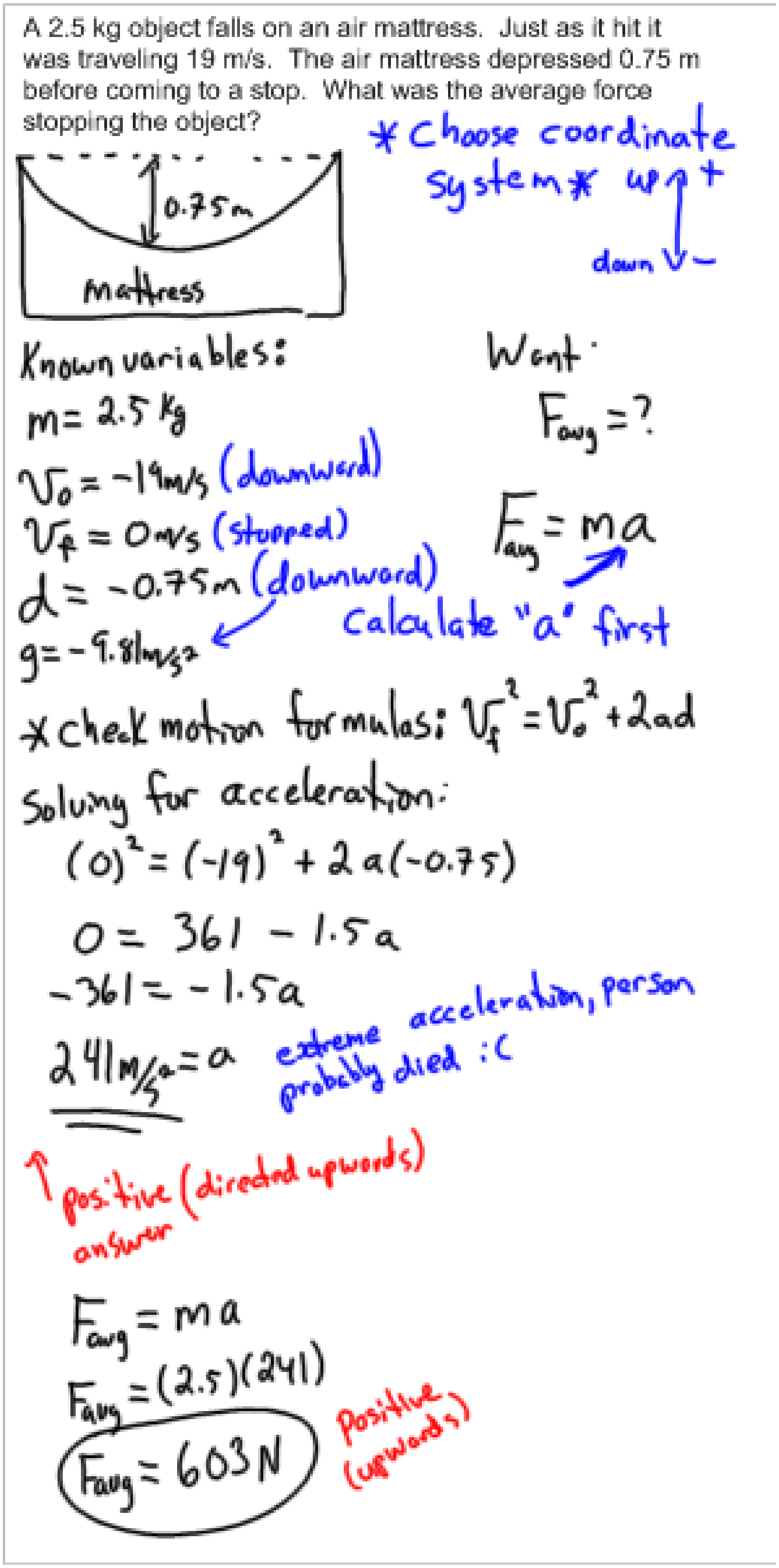
(b) $F_{net} = F_a + F_f$

$F_{net} = 50 - 10$

$\boxed{F_{net} = 40\text{N}}$

(c) $F = ma$

$a = \frac{F}{m} = \frac{40}{8.2} = \boxed{4.9\text{ m/s}^2}$



A 2.5 kg object falls on an air mattress. Just as it hit it was traveling 19 m/s. The air mattress depressed 0.75 m before coming to a stop. What was the average force stopping the object?

* Choose coordinate system * up ↑ +

down ↓ −

Known variables:

$m = 2.5\ kg$

$v_0 = -19\ m/s$ (downward)

$v_f = 0\ m/s$ (stopped)

$d = -0.75\ m$ (downward)

$g = -9.8\ m/s^2$

Want:

$F_{avg} = ?$

$F_{avg} = ma$

Calculate "a" first

* check motion formulas: $v_f^2 = v_0^2 + 2ad$

Solving for acceleration:

$$(0)^2 = (-19)^2 + 2a(-0.75)$$

$$0 = 361 - 1.5a$$

$$-361 = -1.5a$$

$$\underline{241\ m/s^2 = a}$$

extreme acceleration, person probably died :(

↑ positive (directed upwords) answer

$$F_{avg} = ma$$

$$F_{avg} = (2.5)(241)$$

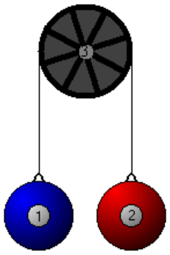
$$\boxed{F_{avg} = 603\ N}$$

Positive (upwards)

3.7 Multiple Connected Masses

Multiple Masses and Finding Net Force

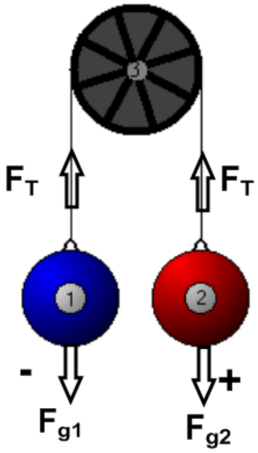
Chapter 10.2 of MHR:
Read Pg 478 - 489
Problems Pg 485 #s 19 - 22, Pg 488 #s 24 - 28



This is an example of a system where there are multiple masses, the Atwood machine.

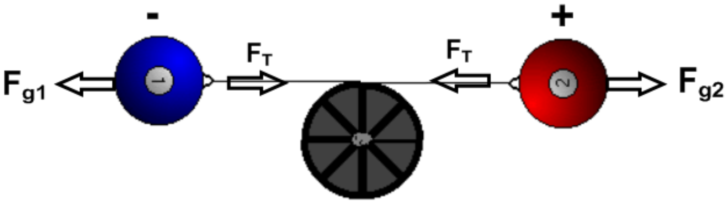
We will apply the concept of forces to determine the resulting acceleration.

Define the Direction of Forces

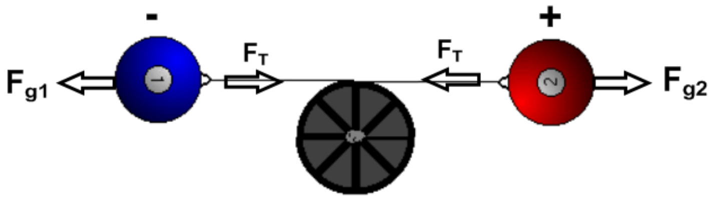


Our problems will not include friction and the pulley will be massless.

You may find it easier to picture, or draw, the system horizontally.



Acceleration of the Masses: Newton's 2nd Law



$$F_{net} = ma$$

$$F_{net} = \sum Forces$$

$$m = \sum masses\ that\ accelerate$$

$$F_{g1} + F_T + F_T + F_{g2} = (m_1 + m_2)a$$

$$-m_1 g + F_T + (-F_T) + m_2 g = (m_1 + m_2)a$$

$$m_2 g - m_1 g = (m_1 + m_2)a$$

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$☆ \quad \vec{a} = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

To Find Tension:

$$F_{g1} + F_T = m_1 a$$

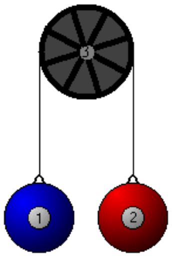
or

$$F_{g2} + F_T = m_2 a$$

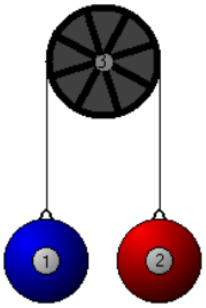
☆ How would the formula for acceleration change if positive was to the left?

Atwood Machine Examples

What is the acceleration of an Atwood machine with masses of 6.5 kg and 15 kg on opposite sides of the pulley? What is the magnitude of the force of tension in the rope?

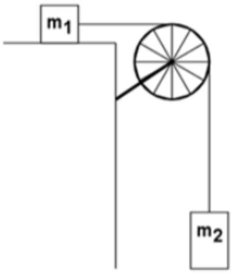


A counter weight of 25 kg is used to help a person of mass 85 kg to do chin ups. What is the force applied by the person if he accelerates at 1.2 m/s^2?



A counterbalance is set up to help someone lift an object. The largest mass a person can lift is 33 kg. What must be the minimum mass of the counter weight for a 55 kg object to be lifted with an acceleration of 1.5 m/s 2 ? (mass = 36 kg)

In the diagram below m$_1$ = 425 g, m$_2$ = 735 g, the coefficient of kinetic friction is 0.34, and there is not friction from the pulley and string. What is the acceleration of the masses?



3.8 Newton's Third Law

Students will learn that Newton's 3rd law holds that for every force there is an equal but opposite reaction force. Important to note that the force and reaction force act on different objects.

Key Equations

$\vec{F} = -\vec{F}'$

Guidance

Newton's 3^{rd} Law states for every force there is an equal but opposite reaction force. To distinguish a third law pair from merely oppositely directed pairs is difficult, but very important. Third law pairs must obey three rules: (1) Third law force pairs must be of the same type of force. (2) Third law force pairs are exerted on two different objects. (3) Third law force pairs are equal in magnitude and oppositely directed. *Example:* A block sits on a table. The Earth's gravity on the block and the force of the table on the block are equal and opposite. **But these are not third law pairs,** because they are both on the same object and the forces are of different types. The proper third law pairs are: (1) earth's gravity on block/block's gravity on earth and (2) table pushes on block/ block pushes on table.

Example 1

Question: Tom and Mary are standing on identical skateboards. Tom and Mary push off of each other and travel in opposite directions.

a) If Tom (M) and Mary (m) have identical masses, who travels farther?

b) If Tom has a bigger mass than Mary, who goes farther?

c) If Tom and Mary have identical masses and Tom pushes twice as hard as Mary, who goes farther?

Solution

a) Neither. Both Tom and Mary will travel the same distance. The forced applied to each person is the same (Newton's Third Law). So

$$\cancel{M}a = \cancel{m}a$$

which cancels to

$$a = a$$

Therefore both people will travel the same distance because the acceleration controls how far someone will travel and Tom and Mary have equal acceleration.

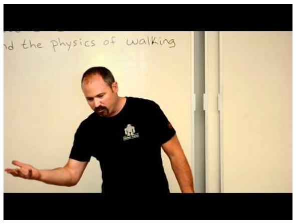
b) Mary will go farther. Again, the same force is applied to both Mary and Tom so

$$Ma = ma$$

Since Tom has the larger mass, his acceleration must be smaller (acceleration and mass are inversely proportional). Finally, because Mary's acceleration is greater, she will travel farther.

c) Neither. Newton's Third Law states that for every action there is an equal and opposite reaction. Therefore if Tom pushes twice as hard as Mary, Mary will essentially be pushing back with the same strength. They will therefore travel the same distance.

Watch this Explanation

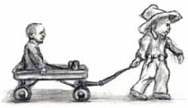


MEDIA

Click image to the left for more content.

Time for Practice

1. You are standing on a bathroom scale. Can you reduce your weight by pulling up on your shoes? (Try it.)
2. A VW Bug hits a huge truck head-on. Each vehicle was initially going 50 MPH.
    a. Which vehicle experiences the greater force?
    b. Which experiences the greater acceleration? Explain briefly.
3. You and your friend are standing on identical skateboards with an industrial-strength compressed spring in between you. After the spring is released, it falls straight to the ground and the two of you fly apart.
    a. If you have identical masses, who travels farther?
    b. If your friend has a bigger mass who goes farther?
    c. If your friend has a bigger mass who feels the larger force?
    d. If you guys have identical masses, even if you push on the spring, why isn't it possible to go further than your friend?
4. Analyze the situation shown here with a big kid pulling a little kid in a wagon. You'll notice that there are a lot of different forces acting on the system. Let's think about what happens the moment the sled begins to move.



    a. First, draw the free body diagram of the big kid. Include all the forces you can think of, including friction. Then do the same for the little kid.
    b. Identify all third law pairs. Decide which forces act on the two body system and which are extraneous.
    c. Explain what conditions would make it possible for the two-body system to move forward.

3.9 References

1. CK-12 Foundation. . CCSA

CHAPTER 4

Waves and Sound

Chapter Outline

MEDIA

Click image to the left for more content.

The Wii Remote was used to measure its change in acceleration with time. That data gives the period of the oscillations. As the Wii Remote vibrates up a down the graph maps the change in acceleration in real time.

4.1 Period and Frequency

Students will learn the concepts of period and frequency and how to read them off a graph and how to calculate them for systems in harmonic motion. In addition, students will learn how to graph simple harmonic motion.

Students will learn the concepts of period and frequency and how to read them off a graph and how to calculate them for systems in harmonic motion. In addition, students will learn how to graph simple harmonic motion.

Key Equations

$$\text{Period Equations}\begin{cases} T = \frac{1}{f} & \text{Period is the inverse of frequency} \\ T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}} & \text{Period of mass } m \text{ on a spring with constant } k \\ T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}} & \text{Period of a pendulum of length } L \end{cases}$$

$$\text{Kinematics of SHM}\begin{cases} x(t) = x_0 + A\cos 2\pi f(t-t_0) & \text{Position of an object in SHM of Amplitude } A \\ v(t) = -2\pi f A\cos 2\pi f(t-t_0) & \text{Velocity of an object in SHM of Amplitude } A \end{cases}$$

Guidance

Example 1

A bee flaps its wings at a rate of approximately 190 Hz. How long does it take for a bee to flap its wings once (down and up)?

Question: $T = ?$ [sec]

Given: $f = 190\ Hz$

Equation: $T = \frac{1}{f}$

Plug n' Chug: $T = \frac{1}{f} = \frac{1}{190\ Hz} = 0.00526\ s = 5.26\ ms$

Answer:

$$\boxed{\textbf{5.26 ms}}$$

Example 2

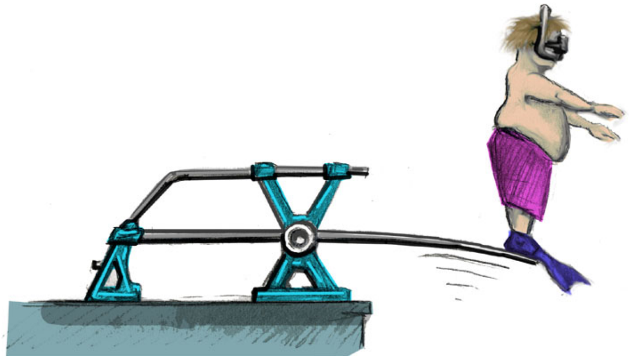
Question: The effective k of a diving board is 800N/m (we say effective because it bends in the direction of motion instead of stretching like a spring, but otherwise behaves the same). A pudgy diver is bouncing up and down at the end of the diving board. The y vs. t graph is shown below.

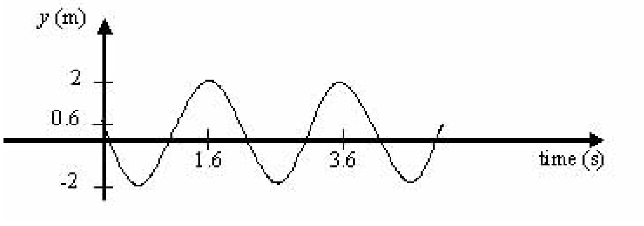
a) What is the distance between the lowest and the highest point of oscillation?

b) What is the Period and frequency of the diver?

c) What is the diver's mass?

d) Write the sinusoidal equation of motion for the diver.





Solution:

a) As we can see from the graph the highest point is 2m and the lowest point is -2m. Therefore the distance is

$$|2\text{m}-(-2\text{m})|=4\text{m}$$

b)We know that

$$f=\frac{1}{T}$$

From the graph we know that the period is 2 seconds, so the frequency is $\frac{1}{2}$hz.

c) To find the diver's mass we will use the equation

$$T=2\pi\sqrt{\frac{m}{k}}$$

and solve for m. Then it is a simple matter to plug in the known values to get the mass.

$$T=2\pi\sqrt{\frac{m}{k}}\Rightarrow\frac{T}{2\pi}=\sqrt{\frac{m}{k}}\Rightarrow(\frac{T}{2\pi})^2=\frac{m}{k}\Rightarrow k(\frac{T}{2\pi})^2=m$$

Now we plug in what we know.

$$m=k(\frac{T}{2\pi})^2=800\frac{N}{m}(\frac{\pi\text{s}}{2\pi})^2=200\text{kg}$$

d) To get the sinusoidal equation we must first choose whether to go with a cosine graph or a sine graph. Then we must find the amplitude (A), vertical shift (D), horizontal shift (C), and period (B). Cosine is easier in this case so we will work with it instead of sine. As we can see from the graph, the amplitude is 2, the vertical shift is 0, and the horizontal shift is $-.4$. We solved for the period already. Therefore, we can write the sinusoidal equation of this graph.

$$AcosB(x-C)+D=2cos\pi(x+.4)$$

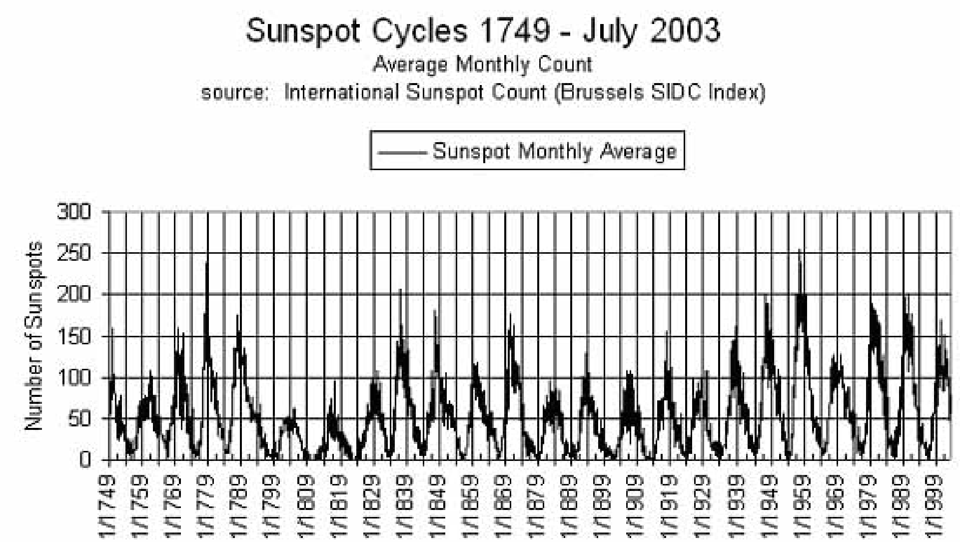
Watch this Explanation

MEDIA

Click image to the left for more content.

Time for Practice

1. While treading water, you notice a buoy way out towards the horizon. The buoy is bobbing up and down in simple harmonic motion. You only see the buoy at the most upward part of its cycle. You see the buoy appear 10 times over the course of one minute.
   a. What kind of force that is leading to simple harmonic motion?
   b. What is the period (T) and frequency (f) of its cycle? Use the proper units.
2. The pitch of a Middle C note on a piano is 263 Hz. This means when you hear this note, the hairs in your ears wiggle back and forth at this frequency.
   a. What is the period of oscillation for your ear hairs?
   b. What is the period of oscillation of the struck wire within the piano?
3. The Sun tends to have dark, Earth-sized spots on its surface due to kinks in its magnetic field. The number of visible spots varies over the course of years. Use the graph of the sunspot cycle above to answer the following questions. (Note that this is real data from our sun, so it doesn't look like a *perfect* sine wave. What you need to do is estimate the *best*sine wave that fits this data.)
   a. Estimate the period T in years.
   b. When do we expect the next "solar maximum?"



Answers to Selected Problems

1. a. Buoyant force and gravity b. $T = 6$ s, $f = 1/6$ Hz
2. a. 0.0038 s b. 0.0038 s
3. a. About 11 years b. About 2014

4.2 Types of Waves

Students will learn the difference between a longitudinal wave a transverse wave. Students will also learn some general wave properties.

"I often think about music. I daydream about music. I see my life in the form of music." - Albert Einstein

The Big Idea

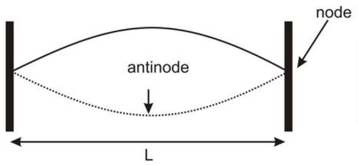
The development of devices to measure time like a pendulum led to the analysis of *periodic motion.*Motion that *repeats itself in equal intervals of time* is called *harmonic motion.* When an object moves back and forth over the *same path* in harmonic motion it is said to be *oscillating.* If the amount of motion of an oscillating object (the distance the object travels) stays the same during the period of motion, it is called *simple* harmonic motion(SHM). A grandfather clock's pendulum and the quartz crystal in a modern watch are examples of SHM.

Objects in motion that return to the same position after a fixed period of time are said to be in *harmonic motion.* Objects in harmonic motion have the ability to transfer some of their energy over large distances. They do so by creating waves in a medium. Imagine pushing up and down on the surface of a bathtub filled with water. Water acts as the medium that carries energy from your hand to the edges of the bathtub. Waves transfer energy over a distance without direct contact of the initial source. In this sense waves are phenomena not objects.

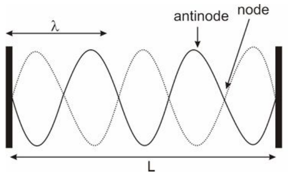
Guidance

- The oscillating object does not lose any energy in SHM. Friction is assumed to be zero.
- In harmonic motion there is always a *restorative force*, which acts in the opposite direction of the velocity. The restorative force changes during oscillation and depends on the position of the object. In a spring the force is the spring force; in a pendulum it is the component of gravity along the path. In both cases, the force on the oscillating object is directly opposite that of the direction of velocity.
- Objects in simple harmonic motion do not obey the "Big Three" equations of motion because the acceleration is not constant. As a spring compresses, the force (and hence acceleration) increases. As a pendulum swings, the tangential component of the force of gravity changes, so the acceleration changes.
- The **period**, T, is the amount of time for the harmonic motion to repeat itself, or for the object to go one full cycle. In SHM, T is the time it takes the object to return to its exact starting point and starting direction. The period of a wave depends on the period of oscillation of the object creating the wave.
- The **frequency**, f, is the number of cycles an object or wave goes through in 1 second. Frequency is measured in Hertz (Hz). 1 Hz = 1 cycle per sec.
- The **amplitude**, A, is the distance from the *equilibrium* (or center) *point* of motion to either its lowest or highest point (*end points*). The amplitude, therefore, is half of the total distance covered by the oscillating object. The amplitude can vary in harmonic motion but is constant in SHM. The amplitude of a wave often determines its strength or intensity; the exact meaning of "strength" depends on the type of wave. For example, a sound wave with a large amplitude is a loud sound and a light wave with a large amplitude is very bright.
- A ***medium*** is the substance through which the wave travels. For example, water acts as the medium for ocean waves, while air molecules act as the medium for sound waves.

- When a wave passes through a medium, the medium is only temporarily disturbed. When an ocean wave travels from one side of the Mediterranean Sea to the other, no actual water molecules move this great distance. Only the *disturbance* propagates (moves) through the medium.
- An object oscillating with frequency f will create waves which oscillate with the same frequency f.
- The **speed** v**and wavelength** λ of a wave depend on the nature of the medium through which the wave travels.
- There are two main types of waves we will consider: **longitudinal** and **transverse** waves.
- In **longitudinal** waves, the vibrations of the medium are in the *same direction* as the wave motion. A classic example is a wave traveling down a line of standing dominoes: each domino will fall in the same direction as the motion of the wave. A more physical example is a sound wave. For sound waves, high and low pressure zones move both forward and backward as the wave moves through them.
- In **transverse** waves, the vibrations of the medium are *perpendicular* to the direction of motion. A classic example is a wave created in a long rope: the wave travels from one end of the rope to the other, but the actual rope moves up and down, and not from left to right as the wave does.
- Water waves act as a mix of longitudinal and transverse waves. A typical water molecule pretty much moves in a circle when a wave passes through it.
- Most wave media act like a series of connected oscillators. For instance, a rope can be thought of as a large number of masses (molecules) connected by springs (intermolecular forces). The speed of a wave through connected harmonic oscillators depends on the distance between them, the spring constant, and the mass. In this way, we can model wave media using the principles of simple harmonic motion.
- The speed of a wave on a string depends on the material the string is made of, as well as the tension in the string. This fact is why *tightening* a string on your violin or guitar will *increase* the frequency, or pitch, of the sound it produces.
- When a wave meets a barrier, it reflects and travels back the way it came. The reflected wave may interfere with the original wave. If this occurs in precisely the right way, a *standing wave* can be created. The types of standing waves that can form depend strongly on the speed of the wave and the size of the region in which it is traveling.
- A couple typical standing waves are shown below. The first is the motion of a simple jump-rope.*Nodes* are the places where the rope doesn't move at all; *antinodes* occur where the motion is greatest. The second is a string in its 5^{th} harmonic



For this wave, the wavelength is λ = 2L. Since v = λf, the frequency of oscillation is f = v/2L



In general, the frequency of oscillation is f = nv / 2L, where n is the number of antinodes, v is the speed the wave is traveling along the string and L is the length of the string; thus f = 5v / 2L The thick and dotted lines represent the wave *envelope*: these are the upper and lower limits to the motion of the string.

Informative Websites

StandingWaves (visual simulation)

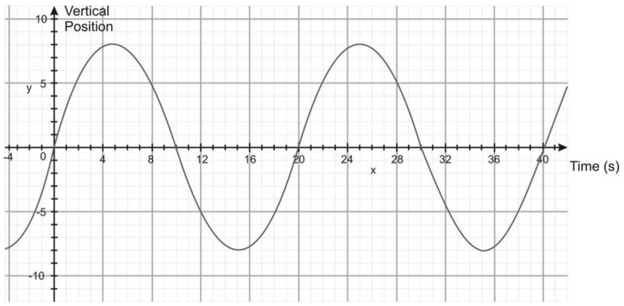
StandingWaves (editing wave math equations)

Time for Practice

1. Reread the difference between *transverse* and *longitudinal* waves. For each of the following types of waves, tell what type it is and why. (Include a sketch for each.)

- sound waves
- water waves in the wake of a boat
- a vibrating string on a guitar
- a swinging jump rope
- the vibrating surface of a drum
- the “wave” done by spectators at a sports event
- slowly moving traffic jams

2. A mass is oscillating up and down on a spring. Below is a graph of its vertical position as a function of time.



   a. Determine the
      a. amplitude,
      b. period and
      c. frequency.
   b. What is the amplitude at $t = 32$ seconds?
   c. At what times is the mass momentarily at rest? How do you know?
   d. Velocity is defined as change in position over time. Can you see that would be the slope of this graph? (slope = rise over run and in this case the ’rise’ is position and the ’run’ is time). Find the instantaneous speed at $t = 20$ sec.

Answers.

2. a. 8 m; 20 s ; 0.05 Hz; b. – 4 m c. $\sim$ 2.3 m/s

4.3 Wave Equation

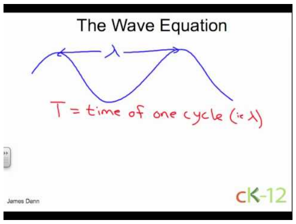
Students will learn how to use the wave equation to solve problems.

Key Equations

$T = \frac{1}{f}$ Wave period

$v = \lambda f$ Wave velocity

Watch this Explanation



MEDIA

Click image to the left for more content.

Guidance

The wave equation is analogous to *distance equals rate multiplied by time* equation for one cycle. Here the distance is a wavelength and the time is the period. We simply divide both sides by the period, to get speed of the wave equals the wavelength mulitiplied by one over the period. This is equivalent to wavelength multiplied by frequency, since frequency equals one of the period by definition.

Example 1

While on vacation in Hawaii you observe waves at the Banzai Pipeline approaching the shore at 6.0 m/s. You also note that the distance between waves is 28 m. Calculate (a) the frequency of the waves and (b) the period.

Question ***a****:* $f = ?\,[Hz]$

Given: $v = 6.0\ m/s$

$\lambda = 28\ m$

Equation: $v = f \cdot \lambda$ therefore $f = \frac{v}{\lambda}$

Plug n' Chug: $f = \frac{v}{\lambda} = \frac{6.0\ m/s}{28\ m} = 0.21\ Hz$

Answer:

$$\boxed{\mathbf{0.21\ Hz}}$$

Question ***b****:* $T = ?\,[s]$

Given: $f = 0.21\ Hz$

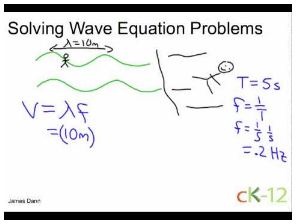
Equation: $T = \frac{1}{f}$

Plug n' Chug: $T = \frac{1}{f} = \frac{1}{0.21\ Hz} = 4.76\ s$

Answer:

$$\boxed{\textbf{4.76 s}}$$

Example 2

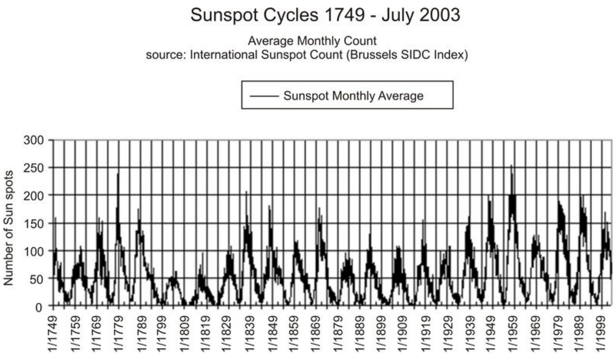


MEDIA

Click image to the left for more content.

Time for Practice

1. Bored in class, you start tapping your finger on the table. Your friend, sitting right next to you also starts tapping away. But while you are tapping once every second, you're friend taps twice for every one tap of yours.
    a. What is the Period and frequency of your tapping?
    b. What is the Period and frequency of your friend's tapping?
    c. Your tapping starts small waves going down the desk. Sort of like hitting a bell with a hammer. The frequency of the sound you hear is 1200 Hz. You know the wave speed in wood is about 3600 m/s. Find the wavelengths generated by your tapping.
2. You're sitting on Ocean Beach in San Francisco one fine afternoon and you notice that the waves are crashing on the beach about 6 times every minute.
    a. Calculate the frequency and period of the waves.
    b. You estimate that it takes 1 wave about 4 seconds to travel from a surfer 30 m off shore to the beach. Calculate the velocity and average wavelengths of the wave.
3. The Sun tends to have dark, Earth-sized spots on its surface due to kinks in its magnetic field. The number of visible spots varies over the course of years. Use the graph of the sunspot cycle below to answer the following questions. (Note that this is real data from our sun, so it doesn't look like a *perfect* sine wave. What you need to do is estimate the *best* sine wave that fits this data.)



    a. Estimate the period T in years.
    b. When do we expect the next "solar maximum?"

4. Human beings can hear sound waves in the frequency range 20 Hz – 20 kHz. Assuming a speed of sound of 343 m/s, answer the following questions.
    a. What is the shortest wavelength the human ear can hear?
    b. What is the longest wavelength the human ear can hear?
5. The speed of sound in hydrogen gas at room temperature is $1270\ \mathrm{m/s}$. Your flute plays notes of $600, 750$, and 800 Hz when played in a room filled with normal air. What notes would the flute play in a room filled with hydrogen gas?
6. The speed of light $c$is 300,000 km/sec.
    a. What is the frequency in Hz of a wave of red light ($\lambda = 0.7 \times 10^{-6}\ m$)?
    b. What is the period T of oscillation (in seconds) of an electron that is bouncing up and down in response to the passage of a packet of red light? Is the electron moving rapidly or slowly?
7. Radio signals are carried by electromagnetic waves (i.e. light waves). The radio waves from San Francisco radio station KMEL (106.1 FM) have a frequency of 106.1 MHz. When these waves reach your antenna, your radio converts the motions of the electrons in the antenna back into sound.
    a. What is the wavelength of the signal from KMEL?
    b. What is the wavelength of a signal from KPOO (89.5 FM)?
    c. If your antenna were broken off so that it was only 2 cm long, how would this affect your reception?
8. A standing wave interference pattern is produced in a rope by a vibrator with a frequency of 45 Hz. If the wavelength is 55 cm,
    (a) what is the distance between successive nodes?
    (b) What is the velocity of the wave?
9. The distance between the first and sixth nodes in a standing wave is 75 cm.
    (a) What is the wavelength of the waves?
    (b) What is the velocity if the source has a frequency of 12 Hz?
10. A standing wave pattern is produced. It is observed to have 10 antinodes with a node at each end. The distance between the first and last node is 75.0 cm and the waves have a velocity of 625 cm/s. What frequency is needed to observe four loops?

Answers

1. a. 1.0s, 1 Hz b. 0.5s, 2 Hz c. 3 m
2. a. 0.1 Hz, 10.0 s b. 7.5 m/s, 75 m
3. a. About 11 years b. About 2014

(see http://solarscience.msfc.nasa.gov/SunspotCycle.shtml for more info)

1. a. 1.7 cm b.17 m
2. $2230\ \mathrm{Hz}; 2780\ \mathrm{Hz}; 2970\ \mathrm{Hz}$
3. $4.3 \times 10^{14}\ Hz$
    a. $2.3 \times 10^{-15}\ s$
4. a. 2.83 m b. 3.35 m c. rule of thumb, antenna should be $\frac{1}{4}\lambda$, thus quality of reception will suffer

4.4 Sound

Students will learn how sound is produced and how it travels through the air to then register a signal in the brain.

Key Equations

$T = \frac{1}{f}$; period and frequency are inversely related

$v \approx 331.4\ m/s + 0.6\ T$; The speed of sound in air, where T is the temperature of the air in Celsius

Guidance

- Sound waves are longitudinal waves. Thus, the molecules of the medium vibrate back and forth in the same direction as the wave is traveling through. The medium for sound is normally air for humans, but sound waves can travel in water, metal, etc. as well.
- An object oscillating with frequency f will create waves which oscillate with the same frequency f.
- The speed of a sound wave in air depends subtly on pressure, density, and temperature, but is about 343 m/s at room temperature.

Example 1

Describe the pressure changes in the air as a sound wave passes a given point, then explain why a very loud sound can damage your tympanic membrane (ear drum).

Answer

As a sound wave passes a certain point, the air pressure at that point alternates between high and low pressure. When sound waves pass into the ear, the alternating pressures cause a pressure difference on either side of the tympanic membrane. The pressure differences between the inside and outside of the membrane cause the membrane to move back and forth with the same frequency as the changes in pressure. What we perceive as very loud sounds are sound waves with very large amplitudes, meaning that the differences in pressure are very large. The larger changes in pressure could cause damage to the membrane by causing it to vibrate too violently.

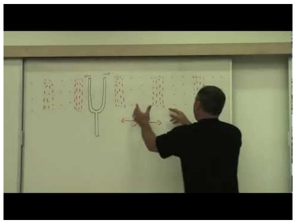
Example 2

What is the speed of sound in air that is 25 ºC?

Example 3

A fighter pilot wants to travel three times the speed of sound. How fast must she travel if the air temperature is 15ºC?

Watch this Explanation



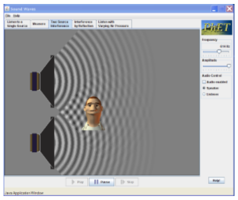
MEDIA

Click image to the left for more content.

MEDIA

Click image to the left for more content.

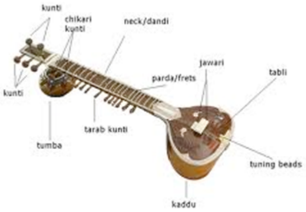
Simulation



Sound (PhET Simulation)

Time for Practice

1. Describe how sound is produce in a speaker, travels to your ear and how your ear converts these sound waves into sound in your brain.
2. The Indian instrument called a "sitar" uses two sets of strings, one above the other. Only one set of strings is played but both make sound. Research the sitar and explain briefly how this works.



For more information about the Sitar, see http://search.creativecommons.org/?q=sitar&sourceid=Mozilla-search.

3. A train, moving at some speed lower than the speed of sound, is equipped with a gun. The gun shoots a bullet forward at precisely the speed of sound, relative to the train. An observer watches some distance down the tracks, with the bullet headed towards him. Will the observer hear the sound of the bullet being fired before being struck by the bullet? Explain.
4. The speed of sound v in air is approximately 331.4 $m/s + 0.6\ T$, where T is the temperature of the air in Celsius. The speed of light c is 300,000 km/sec, which means it travels from one place to another on Earth more or less instantaneously. Let's say on a cool night (air temperature 10° Celsius) you see lightning flash and then hear the thunder rumble five seconds later. How far away (in km) did the lightning strike?

Answers

1. .
2. .
3. struck by bullet first.
4. 1.7 km

4.5 Doppler Effect

Students will learn what the Doppler effect is and how to determine frequencies and speeds using the Doppler shift equations.

Key Equations

Doppler Shifts:

$f_o = f\frac{v+v_o}{v-v_s}$ f_o (observed frequency) is shifted up when source and observer moving closer

$f_o = f\frac{v-v_o}{v+v_s}$ f_o (observed frequency) is shifted down when source and observer moving apart, where

v is the speed of sound, v_s is the speed of the source, and v_o is the speed of the observer

Guidance

The *Doppler Effect* occurs when either the source of a wave or the observer of a wave (or both) are moving. When a source of a wave is moving towards you, the apparent frequency of the wave you detect is higher than that emitted. For instance, if a car approaches you while playing a note at 500 Hz, the sound you hear will be slightly higher frequency. The opposite occurs (the frequency observed is lower than emitted) for a receding wave or if the observer moves away from the source. It's important to note that the speed of the wave does not change –it's traveling through the same medium so the speed is the same. Due to the relative motion between the source and the observer the frequency changes, but not the speed of the wave. Note that while the effect is similar for light and electromagnetic waves the formulas are not exactly the same as for sound.

Example 1

Question: How fast would a student playing an A note (440Hz) have to move towards you in order for you to hear a G note (784Hz)?

Answer: We will use the Doppler shift equation for when the objects are getting closer together and solve for the speed of the student (the source).

$$f_o = f(\frac{v+v_o}{v-v_s}) \Rightarrow f_o \times (v-v_s) = f \times (v+v_o) \Rightarrow vf_o - v_s f_o = f \times (v+v_o) \Rightarrow v_s = -(\frac{f \times (v+v_o) - vf_o}{f_o})$$

Now we plug in the known values to solve for the velocity of the student.

$$v_s = -(\frac{f \times (v+v_o) - vf_o}{f_o}) = -(\frac{440\text{Hz} \times (343\text{m/s} + 0\text{m/s}) - 343\text{m/s} \times 784\text{Hz}}{784\text{Hz}}) = 151\text{m/s}$$

Example 2

What is the observed frequency of a 525 Hz source moving towards a stationary observer at 75 m/s? Take the speed of sound to be 375 m/s.

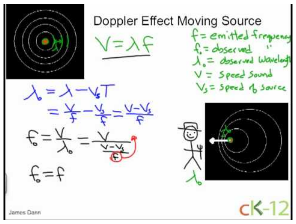
Example 3

How fast must someone be driving so that a stationary observer hears a frequency twice that of the source frequency? Take the speed of sound in air to be 345 m/s. For a reference convert your answer into km/h.

Example 4

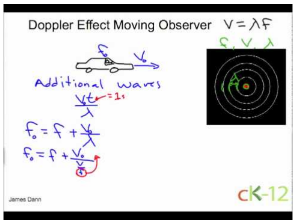
A police siren has a frequency of 1.8×10^4 Hz. A crook in his getaway car drives away from the police at 105 m/s. What frequency is heard by the crook if the police car is driving at 85 m/s? The temperature today is 25 ºC.

Watch this Explanation



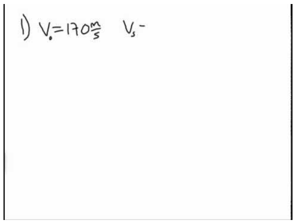
MEDIA

Click image to the left for more content.



MEDIA

Click image to the left for more content.



MEDIA

Click image to the left for more content.

Simulation

http://www.lon-capa.org/ mmp/applist/doppler/d.htm

Time for Practice

1. What changes about a wave in the Doppler effect: Frequency? Wavelength? Speed? Is it correct to say that if an ambulance is moving towards you the speed of the sound from its siren is faster than normal? Discuss.
2. A friend plays an A note (440 Hz) on her flute while hurtling toward you in her imaginary space craft at a speed of 40 m/s. What frequency do you hear just before she rams into you? Assume the speed of sound to be 343 m/s.

3. How fast would a student playing an A note (440 Hz) have to move towards you in order for you to hear a *G* note (784 Hz)? Assume the speed of sound to be 343 m/s

Answers

1. .
2. 498 Hz
3. 150 m/s

CHAPTER 5 Electromagnetic Radiation

Chapter Outline

Multimedia

MEDIA

Click image to the left for more content.

5.1 Nature of Light

Students will gain an understanding of the speed of light and work problems involving light. The distance of a light year is also explained and practiced.

The Big Idea

Light is a *wave* of changing electric and magnetic fields. Light waves are caused by disturbances in the electromagnetic field that permeates the universe, for example, the acceleration of charged particles (such as electrons). Light has a dual nature: at times, it acts like waves; at other times it acts like particles, called *photons*. Light travels through space at the maximum speed allowed by the laws of physics, called the speed of light. Light has no mass, but it carries energy and momentum. Of all possible paths *light rays will always take the path that takes the least amount of time* (not distance). This is known as Fermat's Principle.

The brightness, or **intensity**, of light is inversely proportional to the square of the distance between the light source and the observer. This is just one of many **inverse square laws** in physics; others include gravitational and electrical forces.

Fermat's Principle makes the angle of incident light equal to the angle of reflected light. This is the *law of reflection*.

When light travels from one type of material (like air) into another (like glass), the speed changes due to interactions between photons and electrons. **Transparent** materials transmit the electromagnetic energy at a speed slower than c; **opaque** materials absorb that EM energy and convert it into heat. A material may be transparent to some wavelengths of light but opaque to others.

Polarized light is made of waves oscillating in only one direction: horizontal or vertical. The direction of the oscillation of the light waves is the same as the direction of oscillation of the electron *creating* the light. Unpolarized light can be polarized selectively by reflections from surfaces (glare); the orientation of the reflected light polarization is the same as that of the surface.

Light, more generally known as Electromagnetic Waves (EM Waves), can be produced in many different wavelengths that can be very large to extremely small. EM waves can be polarized when produced or after going through a filter (natural or man-made). Polarization of light, means that the light wave oscillates in only one direction rather than unpolarized light that oscillates in two directions as it moves forward.

The visible range of light (i.e. the range of wavelengths that our eyes can detect) is a very narrow piece of the full EM spectrum. In the visible range our eyes differentiate between the different wavelengths by producing 'color' for them. When we observe something that is green, it is green to us, because the wavelength of the light hitting our eyes is around 500 nm. If the wavelength of light is slightly smaller than this it starts to look red, if it is slightly larger it looks blue. White light is the combination of all the colors. Black light is the absence of EM waves in the visible spectrum for human beings.

Key Equations

$D = ct$; distance is equal to the speed of light multiplied by the time it has traveled

Guidance

- Light travels at 3×10^8m/s. We call this value c. Light can never travel at any other speed. Although when light travels through materials, due to scattering and absorption it appears that light is going slightly slower (see refraction lesson for more detail).
- A light year is the amount of distance light covers in a time of one year. The value of one light year is simply the speed of light multiplied by the number of seconds in a year and is roughly equal to 9.4×10^{15}m .
- Viewing distant stars is looking back in time. The stars we see are many thousands of light years away, which means the light takes many thousands of years to reach us. Thus the stars we see in the sky are how they looked thousands of years ago.

Example 1

Question How long after texting your friend on Mars will he receive it?

Answer Mars is a distance of 3.1×10^{11}m. Thus, using the equation $t = \frac{D}{c} = \frac{3.1 \times 10^{11}\text{m}}{3 \times 10^8\text{m/s}}$ = 1033 seconds or about 17 minutes.

Time for Practice

1. Our sun is 8 "light minutes" from earth.
   a. Using the speed of light as c calculate the earth-sun distance in m, then convert it to miles (one mile = 1609 m)
   b. The nearest galaxy to our Milky Way is the Andromeda Galaxy, 2 million light years away. How far away is the Andromeda Galaxy in miles?
2. The Canis Major Dwarf galaxy is 2.364×10^{20}m.
   a. How far is it in light years?
   b. If the galaxy were to disappear, how long until we notice it vanish on Earth?
3. Light does not travel infinitely fast – its speed in a vacuum is 3×10^8m/s. We measure very large astronomical distances in "light years" (LY) – the distance light travels in one year. The closest star beyond our sun is Alpha Centauri, 4 LY away. Some distant quasars are 2 billion LY away. Why do astronomers say that to look at these distant objects is to look back in time? Explain briefly.

Answers to Selected Problems

1. a. 1.44×10^{11}m b. 8.95×10^7miles
2. about 25,000 cyr

5.2 Electromagnetic Spectrum

Students will learn what an electromagnetic wave is, gain a feel for the main parts of the spectrum and work problems involving basic properties of electromagnetic waves.

Key Equations

$$c = f\lambda$$

; Wave equation for light

$c = 3.00 \times 10^8 \text{m/s}$

Guidance

- When charged particles *accelerate*, changing electric and magnetic fields radiate outward. The traveling electric and magnetic fields of an accelerating (often oscillating) charged particle are known as electromagnetic radiation or light.
- When using the wave equation for light keep in mind that light always travels at the speed of light. So plug in c for v in the wave equation.
- The color of light that we observe is a measure of the wavelength of the light: the *longer* the wavelength, the *redder* the light.
- The spectrum of electromagnetic radiation can be roughly broken into the following ranges:

TABLE 5.1:

EM wave	Wavelength range	Comparison size
gamma-ray ($\gamma-$ ray)	10^{-11} m and shorter	atomic nucleus
$x-$ray	10^{-11} m – 10^{-8} m	hydrogen atom
ultraviolet (UV)	10^{-8} m – 10^{-7} m	small molecule
violet (visible)	$\sim 4 \times 10^{-7}$ m(400 nm)*	typical molecule
blue (visible)	$\sim$ 450 nm	typical molecule
green (visible)	$\sim$ 500 nm	typical molecule
red (visible)	$\sim$ 650 nm	typical molecule
infrared (IR)	10^{-6} m – 1 mm	human hair
microwave	1 mm – 10 cm	human finger
radio	Larger than 10 cm	car antenna

Example 1

Which has a higher frequency, green light or microwaves?

Answer

Green light has a higher frequency than microwaves. It is possible to calculate it, but since the speed of an electromagnetic wave is constant we know that waves with higher wavelengths must have a lower frequency based on the wave equation.

Example 2

Calculate the frequency for an ultraviolet wave of wavelength 10^{-7} m and compare it to the frequency of a radio wave (about 3.00×10^9 Hz). Which type of wave do you think takes more energy to generate?

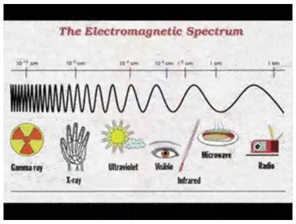
Solution

We'll use the wave equation to determine the wave length of ultraviolet light.

$$\begin{aligned} c &= f\lambda \\ f &= \frac{c}{\lambda} \\ f &= \frac{3.00 \times 10^8 \text{ m/s}}{10^{-7} \text{ m}} \\ f &= 3.00 \times 10^{14} \text{ Hz} \end{aligned}$$

The oscillating charged particles that create UV light are vibrating much more violently than the ones that create radio waves so they take more energy to generate.

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. Which corresponds to light of longer wavelength, UV rays or IR rays?
2. Which corresponds to light of lower frequency, $x-$rays or millimeter-wavelength light?
3. Approximately how many blue wavelengths would fit end-to-end within a space of one millimeter?
4. Approximately how many short ("hard") $x-$rays would fit end-to-end within the space of a single red wavelength?
5. Calculate the frequency in Hz of a typical green photon emitted by the Sun. What is the physical interpretation of this (very high) frequency? (That is, what is oscillating?)

6. FM radio stations list the frequency of the light they are emitting in MHz, or millions of cycles per second. For instance, 90.3 FM would operate at a frequency of 90.3×10^6 Hz. What is the wavelength of the radio-frequency light emitted by this radio station? Compare this length to the size of your car's antenna, and make an argument as to why the length of a car's antenna should be about the wavelength of the light you are receiving.

Answers to Selected Problems

1. .
2. .
3. 2200 blue wavelengths
4. 65000 $x-$rays
5. 6×10^{14} Hz
6. 3.3 m

5.3 Color

Students will learn how our eye detects color and how color addition works. Reflection and transmission of color in various situations is also covered.

Key Equations

Color Addition

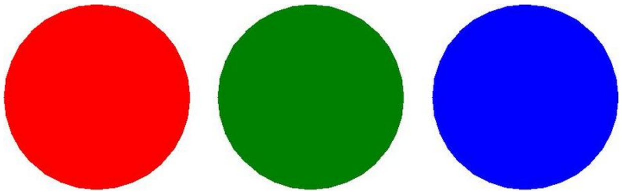


TABLE 5.2:

Red	*Green*	*Blue*	*Perceived color*
✓	✓	✓	white
			black
✓		✓	magenta
✓	✓		yellow
	✓	✓	cyan

Guidance

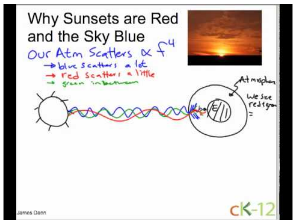
- White light consists of a mixture of all the visible colors: red, orange, yellow, green, blue, indigo, and violet (ROYGBIV). Our perception of the color black is tied to the *absence* of light.
- Our eyes include color-sensitive and brightness-sensitive cells. The three different color-sensitive cells (cones) can have sensitivity in three colors: red, blue, and green. Our perception of other colors is made from the *relative amounts* of each color that the cones register from light reflected from the object we are looking at. Our brightness-sensitive cells (rods) work well in low light. This is why things look 'black and white' at night.
- The chemical bonds in pigments and dyes – like those in a colorful shirt – absorb light at frequencies that correspond to certain colors. When you shine white light on these pigments and dyes, some colors are absorbed and some colors are reflected. We only see the colors that objects *reflect*.
- *Beautiful sunsets* are another manifestation of Rayleigh scattering that occurs when light travels long distances through the atmosphere. The blue light and some green is scattered away, making the sun appear red.

Example 1

Question: In a dark room a blue light is shined on a magenta shirt what color does it appear? How about a yellow shirt?

Answer: Since magenta is the combination of red and blue and only blue light is available to be reflected, the shirt will appear blue. As for the yellow shirt, there is no blue light in yellow (yellow is the combination of red and green), thus the shirt will appear black since no light is reflected off it.

Watch this Explanation



Why Sunsets are Red and the Sky Blue

Our Atm Scatters $\propto f^4$

→ blue scatters a lot

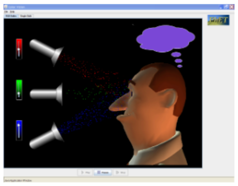
→ red scatters a little

→ green in between

MEDIA

Click image to the left for more content.

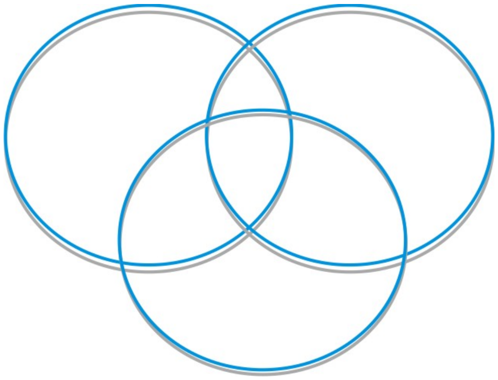
Simulation



Color Vision(PhET Simulation)

Time for Practice

1. Consult the color table for human perception under the 'Key Concepts' section and answer the questions which follow.
   a. Your coat looks magenta in white light. What color does it appear in blue light? In green light?
   b. Which secondary color would look black under a blue light bulb?
   c. You look at a cyan-colored ribbon under white light. Which of the three primary colors is your eye *not* detecting?
2. The **primary** colors of light are red, green, and blue. This is due to the way our eyes see color and the chemical reactions in the cone cells of the retina. Mixing these colors will produce the **secondary**, or complementary colors: magenta, cyan and yellow, as well as white. On the diagram below label colors to show how this works:



3. Color printers use a different method of color mixing: color mixing by **subtraction**. What colors are used in this process? (HINT: look at the ink colors in an ink-jet printer, CYMB). The idea here is that each color **subtracts** from the reflected light – for example, cyan ink reflects blue and green but SUBTRACTS red. Explain with diagrams how CYM can be combined to produce red, blue, and green.
4. Answer the following light transmission questions

a. A beam of cyan light passes into a yellow filter. What color emerges?
b. A beam of yellow light passes into a magenta filter. What color emerges?
c. What color results when two beams of light, one cyan and one magenta, are made to overlap on a white screen?
d. White light passes through a cyan filter followed by a magenta filter. What color emerges?

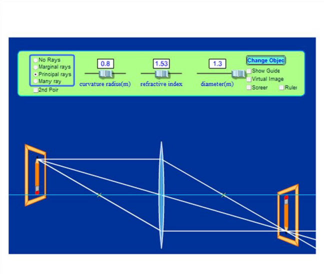
Answers to Selected Problems

1. a. blue, black b. yellow c. red
2. .
3. .
4. a. green b. red c. blue d. blue

CHAPTER 6

Optics

Chapter Outline



○ No Rays
○ Marginal rays
● Principal rays
○ Many ray
☐ 2nd Poir

curvature radius(m) 0.8

refractive index 1.53

diameter(m) 1.3

☐ Show Guide
☐ Virtual Image
☐ Screer
☐ Ruler

FIGURE 6.1

6.1 Mirrors

Students will learn how light reflects off mirrors. Flat mirrors, concave mirrors and convex mirrors are all covered.

Key Equations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

; The len's maker's equation

Where f is the focal length of the mirror, d_o is the distance of the object from the mirror and d_i is the distance the image is formed from the mirror.

$$M = \frac{-d_i}{d_0} = \frac{h_i}{h_o}$$

The size of an object's image is larger (or smaller) than the object itself by its magnification, M. The level of magnification is proportional to the ratio of d_i and d_o. An image that is double the size of the object would have magnification $M = 2$.

$$R = 2f$$

The radius of curvature of a mirror is twice its focal length

Guidance

- *Mirrors* are made from highly reflective metal that is applied to a curved or flat piece of glass. Converging mirrors can be used to focus light – headlights, telescopes, satellite TV receivers, and solar cookers all rely on this principle.

- Converging mirrors (also known as concave mirrors) are curved towards the incoming light and focus parallel rays at the focal point. Diverging mirrors (also known as convex mirrors) are curved away from the incoming light and do not focus, but scatter the light instead.

- The *focal length*, f, of a lens or mirror is the distance from the surface of the lens or mirror to the place where the light is focused. This is called the *focal point* or *focus*. For diverging mirrors, there is no true focal point (i.e. real light does not focus), so the focal length is negative.

- When light rays converge in front of a mirror, a *real* image is formed. Real images are useful in that you can place photographic film at the physical location of the real image, expose the film to the light, and make a two-dimensional representation of the world, a photograph.

- When light rays diverge in front of a mirror, a *virtual* image is formed. A virtual image is formed by your brain tracing diverging rays backwards and is kind of a trick, like the person you see "behind" a mirror's surface when you brush your teeth (there's obviously no real light focused *behind* a mirror!). Since virtual images aren't actually "anywhere," you can't place photographic film anywhere to capture them.

- Real images are upside-down, or *inverted*. You can make a real image of an object by putting it farther from a mirror or lens than the focal length. Virtual images are typically right-side-up. You can make virtual images by moving the mirror or lens closer to the object than the focal length.

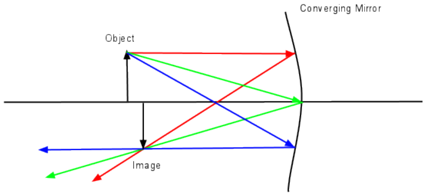
In the problems below that consider converging or diverging mirrors, you will do a careful ray tracing with a ruler (including the extrapolation of rays for virtual images). It is best if you can use different colors for the three different ray tracings. When sketching diverging rays, you should use dotted lines for the extrapolated lines behind a mirror in order to produce the virtual image. When comparing measured distances and heights to calculated distances and heights, values within 10% *are considered "good." Use the* **Table** (6.3) as your guide.

TABLE 6.1:

Mirror type	Ray tracings
Converging mirrors (**concave**)	Ray #1: Leaves tip of candle, travels parallel to optic axis, reflects back through focus. Ray #2: Leaves tip, travels through focus, reflects back parallel to optic axis. Ray #3: Leaves tip, reflects off center of mirror with an angle of reflection equal to the angle of incidence.
Diverging mirrors (**convex**)	Ray #1: Leaves tip, travels parallel to optic axis, reflects OUTWARD by lining up with focus on the OPPOSITE side as the candle. Ray #2: Leaves tip, heads toward the focus on the OPPOSITE side, and emerges parallel to the optic axis. Ray #3: Leaves tip, heads straight for the mirror center, and reflects at an equal angle.

Example 1

In the situation illustrated below, the object is set up .5 m away from a converging mirror. If the focal length of the lens is .2 m, determine (a) the location of the real image, and (b) the magnification of the image.



Solution

(a): In order to determine the location of the image, we'll use the lens maker's equation.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$
$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$
$$\frac{1}{d_i} = \frac{1}{.2\text{ m}} - \frac{1}{.5\text{ m}}$$
$$d_i = .33\text{ m}$$

(b): Now that we have the location of the image, we can find the magnification.

$$M = \frac{-d_i}{d_o}$$
$$M = \frac{-.33\text{ m}}{.5\text{ m}}$$
$$M = -\frac{2}{3}$$

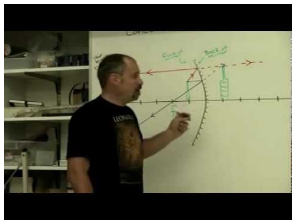
Example 2

1. (a) Calculate where an object should be placed in front of a converging mirror with f = 18 cm to produce:
    i. A real image 6 times larger than the object.
    ii. A real image 6 times smaller than the object.
    iii. A virtual image 6 times larger than the object.
    iv. A virtual image 6 times smaller than the object.

Watch this Explanation

MEDIA

Click image to the left for more content.

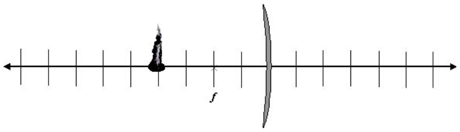


MEDIA

Click image to the left for more content.

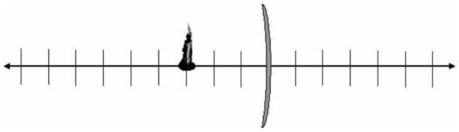
Time for Practice

1. Here's an example of the "flat mirror problem." Marjan is looking at herself in the mirror. Assume that her eyes are 10 cm below the top of her head, and that she stands 180 cm tall. Calculate the minimum length flat mirror that Marjan would need to see her body from eye level all the way down to her feet. Sketch at least 3 ray traces from her eyes showing the topmost, bottommost, and middle rays.
2. Consider a concave mirror with a focal length equal to two units, as shown below.



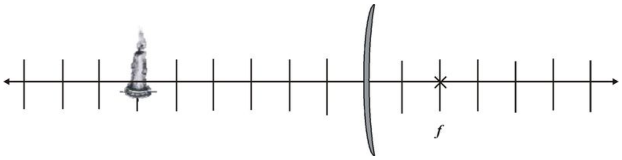
   a. *Carefully* trace three rays coming off the top of the object in order to form the image.
   b. Measure d_o and d_i.
   c. Use the mirror/lens equation to calculate d_i.
   d. Find the percent difference between your measured d_i and your calculated d_i.
   e. Measure the magnification M and compare it to the calculated magnification.

3. Consider a concave mirror with unknown focal length that produces a virtual image six units behind the mirror.

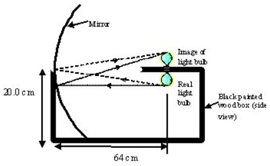


   a. Calculate the focal length of the mirror and draw an × at the position of the focus.
   b. *Carefully* trace three rays coming off the top of the object and show how they converge to form the image.
   c. Does your image appear bigger or smaller than the object? Calculate the expected magnification and compare it to your sketch.

4. Consider a convex mirror with a focal length equal to two units.



   a. *Carefully* trace three rays coming off the top of the object and form the image.
   b. Measure d_o and d_i.
   c. Use the mirror/lens equation to calculate d_i.
   d. Find the percent difference between your measured d_i and your calculated d_i.
   e. Measure the magnification M and compare it to the calculated magnification.



5. Above is a diagram showing how to make a "ghost light bulb." The real light bulb is below the box and it forms an image of the exact same size right above it. The image looks very real until you try to touch it. What is the focal length of the concave mirror?

Answers to Selected Problems

1. 85 cm
2. C. +4 units e. $M = -1$
3. a. 6 units c. bigger; $M = 3$
4. c. 1.5 units e. $M = 2/3$
5. 32 cm

6.2 Refraction

Students will learn why light refracts when entering different substances like water, glass, etc. and how to calculate the angle of refraction using Snell's Law.

Key Equations

$c = 300,000,000 \text{ m/s}$; the speed of light

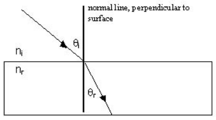
$n = \frac{c}{v}$

The index of refraction, n, is the ratio of its speed (c) in a vacuum to the slower speed (v) it travels in a material. n can depend slightly on wavelength.

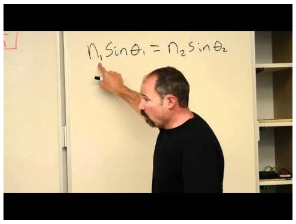
$n_i \sin \theta_i = n_R \sin \theta_R$; Snell's Law

Guidance

Fermat's principle states that light will always take the path that takes the least amount of time (not distance). Refraction follows from this. When light travels from air into another material (like glass), its speed through the material is reduced due to interactions between photons that make up the light ray and the densely packed atoms of the material. Because light is effectively moving slower in the glass, for example, as compared to air, the light ray bends in order to get out quicker and satisfy Fermat's Principle of least time. This is called refraction. The figure below demonstrates the refraction a light ray experiences as it passes from air into a rectangular piece of glass and out again.



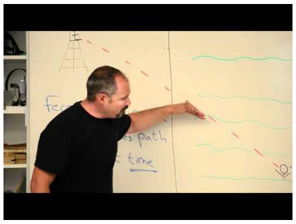
Example 1



MEDIA

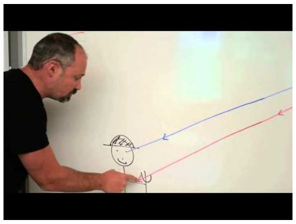
Click image to the left for more content.

Watch this Explanation



MEDIA

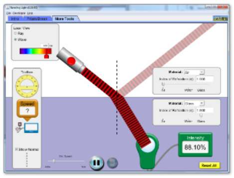
Click image to the left for more content.



MEDIA

Click image to the left for more content.

Simulation



Bending Light (PhET Simulation)

Time for Practice

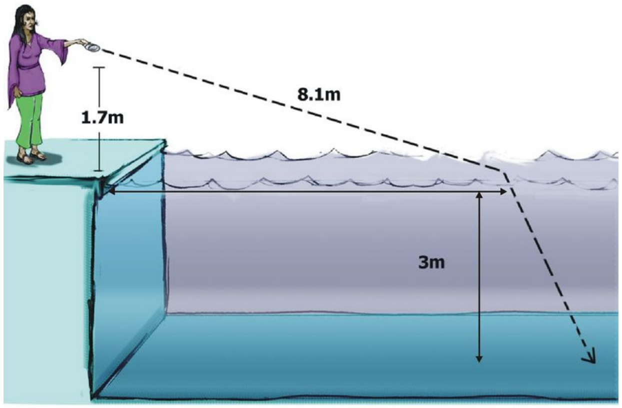
1. Using the **Table** (6.2), which states the indices of refraction for a number of materials, answer the following questions:
    a. For which of these materials is the speed of light *slowest*?
    b. Which two materials have the most similar indices of refraction?
    c. What is the speed of light in cooking oil?

TABLE 6.2:

Material	n
vacuum	1.00000
air	1.00029
water	1.33
typical glass	1.52
cooking oil	1.53
heavy flint glass	1.65
sapphire	1.77
diamond	2.42

2. A certain light wave has a frequency of 4.29×10^{14} Hz. What is the wavelength of this wave in empty space? In water?
3. A light ray bounces off a fish in your aquarium. It travels through the water, into the glass side of the aquarium, and then into air. Draw a sketch of the situation, being careful to indicate how the light will change directions

when it refracts at each interface. Include a brief discussion of why this occurs.

4. A light ray goes from the air into the water. If the angle of incidence is 34°, what is the angle of refraction?
5. 

6. Nisha stands at the edge of an aquarium 3.0 m deep. She shines a laser at a height of 1.7 m that hits the water of the pool 8.1 mfrom the edge.
   a. Draw a diagram of this situation. Label all known lengths.
   b. How far from the edge of the pool will the light hit bottom?
   c. If her friend, James, were at the bottom and shined a light back, hitting the same spot as Nisha's, how far from the edge would he have to be so that the light never leaves the water?
7. You are to design an experiment to determine the index of refraction of an unknown liquid. You have a small square container of the liquid, the sides of which are made of transparent thin plastic. In addition you have a screen, laser, ruler and protractors. Design the experiment. Give a detailed procedure; include a diagram of the experiment. Tell which equations you would use and give some sample calculations. Finally, tell in detail what level of accuracy you can expect and explain the causes of lab error in order of importance.

Answers to Selected Problems

1. b. vacuum & air c. 1.96×10^8 m/s
2. 6.99×10^{-7} m; $5.26 \times 10^{-}7$ m
3. .
4. 25°
5. b. 11.4 m c. 11.5 m
6. .

6.3 Total Internal Reflection

Students will learn about total internal reflection. More specifically, they will learn what the critical angle is, how it is derived and how to solve for it in real life applications.

Key Equations

$\theta_C = sin^{-1}\frac{n_R}{n_i}$; where θ_C is the critical angle, n_i is the index of refraction of the material where the light emanates from and n_R is the index of the material outside.

Guidance

Total internal reflection occurs when light goes from a slow (high index of refraction) medium to a fast (low index of refraction) medium. With total internal reflection, light refracts so much it actually refracts back into the first medium. This is how fiber optic cables work: no light leaves the wire.

Example 1

You have some unknown material and you would like to determine it's index of refraction. You find that you are able to create total internal reflection when the material submerged in water, but not when submerged in cooking oil. (a) Can you give a range for the index of refraction? (b) you are able to determine the critical angle in water to be 71.8 degrees; what is the index of refraction of this material?

Solution

(a): Since it is not possible to create total internal refraction when going from a material with a higher index of refraction to a lower index of refraction, we know that the index of refraction of this material must be between 1.33 (water) and 1.53 (cooking oil).

(b): We can use the equation given above to determine the index of refraction of the unknown material.

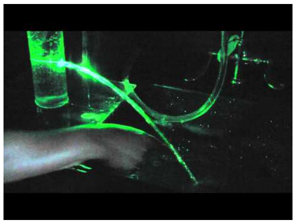
$$\theta_c = sin^{-1}(\frac{n_R}{n_i})$$

$$n_i = \frac{n_R}{\sin(\theta_c)}$$

$$n_i = \frac{1.33}{\sin(71.8)}$$

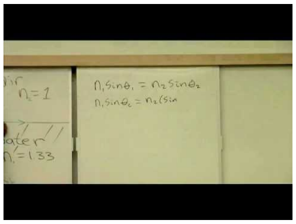
$$n_i = 1.40$$

Watch this Explanation



MEDIA

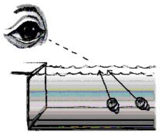
Click image to the left for more content.



MEDIA

Click image to the left for more content.

Time for Practice

1. Imagine a thread of diamond wire immersed in water. Can such an object demonstrate total internal reflection? If so, what is the critical angle? Draw a picture along with your calculations.
2. 
3. A light source sits in a tank of water, as shown.
    a. If one of the light rays coming from inside the tank of water hits the surface at 35.0°, as measured from the normal to the surface, at what angle will it enter the air?
    b. Now suppose the incident angle in the water is 80° as measured from the normal. What is the refracted angle? What problem arises?
    c. Find the *critical angle* for the water-air interface. This is the incident angle that corresponds to the largest possible refracted angle, 90°.

Answers

1. 33.3°
2. a. 49.7° b. No such angle c. 48.8°

6.4 Lenses

Students will learn how light behaves when passing through converging and diverging lenses. Students will also learn how to do ray tracing diagrams and calculate image distances and magnification using the lens' maker's equation.

Key Equations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \text{The thin lens equation}$$

Where f is the focal length of the lens, d_o is the distance of the object from the lens and d_i is the distance the image is formed from the lens.

$$M = \frac{-d_i}{d_0} = \frac{h_i}{h_o} \quad \text{The magnification equations}$$

The size of an object's image is larger (or smaller) than the object itself by its magnification, M. The level of magnification is proportional to the ratio of d_i and d_o. An image that is double the size of the object would have magnification $M = 2$.

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_o} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{This is the lens maker's formula}$$

Where n_{lens} is the index of refraction of the lens material, n_o is that of the surrounding medium, and R_1 and R_2 are the radius of curvature of each side of the lens. Convex sides have a positive radius of curvature as light is refracted to have a real focus. While concave sides have a negative radius of curvature as light is refracted to have a virtual focus. A lens with a flat side has an infinite radius (making the term equate to zero).

Guidance

- For lenses, the distance from the center of the lens to the focus is f. Focal lengths for foci behind the lens are positive in sign. The distance from the center of the lens to the object in question is d_0, where distances to the left of the lens are positive in sign. The distance from the center of the lens to the image is d_i. This number is positive for real images (formed to the right of the lens), and negative for virtual images (formed to the left of the lens).
- *Lenses*, made from curved pieces of glass, focus or de-focus light as it passes through. Lenses that focus light are called *converging* lenses, and these are the ones used to make telescopes and cameras. Lenses that de-focus light are called *diverging* lenses.
- Lenses can be used to make visual representations, called *images*.
- The *focal length*, f, of a lens or mirror is the distance from the surface of the lens to the place where the light is focused. This is called the *focal point* or *focus*. For diverging lenses, the focal length is negative.

- For converging lens, one can find the focal point by simply holding a piece of paper near the lens until a distant image is formed. The distance from the paper to the lens is the focal point.

Converging lens

- When light rays converge behind a lens, a *real* image is formed. Real images are useful in that you can place photographic film at the physical location of the real image, expose the film to the light, and make a two-dimensional representation of the world, a photograph.

Diverging lens

- When light rays diverge behind a lens, a *virtual* image is formed. A virtual image is a manifestation of your brain (it traces the diverging rays backwards and forms an image), like the person you see "behind" a mirror's surface when you brush your teeth (there's obviously no real light focused *behind* a mirror!). Since virtual images aren't actually "anywhere," you can't place photographic film anywhere to capture them.

- Real images are upside-down, or *inverted*. You can make a real image of an object by putting it farther from a mirror or lens than the focal length. Virtual images are typically right-side-up. You can make virtual images by moving the lens closer to the object than the focal length.

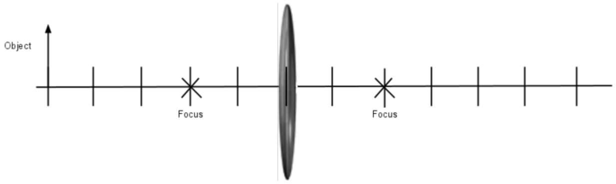
In ray tracing problems, you will do a careful ray tracing with a ruler (including the extrapolation of rays for virtual images). It is best if you can use different colors for the three different ray tracings. When sketching diverging rays, you should use dotted lines for the extrapolated lines in front of a lens in order to produce the virtual image. When comparing measured distances and heights to calculated distances and heights, values within 10% *are considered "good." Use the* **Table** (6.3) as your guide.

TABLE 6.3:

Mirror type	Ray tracings
Converging lenses (**convex**)	Ray #1: Leaves tip, travels parallel to optic axis, refracts and travels through to the focus. Ray #2: Leaves tip, travels through focus on same side, travels through lens, and exits lens parallel to optic axis on opposite side. Ray #3: Leaves tip, passes straight through center of lens and exits without bending.
Diverging lenses (**concave**)	Ray #1: Leaves tip, travels parallel to optic axis, refracts OUTWARD by lining up with focus on the SAME side as the candle. Ray #2: Leaves tip, heads toward the focus on the OPPOSITE side, and emerges parallel from the lens. Ray #3: Leaves tip, passes straight through the center of lens and exits without bending.

Example 1

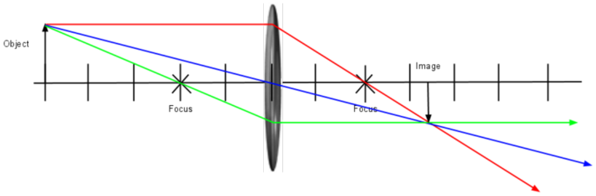
You have a converging lens of focal length 2 units. If you place an object 5 units away from the lens, (a) draw a ray diagram of the situation to estimate where the image will be and (b) list the charactatistics of the image. Finally (c) calculate the position of the image. A diagram of the situation is shown below.



Solution

(a): To draw the ray diagram, we'll follow the steps laid out above for converging lenses.

First we draw the a ray that travels parallel to the principle axis and refracts through the focus on the other side (the red ray). Next we draw a ray through the focus on the same side that refracts out parallel (the green ray). Finally we draw the ray that travels straight through the center of the lens without refracting (the blue ray). The result is shown below.



(b): Based on the ray diagram and the initial position of the object, we know that the image is a real, inverted, and smaller than the original object.

(c): To calculate the exact position of the object, we can use the lens maker's equation.

$$\begin{aligned}
\frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i} \\
\frac{1}{d_i} &= \frac{1}{f} - \frac{1}{d_o} \\
\frac{1}{d_i} &= \frac{1}{2 \text{ units}} - \frac{1}{5 \text{ units}} \\
\frac{1}{d_i} &= \frac{3}{10 \text{ units}} \\
d_i &= 3.33 \text{ units}
\end{aligned}$$

Example 2 - Lens Makers Formula

What is the focal length of a Plexiglas plano-convex lens that has a radius of 15.7 cm?

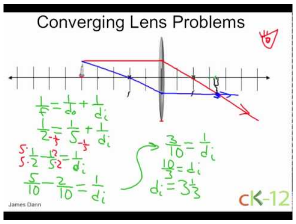
Example 3

What is the measure of each radius of a double concave ruby lens that has a focal length of –25 cm?

Example 4

In air a flint glass converging meniscus lens has a focal length of 14 cm. What will be the focal length of the lens if it is submerged in water?

Watch this Explanation

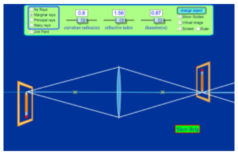


MEDIA

Click image to the left for more content.

Simulation

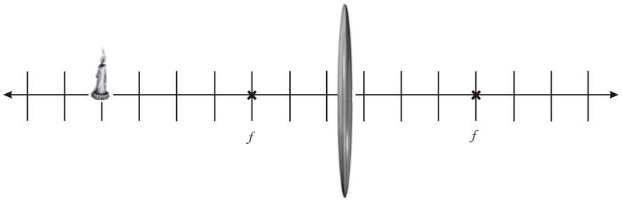
Note: this simulation only shows the effects of a convex lens



Geometric Optics(PhET Simulation)

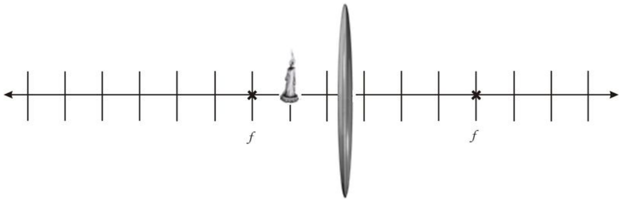
Time for Practice

1. Consider a converging lens with a focal length equal to three units and an object placed outside the focal point.



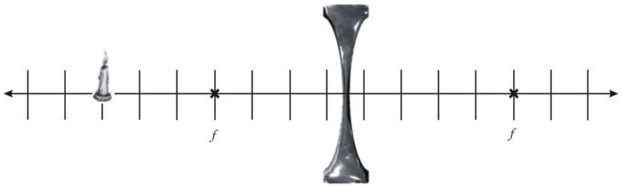
   a. *Carefully* trace three rays coming off the top of the object and form the image.
   b. Measure d_o and d_i.
   c. Use the mirror/lens equation to calculate d_i.
   d. Find the percent difference between your measured d_i and your calculated d_i.
   e. Measure the magnification M and compare it to the calculated magnification.

2. Consider a converging lens with a focal length equal to three units, but this time with the object placed inside the focal point.



   a. *Carefully* trace three rays coming off the top of the object and form the image.
   b. Measure d_o and d_i.
   c. Use the mirror/lens equation to calculate d_i.
   d. Find the percent difference between your measured d_i and your calculated d_i.
   e. Measure the magnification M and compare it to the calculated magnification.

3. Consider a diverging lens with a focal length equal to four units.



   a. *Carefully* trace three rays coming off the top of the object and show where they converge to form the image.
   b. Measure d_o and d_i.
   c. Use the mirror/lens equation to calculate d_i.
   d. Find the percent difference between your measured d_i and your calculated d_i.
   e. Measure the magnification M and compare it to the calculated magnification.

4. A piece of transparent goo falls on your paper. You notice that the letters on your page appear smaller than they really are. Is the goo acting as a converging lens or a diverging lens? Explain. Is the image you see real or virtual? Explain.
5. An object is placed 30 mm in front of a lens. An image of the object is located 90 mmbehind the lens.
   a. Is the lens converging or diverging? Explain your reasoning.
   b. What is the focal length of the lens?
6. Little Red Riding Hood (*aka* $R-$Hood) gets to her grandmother's house only to find the Big Bad Wolf (*aka* BBW) in her place. $R-$Hood notices that BBW is wearing her grandmother's glasses and it makes the wolf's eyes look magnified (bigger).
   a. Are these glasses for near-sighted or far-sighted people? For full credit, explain your answer thoroughly. You may need to consult some resources online.
   b. Create a diagram of how these glasses correct a person's vision.

Answers to Selected Problems

1. c. 3 units e. `[U+0080][U+0093]`2/3
2. c. -6 (so 6 units on left side) e. 3 times bigger
3. c. -2.54 units (2.54 units on the left) e. $M = .36$
4. .
5. b. 22.5 mm
6. .

6.5 Diffraction

Students will learn about diffraction of light and the interference patterns that are produced through constructive and destructive interference of the waves. Students will also learn to calculate diffraction pattern spacing or work backwards to calculate the wavelength of light emitted.

Key Equations

$$m\lambda = d\sin\theta$$

Double slit interference maxima. m is the order of the interference maximum in question, d is the distance between slits. and θ is the angular position of the maximum.

$$m\lambda = d\sin\theta$$

Single slit interference maxima. m and θ are defined as above and d is the width of the slit.

$$m\lambda = d\sin\theta$$

Diffraction grating interference maxima. m and θ are defined as above and d is the distance between the lines on the grating.

$$m\lambda = 2nd$$

Thin film interference: n is the index of refraction of the film, d is the thickness of the film, and m is an integer. In the film interference, there is a $\lambda/2$ delay (phase change) if the light is reflected from an object with an index of refraction greater than that of the incident material.

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

Guidance

- Waves are characterized by their ability to constructively and destructively *interfere*. Light waves which interfere with themselves after interaction with a small aperture or target are said to *diffract*.
- Light creates interference patterns when passing through holes ("slits") in an obstruction such as paper or the surface of a CD, or when passing through a thin film such as soap.

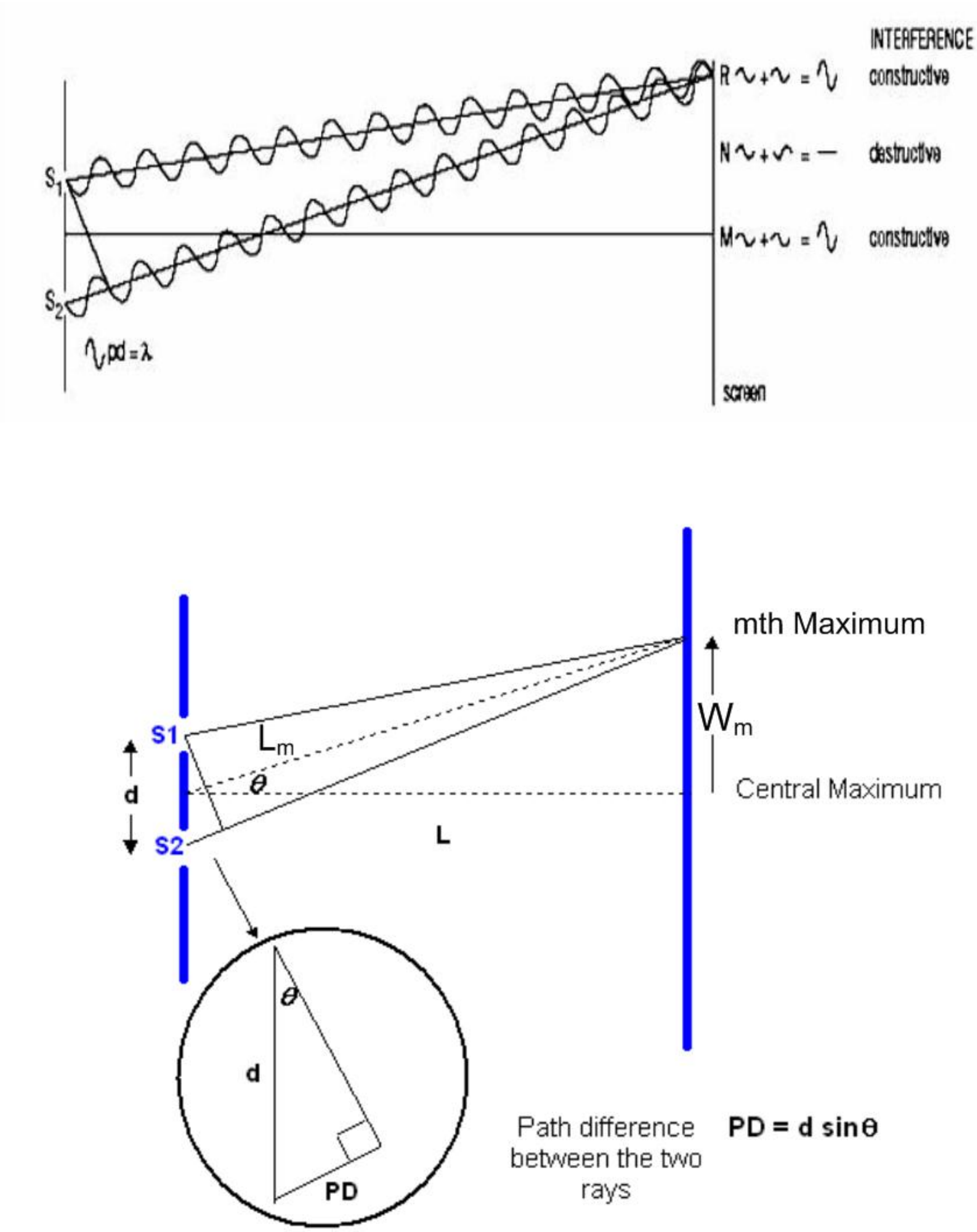


FIGURE 6.2

The top image shows the theory behind the interference pattern and the bottom image shows the geometry.

Example 1

A typical experimental setup for an interference experiment will look something like this:

Recall: for constructive interference we require a path difference of $m\lambda = d \sin\theta$

d = spacing of slits

m = number of maximum (i.e. m = 0 is the central maximum, m = 1 is the 1st maximum and thus the first dot to the right and le

L = distance form diffraction grating to screen

$W_m =$ distance from mth spot out from the center (if m = 1 then it is the 1st spot)

Definitions:

Maximum = place where waves constructively interfere

Minimum = place where waves destructively interfere

Because the screen distance L is much larger than the slit distance d, one can see that

$\frac{W_m}{L} = \tan\theta \approx \sin\theta.$

Thus, the condition for a first maximum becomes

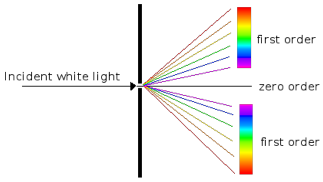
$\lambda = d\sin\theta = d\frac{W_m}{L}$

One can now easily calculate where the first maximum should appear if given the wavelength of the laser light, the distance to the screen and the distance between slits.

First Maximum: $W_m = \frac{L\lambda}{d}$

Example 2

White light (which is comprised of all wavelengths and thus all colors) separates into a rainbow pattern as shown below.



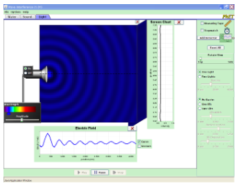
Each wavelength of light has a unique interference pattern given by the equation above. Thus all the wavelengths (i.e. colors) have a unique W_m based on the equation given at the end of Example 1. This is how white light separates out into its individual wavelengths producing a rainbow after going through a diffraction grating.

Watch this Explanation

MEDIA

Click image to the left for more content.

Simulation



WaveInterference (PhET Simulation)

Time for Practice

1. In your laboratory, light from a 650 nm laser shines on two thin slits. The slits are separated by 0.011 mm. A flat screen is located 1.5 mbehind the slits.
    a. Find the angle made by rays traveling to the third maximum off the optic axis.
    b. How far from the center of the screen is the third maximum located?
    c. How would your answers change if the experiment was conducted underwater?
2. Again, in your laboratory, 540 nm light falls on a pinhole 0.0038 mm in diameter. Diffraction maxima are observed on a screen 5.0 maway.
    a. Calculate the distance from the central maximum to the first interference maximum.
    b. Qualitatively explain how your answer to (a) would change if you:
        a. move the screen closer to the pinhole
        b. increase the wavelength of light
        c. reduce the diameter of the pinhole
3. Students are doing an experiment with a Helium-neon laser, which emits 632.5 nm light. They use a diffraction grating with 8000 lines/cm. They place the laser 1 m from a screen and the diffraction grating, initially, 95 cmfrom the screen. They observe the first and then the second order diffraction peaks. Afterwards, they move the diffraction grating closer to the screen.
    a. Fill in the **Table** (6.4) with the *expected* data based on your understanding of physics. Hint: find the general solution through algebra *before* plugging in any numbers.
    b. Plot a graph of the first order distance as a function of the distance between the grating and the screen.
    c. How would you need to manipulate this data in order to create a *linear* plot?
    d. In a real experiment what could cause the data to deviate from the expected values? Explain.
    e. What safety considerations are important for this experiment?
    f. Explain how you could use a diffraction grating to calculate the unknown wavelength of another laser.

TABLE 6.4:

Distance of diffraction grating to screen (cm)	Distance from central maximum to first order peak (cm)
95	
75	
55	
35	
15	

4. A crystal of silicon has atoms spaced 54.2 nm apart. It is analyzed as if it were a diffraction grating using an $x-$ray of wavelength 12 nm. Calculate the angular separation between the first and second order peaks from the central maximum.

5. Laser light shines on an oil film $(n = 1.43)$ sitting on water. At a point where the film is 96 nm thick, a 1^{st} order dark fringe is observed. What is the wavelength of the laser?
6. You want to design an experiment in which you use the properties of thin film interference to investigate the variations in thickness of a film of water on glass.
    a. List all the necessary lab equipment you will need.
    b. Carefully explain the procedure of the experiment and draw a diagram.
    c. List the equations you will use and do a sample calculation using realistic numbers.
    d. Explain what would be the most significant errors in the experiment and what effect they would have on the data.

Answers to Selected Problems

1. a. 10.2° b. 27 cm c. 20 cm
2. a. 0.72 m
3. $54\text{ cm}, 44\text{ cm}, 21\text{ cm}, 8.8\text{ cm}$
4. 13.5°
5. 549 nm
6. .

6.6 References

1. CK-12 Foundation. . CCSA
2. CK-12 Foundation. . CCSA

CHAPTER 7

Dynamics in 2D

Chapter Outline

7.1 Vectors

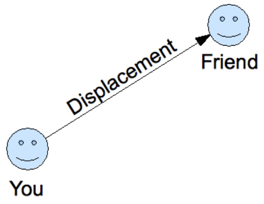
In order to solve two dimensional problems it is necessary to break all vectors into their x and y components. Different dimensions do not 'talk' to each other. Thus one must use the equations of motion once for the x-direction and once for the y-direction. For example, when working with the x-direction, one only includes the x-component values of the vectors in the calculations. Note that if an object is 'launched horizontally', then the full value is in the x-direction and there is no component in the y-direction.

In order to solve two dimensional problems it is necessary to break all vectors into their x and y components. Different dimensions do not 'talk' to each other. Thus one must use the equations of motion once for the x-direction and once for the y-direction. For example, when working with the x-direction, one only includes the x-component values of the vectors in the calculations. Note that if an object is 'launched horizontally', then the full value is in the x-direction and there is no component in the y-direction.

Key Equations

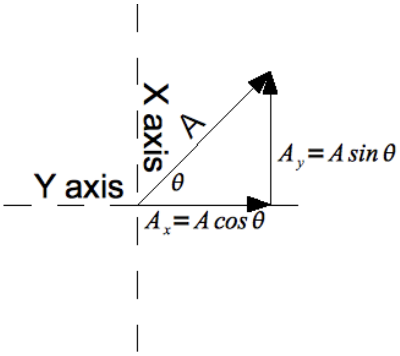
Vectors

The first new concept introduced here is that of a vector: a scalar magnitude with a direction. In a sense, we are almost as good at natural vector manipulation as we are at adding numbers. Consider, for instance, throwing a ball to a friend standing some distance away. To perform an accurate throw, one has to figure out both where to throw and how hard. We can represent this concept graphically with an arrow: it has an obvious direction, and its length can represent the distance the ball will travel in a given time. Such a vector (an arrow between the original and final location of an object) is called a displacement:



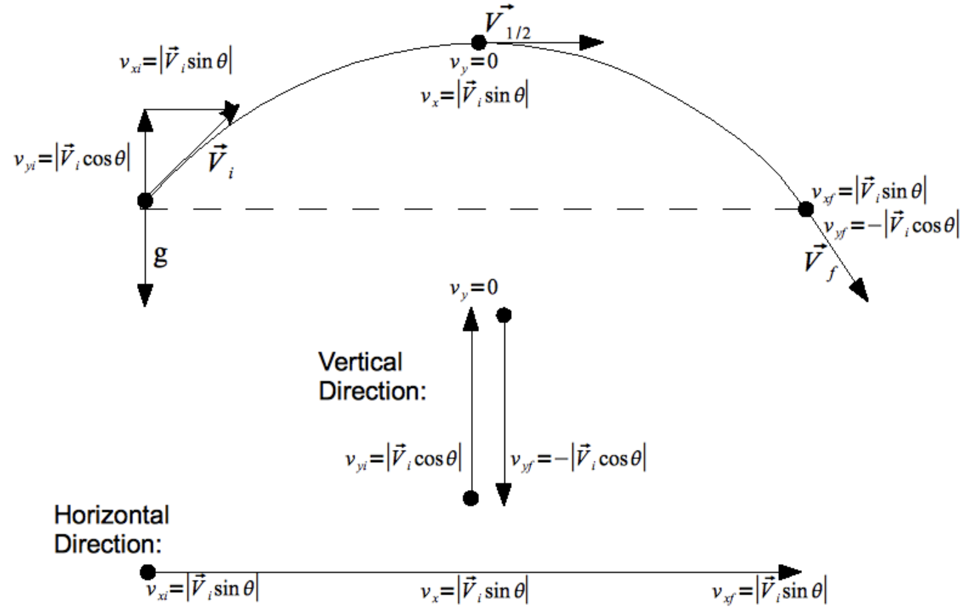
Vector Components

From the above examples, it should be clear that two vectors add to make another vector. Sometimes, the opposite operation is useful: we often want to represent a vector as the sum of two other vectors. This is called breaking a vector into its components. When vectors point along the same line, they essentially add as scalars. If we break vectors into components along the same lines, we can add them by adding their components. The lines we pick to break our vectors into components along are often called a **basis**. Any basis will work in the way described above, but we usually break vectors into *perpendicular* components, since it will frequently allow us to use the Pythagorean theorem in time-saving ways. Specifically, we usually use the x and y axes as our basis, and therefore break vectors into what we call their x and y components:



A final reason for breaking vectors into perpendicular components is that they are in a sense independent: adding vectors along a component perpendicular to an original component one will *never* change the original component, just like changing the y-coordinate of a point can never change its x-coordinate.

Break the Initial Velocity into its Components



Example 1

A tennis ball is launched 32° above the horizontal at a speed of 7.0 m/s. What are the horizontal and vertical velocity components?

Question: v_x and $v_y = ?\ [m/s]$

Given: $v = 7.0\ m/s$

$\theta = 32°$

Equation: $v_x = v\cos\theta \qquad v_y = v\sin\theta$

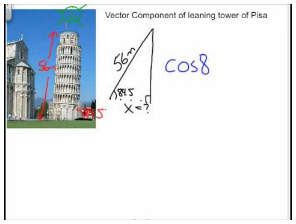
Plug n' Chug: $v_x = v\cos\theta = (7.0\ m/s)\cos(32^\circ) = 5.9\ m/s$

$$v_y = v\sin\theta = (7.0\ m/s)\sin(32^\circ) = 3.7\ m/s$$

Answer:

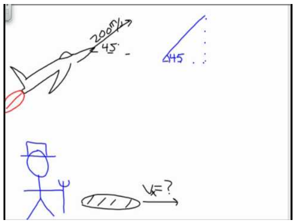
$$\boxed{\mathbf{5.9\ m/s, 3.7\ m/s}.}$$

Watch this Explanation



MEDIA

Click image to the left for more content.



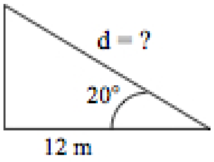
MEDIA

Click image to the left for more content.

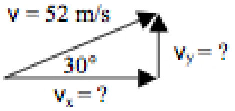
Time for Practice

1. Find the missing legs or angles of the triangles shown.

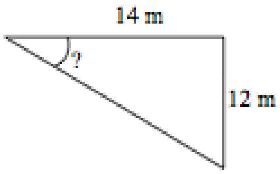
a.



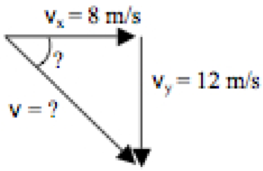
c.



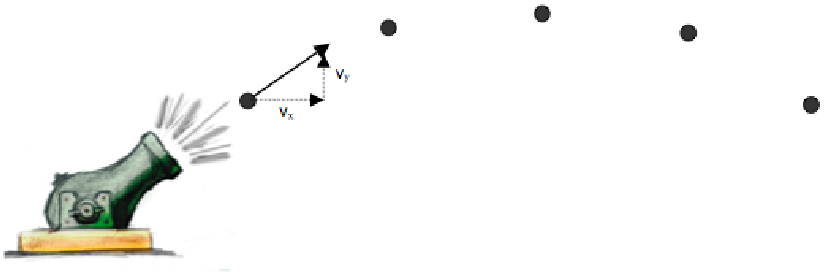
b.



d.

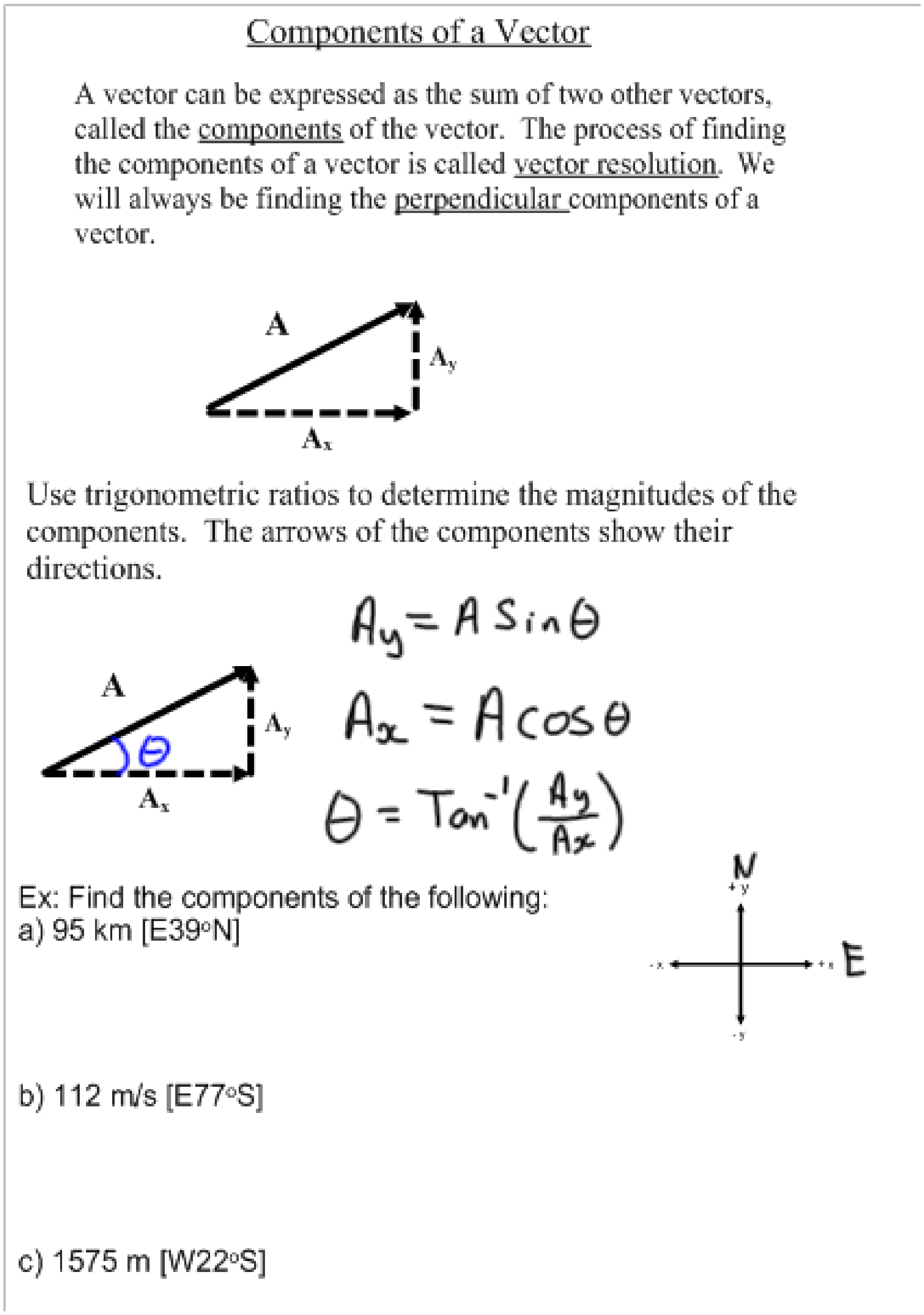


2. Draw in the $x-$ and $y-$velocity components for each dot along the path of the cannonball. The first one is done for you.


v_y
v_x

7.2 Applications of Vectors

Last year you used scale diagrams to determine resultants; now you will learn how to calculate them. You can review the concept of vectors in chapter 2.6.



Components of a Vector

A vector can be expressed as the sum of two other vectors, called the components of the vector. The process of finding the components of a vector is called vector resolution. We will always be finding the perpendicular components of a vector.

Use trigonometric ratios to determine the magnitudes of the components. The arrows of the components show their directions.

$$A_y = A\sin\theta$$

$$A_x = A\cos\theta$$

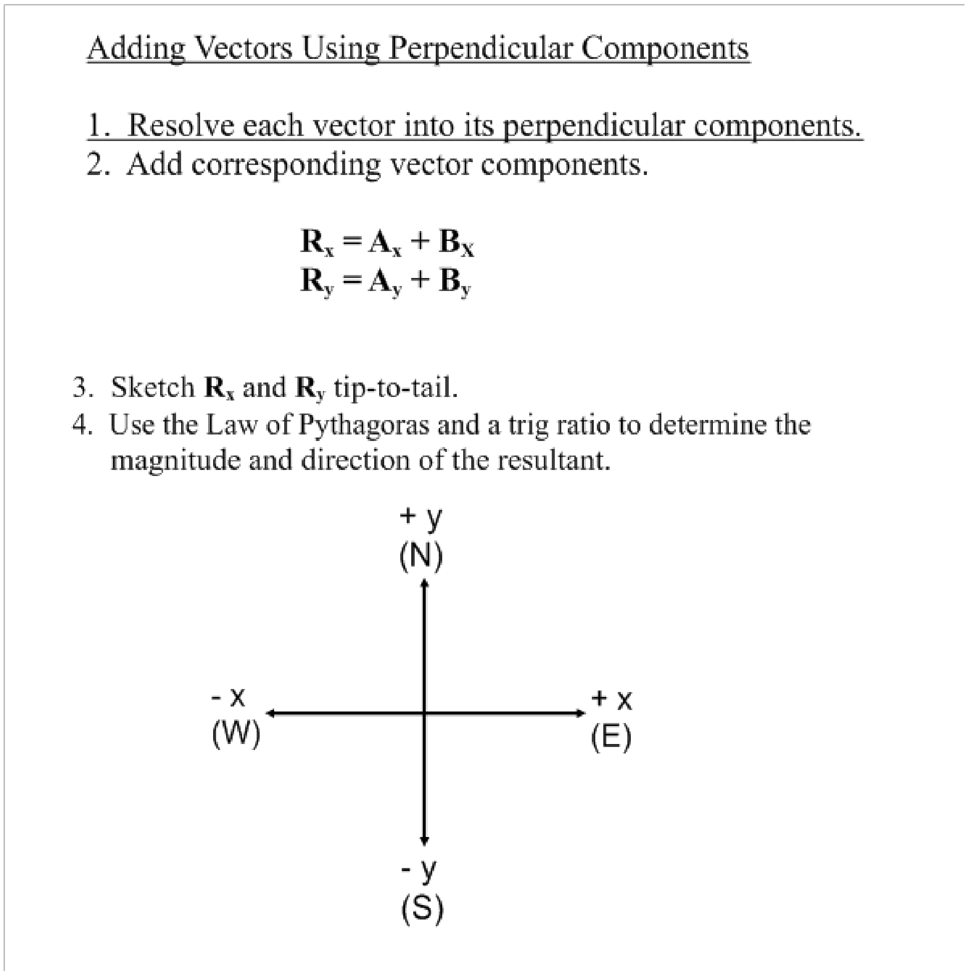
$$\theta = \mathrm{Tan}^{-1}\left(\frac{A_y}{A_x}\right)$$

Ex: Find the components of the following:

a) 95 km [E39°N]

b) 112 m/s [E77°S]

c) 1575 m [W22°S]

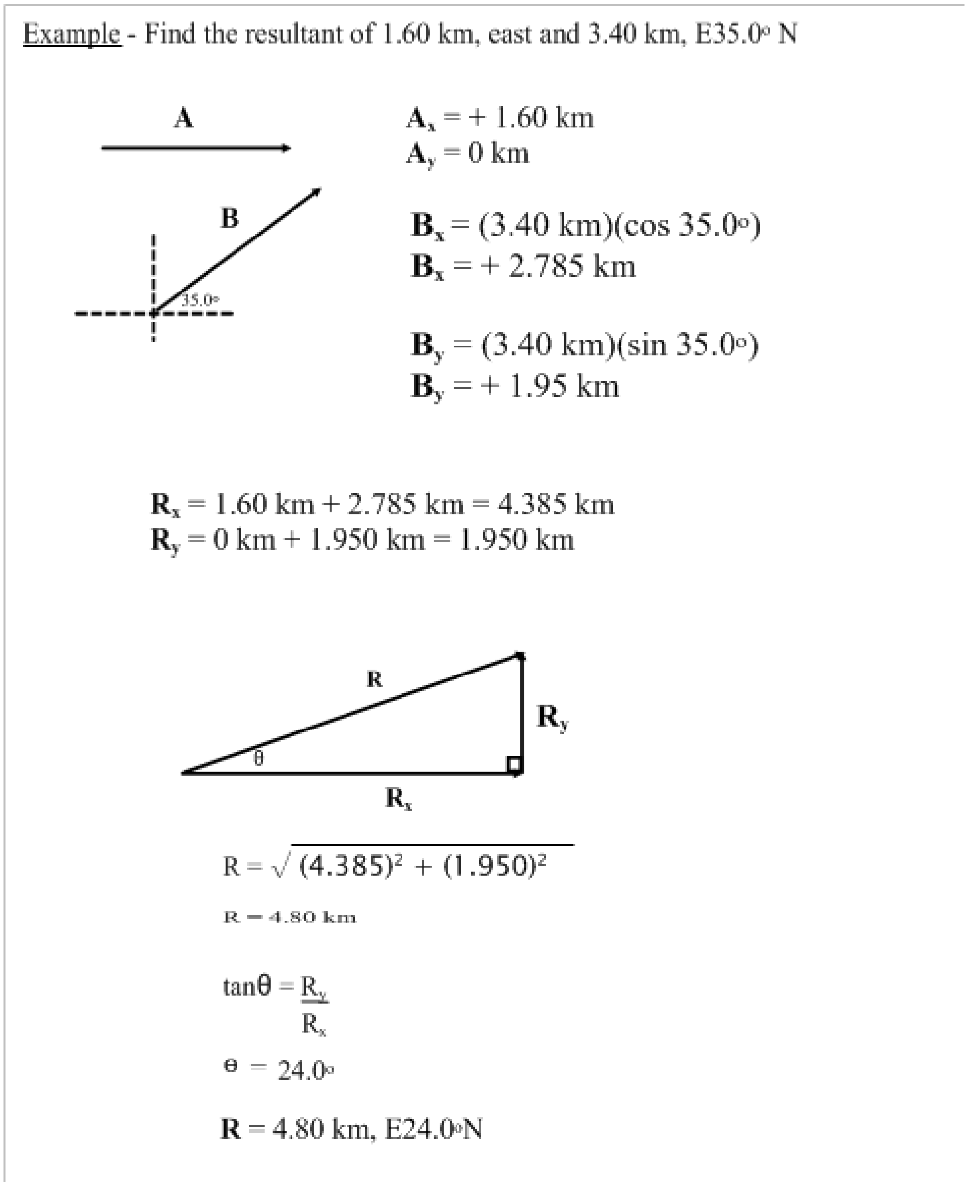


Adding Vectors Using Perpendicular Components

1. Resolve each vector into its perpendicular components.
2. Add corresponding vector components.

$$\mathbf{R_x} = \mathbf{A_x} + \mathbf{B_X}$$
$$\mathbf{R_y} = \mathbf{A_y} + \mathbf{B_y}$$

3. Sketch $\mathbf{R_x}$ and $\mathbf{R_y}$ tip-to-tail.
4. Use the Law of Pythagoras and a trig ratio to determine the magnitude and direction of the resultant.



Example - Find the resultant of 1.60 km, east and 3.40 km, E35.0° N

$A_x = + 1.60$ km
$A_y = 0$ km

$B_x = (3.40 \text{ km})(\cos 35.0°)$
$B_x = + 2.785$ km

$B_y = (3.40 \text{ km})(\sin 35.0°)$
$B_y = + 1.95$ km

$R_x = 1.60 \text{ km} + 2.785 \text{ km} = 4.385 \text{ km}$
$R_y = 0 \text{ km} + 1.950 \text{ km} = 1.950 \text{ km}$

$R = \sqrt{(4.385)^2 + (1.950)^2}$

$R = 4.80$ km

$\tan\theta = \frac{R_y}{R_x}$

$\theta = 24.0°$

R = 4.80 km, E24.0°N

1. Calculate the average velocity of a person who walks 325 m [N] and then 428 m [E 20^o N] in 185 seconds.

$\vec{v}_{avg} = \frac{\vec{d}}{\Delta t}$
$\vec{d} = \Sigma$ vectors
$\vec{d} = \vec{d}_1 + \vec{d}_2$

East Direction
$\vec{d}_E = d_{1E} + d_{2E}$
$d_{1E} = 325 \cos 90° = 0.0\ m$
$d_{2E} = 428 \cos 20° = 402\ m$
$\vec{d}_E = 0\ m + 402\ m = 402\ m \leftarrow$ positive value, so eastward

North Direction
$\vec{d}_N = d_{1N} + d_{2N}$

$d_{1N} = 325\sin 90^\circ = 325\ m$
$d_{2N} = 428\sin 20^\circ = 146\ m$
$\vec{d}_N = 325\ m + 146\ m \leftarrow$ positive, so northward

Use Pythagorean Theorem and Trigonometry
$d^2 = d_E^2 + d_N^2$
$d^2 = (402)^2 + (471)^2$
$|d| = 619\ m$
$\theta = \tan^{-1}|\frac{d_N}{d_E}|$
$\theta = \tan^{-1}|\frac{471}{402}| = 50^\circ$
$\vec{d} = 619\ m[E50^\circ N]$

$\vec{v}_{avg} = \frac{619\ m}{185\ s}[E50^\circ N]$
$\vec{v}_{avg} = 3.35\ \frac{m}{s}[E50^\circ N]$

2. Calculate the average velocity of a car that drives 66 km [E], 52 km [W33^{o}N], and 45 km [W73^{o}S] in 3.1 hours.

3. Calculate the acceleration of a glider that goes from 10 m/s [N] to 10 m/s [E] in 2.5 seconds.

$\vec{a} = \frac{\overrightarrow{v}_f - \vec{v}_o}{\Delta t}$ Use this equation twice; once for each direction

$a_E = \frac{v_{fE} - v_{oE}}{\Delta t}$ $a_N = \frac{v_{fN} - v_{oN}}{\Delta t}$ then use: $a^2 = a_E^2 + a_N^2$

$v_{fE} = 10\ m/s$ $v_{oE} = 0.0\ m/s$
$v_{fN} = 0.0\ m/s$ $v_{oN} = 10\ m/s$

$a_E = \frac{10\ m/s - 0}{2.5\ s} = 4.0\ m/s^2$
$a_N = \frac{0 - 10\ m/s}{2.5\ s} = -4.0\ m/s^2$

$|a| = \sqrt{4^2 + (-4)^2} = 5.7\ m/s^2$
$\theta = \tan^{-1}|\frac{a_N}{a_E}| = \tan^{-1}|\frac{-4.0}{4.0}| = 45^\circ$

$\vec{a} = 5.7\ m/s^2[E45^\circ S]$

4. Calculate the average force on the glider (from the previous question) if it has a mass of 92kg.

$\vec{F}_{avg} = m\vec{a}$
$\vec{F}_{avg} = (92\ kg)(5.7\ m/s^2[E45^\circ S])$
$\vec{F}_{avg} = 524\ N[E45^\circ S]$

5. An 18 kg object experiences two forces. F_1 = 35 N [E25^{o}N] and F_2 = 46 N [E75^{o}N]. What is the acceleration of the object?

6. An object initially has a velocity of 25 m/s [E62^{o}N] and accelerates at 5.5 m/s^2 [E12^{o}N] for 15 seconds. What is the displacement in that time?

$\vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$ Use this equation in each direction.

$v_{oE} = 25\cos 65^\circ = 10.6\ m/s$ $v_{oN} = 25\sin 65^\circ = 22.7\ m/s$
$a_E = 5.5\cos 75^\circ = 1.42\ m/s^2$ $a_N = 5.5\sin 75^\circ = 5.31\ m/s^2$

$d_E = v_{oE}t + \frac{1}{2}a_E t^2$

$$d_E = (10.6)(15)+\tfrac{1}{2}(1.42)(15)^2$$
$$d_E = 159+159.75 = 318.75\ m$$

$$d_N = v_{oN}t+\tfrac{1}{2}a_N t^2$$
$$d_N = (22.7)(15)+\tfrac{1}{2}(5.31)(15)^2$$
$$d_N = 340.5+597.38 = 937.88\ m$$

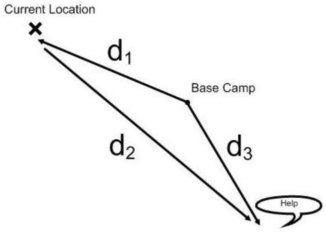
$$|d| = \sqrt{d_E^2+d_N^2}$$
$$|d| = \sqrt{(318.75)^2+(937.88)^2}$$
$$|d| = 990\ m$$

$$\theta = \tan^{-1}|\tfrac{d_N}{d_E}| = \tan^{-1}|\tfrac{937.88}{318.75}|$$
$$\theta = 71^\circ$$

$$\vec{d} = 990\ m\ [E\ 71^\circ\ N]$$

7. An object initially has a velocity of 15 m/s [W42^{o}N] and experiences an acceleration of 3.1 m/s^2 [E75^{o}S] for 12.6 seconds. What is the displacement in that time?

8. As you hike along a trail you track your location from base camp. When you are 8.4 km [W18^{o}N] a call for help comes in from a location 5.5 km [E65^{o}S]. How far are the stranded hikers from you? What heading should you set to go help them?



The information is given relative to base camp. So relative to base camp the people in need of help are located by calculating the vector sum of your current displacement and the path you must take. d$_2$ is the objective of this question.

$$\vec{d_3} = \vec{d_1}+\vec{d_2}$$
$$\vec{d_2} = \vec{d_3}-\vec{d_1}$$

$d_{3E} = +5.5\cos 65^\circ = 2.3\ km$ $d_{3N} = -5.5\sin 65^\circ = -4.98\ km$
$d_{1E} = -8.4\cos 18^\circ = -7.99\ km$ $d_{1N} = +8.4\sin 18^\circ = 2.6\ km$

$$d_{2E} = d_{3E}-d_{1E} = (2.3)-(7.99) = 10.3\ km$$
$$d_{2N} = d_{3N}-d_{1N} = (-4.98)-(2.6) = -7.58$$

$$|d_2| = \sqrt{(10.3)^2+(-7.58)^2} = 12.8\ km$$
$$\theta = \tan^{-1}|\tfrac{-7.58}{10.3}| = 36^\circ$$

$$\vec{d_2} = 12.8\ km\ [E\ 36^\circ\ S]$$

9. An inept boating tour guide takes you to a point 26 km [E33^{o}N] from port when in fact you should be located 30 km [E33^{o}S]. To get to your proper destination in 0.75 hours, with what velocity should the tour boat travel?

7.3 Forces in 2D

This section covers three types of force problems involving vectors in two dimensions.

Three Types of Force Problems

1. Pushing or pulling an object along a horizontal surface.
2. Tension and hanging signs
3. Objects on an incline.

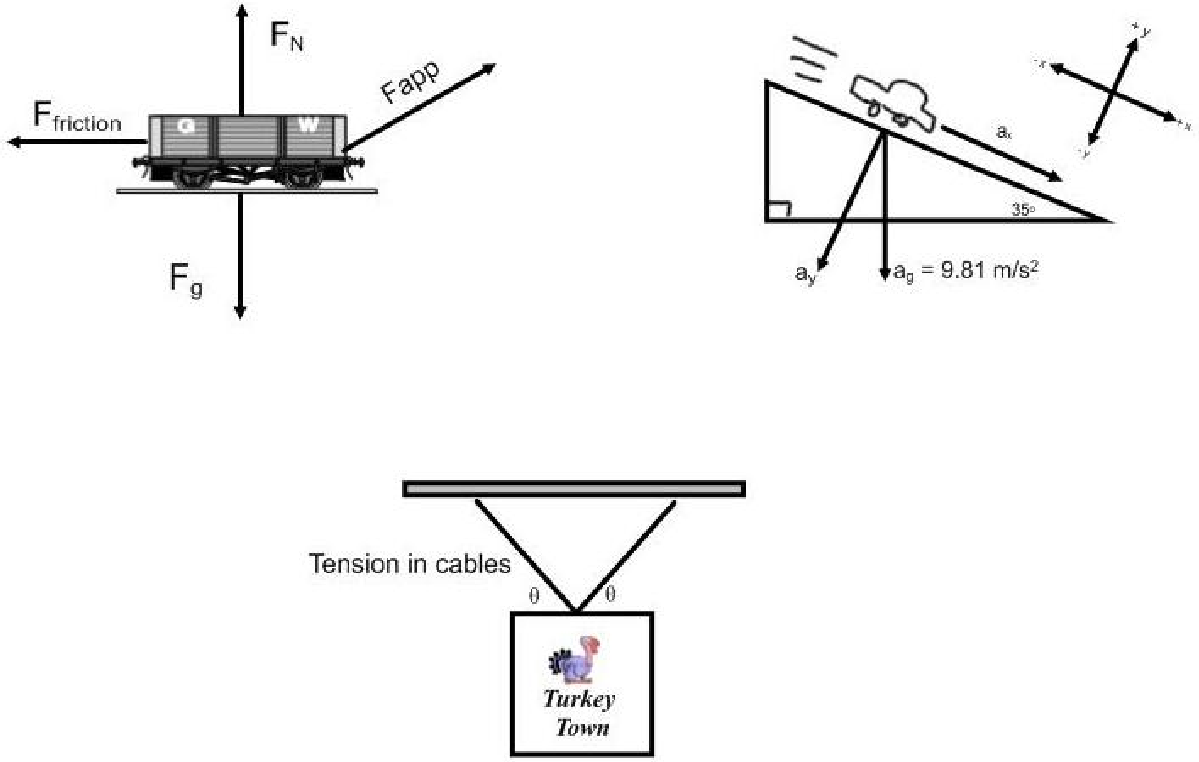


FIGURE 7.1

Diagrams of the three types of force problems.

Type 1 Force Problems

Example 1

A 55 kg snow blower is pushed along the ground at an angle of 35o to the horizontal with an applied force of 175 N.

A. Find the F_{ax} and F_{ay}.
B. Calculate F_N.
C. Find the force of friction if $\mu = 0.19$.
D. Find the F_{netx}.
E. Find a_x.

A.

$F_a = 175\ N$ at 35° with the x-axis
$F_{ax} = 175 \cos 35°$
$F_{ax} = 143\ N$

B.

$F_N =$ Normal Force
$F_{nety} = \Sigma$ Forces in the y - direction
$F_{nety} = F_g + F_{ay} + F_N$
$0 = -(55)(9.81) + (-100) + F_N$
$0 = -540\ N - 100\ N + F_N$
$640\ N = F_N$

C.

$F_f = \mu F_N$
$F_f = 0.19(640)$
$F_f = 122\ N \Leftarrow$ This is the magnitude of F_f

D.

$F_{netx} = \Sigma$ Forces in x-direction
$F_{netx} = F_{ax} + F_f$
$F_{netx} = 143 + (-122)$
$F_{netx} = 21\ N$

E.

$a_x = ?$
$F_{netx} = ma_x$
$21 = 55a_x$
$0.38 \frac{m}{s^2} = a_x$

Example 2

A 35 kg wagon is pulled along the ground at an angle of 25o to the horizontal with an applied force of 97 N.

A. Find the F_{ax} and F_{ay}.
B. Calculate F_N.
C. Find the force of friction if $\mu = 0.22$.
D. Find the F_{netx}.

E. Find a$_x$.

A.

$$F_{ax} = 97\cos 25^\circ$$
$$F_{ax} = 88\ N$$

B.

$$F_{nety} = \Sigma \text{ Forces in the y - direction}$$
$$F_{nety} = F_g + F_{ay} + F_N$$
$$0 = -(35)(9.81) + (+41) + F_N$$
$$0 = -343\ N + 41\ N + F_N$$
$$302\ N = F_N$$

C.

$$F_f = \mu F_N$$
$$F_f = 0.22(302)$$
$$F_f = 66\ N \Leftarrow \text{ This is the magnitude of } F_f$$

D.

$$F_{netx} = \Sigma \text{ Forces in x-direction}$$
$$F_{netx} = F_{ax} + F_f$$
$$F_{netx} = 88 + (-66)$$
$$F_{netx} = 22\ N$$

E.

$$a_x = ?$$
$$F_{netx} = ma_x$$
$$22 = 35a_x$$
$$0.63\frac{m}{s^2} = a_x$$

Type 2 - Force of Tension and Hanging Signs

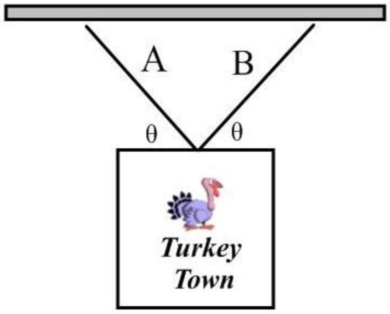
- If an object is hung by a rope (wire, chain, etc.) we can resolve the force of tension.

Example 1

A. If the mass of the traffic light is 65 kg, calculate the magnitude of the force that each cable exerts on the light to prevent it from falling?
B. Calculate the tension in each cable?

A. The y-component of the tension in each cable must add together to support the light's weight; the light is in static equilibrium. Since the angles are the same tension in each cable and their components are the same.

$$F_{nety} = 0\ N \leftarrow \text{ Static equilibrium}$$



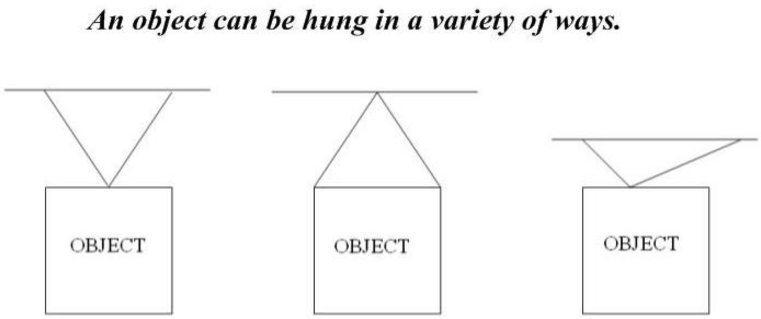


FIGURE 7.2
There are three ways we will hang signs in our problems.

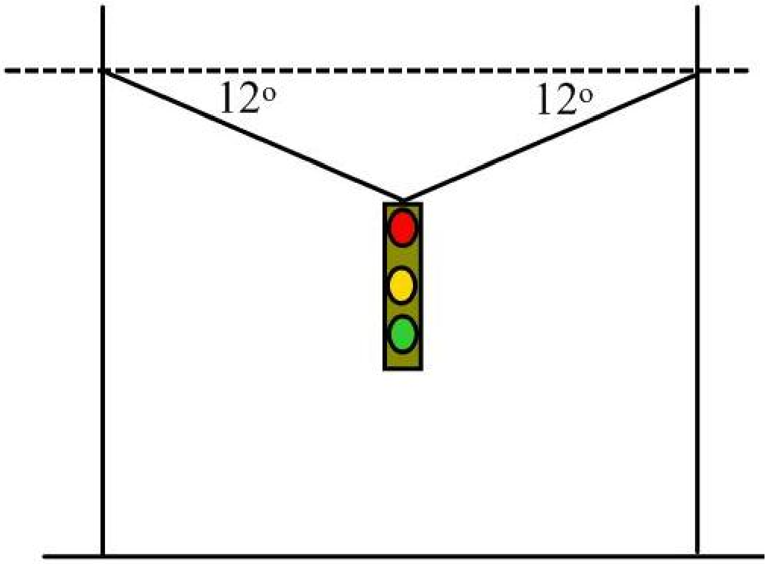


FIGURE 7.3
Traffic light suspended over a road.

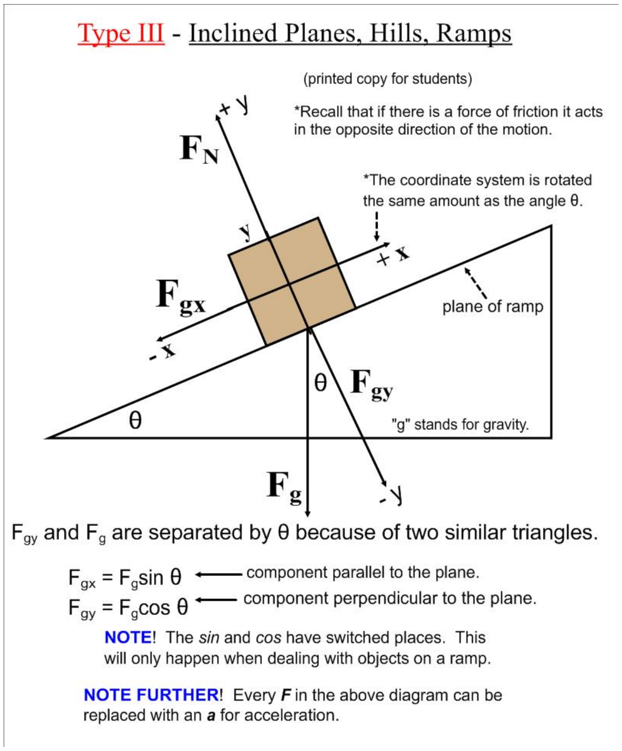
$F_{nety} = T_y + T_y + F_g$
$0 = 2T - (65)(9.81)$
$\frac{(65)(9.81)}{2} = T_y$
$319\,N = T_y$

B. Use tigonometry to solve for the tension in each cable

$T \sin 12^\circ = 319$

$T = \frac{319}{\sin 12^{\circ}}$
$T = 1534\ N$

Type III - Incline Planes, Ramps, and Hills



Type III - Inclined Planes, Hills, Ramps

(printed copy for students)

*Recall that if there is a force of friction it acts in the opposite direction of the motion.

*The coordinate system is rotated the same amount as the angle θ.

F_{gy} and F_g are separated by θ because of two similar triangles.

$F_{gx} = F_g \sin\theta$ ⟵ component parallel to the plane.
$F_{gy} = F_g \cos\theta$ ⟵ component perpendicular to the plane.

NOTE! The *sin* and *cos* have switched places. This will only happen when dealing with objects on a ramp.

NOTE FURTHER! Every ***F*** in the above diagram can be replaced with an ***a*** for acceleration.

Example 1

A 55 kg block is sliding down an incline. The coefficient of kinetic friction is 0.13 and the incline makes an angle of 35^o with the ground. What applied force up the ramp is necessary so the block accelerates with a magnitude of 0.83 m/s^2 down the ramp?

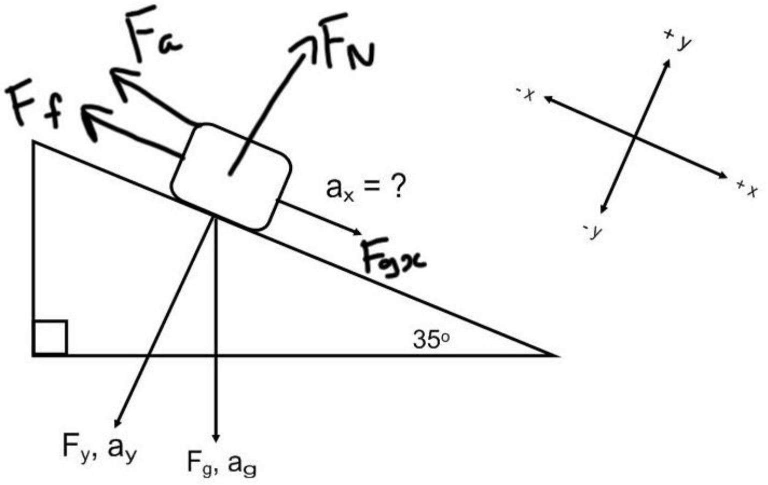


FIGURE 7.4

Diagram for ramp question example 1. Note the declaration of the positive and negative directions.

Step 1. Find F$_{netx}$

$F_{netx} = F_{gx} + F_f + F_a$
$F_{netx} = ma_x$
$F_{netx} = (55)(0.83)$
$F_{netx} = 45.7\ N$

Step 2. Find F$_f$

$F_f = \mu F_N$
$F_N = F_{gy}$
$F_f = 0.13F_{gy}$
$F_f = 0.13F_g \cos 35^\circ$
$F_f = 0.13(55)(9.81)\cos 35^\circ$
$F_f = 57.5\ N$

Step 3. Find F$_{gx}$

$F_{gx} = F_g \sin 35^\circ$
$F_{gx} = (55)(9.81)\sin 35^\circ$
$F_{gx} = 309\ N$

Step 4. Calculate F$_a$

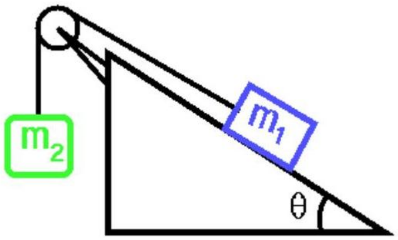
$F_{netx} = F_{gx} + F_f + F_a$
$45.7 = 309 + (-57.5) + F_a$
$-206\ N = F_a$ (negative answer means up the ramp)

Example 2

An inclined ramp is to be used to slide down an object at a constant speed. The coefficient of kinetic friction is 0.16. Calculate the angle the ramp must make with the ground for this to happen.

$\theta = ?$
$F_{netx} = 0$ (constant speed)
$0 = F_f + F_{gx}$
$a_{netx} = a_f + a_{gx}$
$a_g = 9.81\frac{m}{s^2}$
$a_f = \mu a_N$
$a_f = \mu a_{gy}$
$a_f = \mu a_g \cos\theta$
$a_{gx} = a_g \sin\theta$
$a_{netx} = -\mu a_g \cos\theta + a_g \sin\theta$
$0 = -0.16(9.81)\cos\theta + 9.81\cos\theta$
$0 = -0.16\cos\theta + \sin\theta$
$-\sin\theta = -0.16\cos\theta$
$\frac{-\sin\theta}{-\cos\theta} = 0.16$
$\tan\theta = 0.16$
$\theta = \tan^{-1} 0.16$
$\theta = 9.1^\circ$

Objects Connected at an Angle



We approach this problem as discussed in Chapter 3.7; apply Newton's second law.

Example 1

A counterweight is used to slide an object up an inclined plane of 20^o. The counterweight has a mass of 25 kg and is suspended with a mass-less string and a frictionless pully. The coefficient of friction on the plane is 0.19. Calculate the acceleration of a 16 kg object.

$$
\begin{aligned}
&M_1 = 16\ kg \qquad M_2 = 25\ kg\\
&\theta = 20^\circ \qquad\quad \mu = 0.19\\
&\Sigma \text{ Forces} = \Sigma \text{ masses} \times a\\
&F_{gx} + F_f + F_{g2} = (M_1 + M_2)a\\
&F_{gx} = F_g \sin\theta\\
&F_f = \mu F_N = \mu F_g \cos\theta\\
&F_{g2} = M_2 g\\
&-(16)(9.81)\sin 20^\circ + [-(0.19)(16)(9.81\cos 20^\circ)] + (25)(9.81) = (16+25)a\\
&-53.7 - 280 + 245 = 41a\\
&163.55 = 41a\\
&4.0\tfrac{m}{s^2} = a
\end{aligned}
$$

Try this one:

A counterweight is used to slide an object up an inclined plane that makes a 40^o angle with the horizontal. The counterweight has a mass of 35 kg and is suspended with a mass-less string and a frictionless pully. The coefficienct of kinetic friction is 0.23.

a) For the acceleration of the object not to exceed 0.42 m/s^2 up the ramp, calculate the minimum mass of the object. (answer: 39 kg)

b) Calculate the mass would result in an acceleration of 0.21 m/s^2 down the ramp. (answer: 80 kg)

7.4 Impulse

Students will learn the meaning of impulse force and how to calculate both impulse and impulse force in various situations.

Key Equations

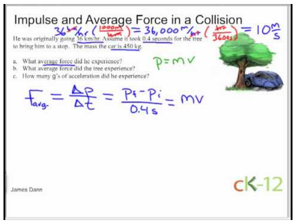
$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$\Delta\vec{p} = \vec{F}_{net}\Delta t$

Guidance

- The force imparted on an object is equal to the change in momentum divided by the time interval over which the objects are in contact.
- **Internal forces** are forces for which both Newton#8217;s Third Law force pairs are contained within the system. For example, consider a two-car head-on collision. Define the *system* as just the two cars. In this case, internal forces include that of the fenders pushing on each other, the contact forces between the bolts, washers, and nuts in the engines, etc.
- **External forces** are forces that act on the system from outside. In our previous example, external forces include the force of gravity acting on both cars (because the other part of the force pair, the pull of gravity the Earth experiences coming from the cars, is not included in the system) and the forces of friction between the tires and the road.
- If there are no external forces acting on a system of objects, the initial momentum of the system will be the same as the final momentum of the system. Otherwise, the final momentum will change by $\Delta\vec{p} = \vec{F}\Delta t$. We call such a change in momentum $\Delta\vec{p}$ an **impulse**.

Example 1



Impulse and Average Force in a Collision

$36\,km/hr\left(\frac{1000m}{km}\right) = 36{,}000\,m/hr\left(\frac{hr}{3600s}\right) = 10\frac{m}{s}$

He was originally going 36 km/hr. Assume it took 0.4 seconds for the tree to bring him to a stop. The mass the car is 450 kg.

a. What average force did he experience?
b. What average force did the tree experience?
c. How many g's of acceleration did he experience?

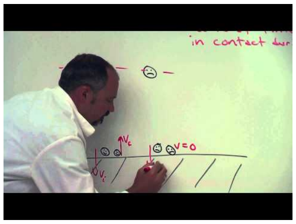
$p = mv$

$F_{avg.} = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{0.4s} = mv$

MEDIA

Click image to the left for more content.

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. You jump off of the top of your house and hope to land on a wooden deck below. Consider the following possible outcomes:

    a. You hit the deck, but it isn#8217;t wood! A camouflaged trampoline slows you down over a time period of 0.2 seconds and sends you flying back up into the air.
    b. You hit the deck with your knees locked in a straight-legged position. The collision time is 0.01 seconds.
    c. You hit the deck and bend your legs, lengthening the collision time to 0.2 seconds.
    d. You hit the deck, but it isn#8217;t wood! It is simply a piece of paper painted to look like a deck. Below is an infinite void and you continue to fall, forever.

        a. Which method will involve the greatest force acting on you?
        b. Which method will involve the least force acting on you?
        c. Which method will land you on the deck in the least pain?
        d. Which method involves the least impulse delivered to you?
        e. Which method involves the greatest impulse delivered to you?

2. You punch the wall with your fist. Clearly your fist has momentum before it hits the wall. It is equally clear that after hitting the wall, your fist has no momentum. But momentum is always conserved! Explain.
3. You look up one morning and see that a 30 kg chunk of asbestos from your ceiling is falling on you! Would you be better off if the chunk hit you and stuck to your forehead, or if it hit you and bounced upward? Explain your answer.
4. A baseball player faces a 80.0 $\mathrm{m/s}$ pitch. In a matter of .020 seconds he swings the bat, hitting a 50.0 $\mathrm{m/s}$ line drive back at the pitcher. Calculate the force on the bat while in contact with the ball.
5. A place kicker applies an average force of 2400 N to a football of 0.40 kg. The force is applied at an angle of 20.0 degrees from the horizontal. Contact time is .010sec.

    a. Find the velocity of the ball upon leaving the foot.
    b. Assuming no air resistance find the time to reach the goal posts 40.0 m away.
    c. The posts are 4.00 m high. Is the kick good? By how much?

6. Your author#8217;s Italian cousin crashed into a tree. He was originally going 36 $\mathrm{km/hr}$. Assume it took 0.40 seconds for the tree to bring him to a stop. The mass of the cousin and the car is 450 kg.

a. What average force did he experience? Include a direction in your answer.
b. What average force did the tree experience? Include a direction in your answer.
c. Express this force in pounds.
d. How many g#8217;s of acceleration did he experience?

7. Serena Williams volleys a tennis ball hit to her at 30 m/s. She blasts it back to the other court at 50 m/s. A standard tennis ball has mass of 0.057 kg. If Serena applied an average force of 500 N to the ball while it was in contact with the racket, how long did the contact last?

Answers to Selected Problems

1. .
2. .
3. .
4. a. 60 m/s b. .700 sec c. yes, 8.16 m

5. .
6. a. 11000 N to the left b. tree experienced same average force of 11000 N but to the right c. 2500 lb. d. about 2.5 #8220;g#8221;s of acceleration
7. a. 0.00912 s

7.5 Momentum

Students will learn what momentum is and how to calculate momentum of objects. In addition, students will learn how to use conservation of momentum to solve basic problems.

Key Equations

$p = mv$ Momentum is equal to the objects mass multiplied by its velocity

$\sum p_{\text{initial}} = \sum p_{\text{final}}$ The total momentum does not change in closed systems

Guidance

- Momentum is a vector that points in the direction of the velocity vector. The magnitude of this vector is the product of mass and speed.
- The total momentum of the universe is always the same and is equal to zero. The total momentum of an isolated system never changes.
- Momentum can be transferred from one body to another. In an isolated system in which momentum is transferred internally, the total initial momentum is the same as the total final momentum.
- Momentum conservation is especially important in collisions, where the total momentum just before the collision is the same as the total momentum after the collision.

Example 1

A truck with mass 500 kg and originally carrying 200 kg of dirt is rolling forward with the transmission in neutral and shooting out the dirt backwards at 2 m/s (so that the dirt is at relative speed of zero compared with the ground). If the truck is originally moving at 2 m/s, how fast will it be moving after it has shot out all the dirt. You may ignore the effects of friction.

Solution

To solve this problem we will apply conservation of momentum to the truck when it is full of dirt and when it has dumped all the dirt.

$$\begin{aligned}
m_o v_o &= m_f v_f && \text{start by setting the initial momentum equal to the final momentum} \\
(m_t + m_d) v_o &= m_t v_f && \text{substitute the mass of the truck plus the mass of the dirt in the truck at the initial and} \\
v_f &= \frac{(m_t + m_d) v_o}{m_t} && \text{solve for the final velocity} \\
v_f &= \frac{(500\text{ kg} + 200\text{ kg}) * 2\text{ m/s}}{500\text{ kg}} && \text{plug in the numerical values} \\
v_f &= 2.8\text{ m/s}
\end{aligned}$$

Example 2

John and Bob are standing at rest in middle of a frozen lake so there is no friction between their feet and the ice. Both of them want to get to shore so they simultaneously push off each other in opposite directions. If John's mass is 50 kg and Bob's mass is 40 kg and John moving at 5 m/s after pushing off Bob, how fast is Bob moving?

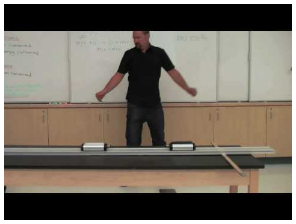
Solution

For this problem, we will apply conservation of momentum to the whole system that includes both John and Bob. Since both of them are at rest to start, we know that the total momentum of the whole system must always be zero. Therefore, we know that the sum of John's and Bob's momentum after they push off each other is also zero. We can use this to solve for Bob's velocity.

$$0 = m_j v_j + m_b v_b$$
$$-m_b v_b = m_j v_j$$
$$v_b = -\frac{m_j v_j}{m_b}$$
$$v_b = -\frac{50\,\text{kg} * 5\,\text{m/s}}{40\,\text{kg}}$$
$$v_b = -6.25\text{ m/s}$$

The answer is negative because Bob is traveling in the opposite direction to John.

Watch this Explanation



MEDIA

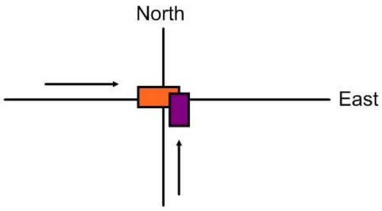
Click image to the left for more content.

Two Dimensional Collisions

In order to solve two dimensional collision problems, write a conservation of momentum equation for the horizontal components of the momenta and a conservation of momentum equation for the vertical components of the momenta.

Example 1

A 1325 kg car moving north at 27.0 m/s collides with a 2165 kg car moving east at 17.0 m/s. They stick together. In what direction and with what speed do they move after the collision?



Calculate the momentum in each direction and then use the Pythagorean Theorem and trigonometry.

x-direction
$v_{1x} = 17\ m/s \qquad v_{2x} = 0.0\ m/s$
$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$ and $v'_{1x} = v'_{2x} = v_{fx}$ (because they stick together)
$(2165)(17) + 0.0 = (2165)v_{fx} + (1325)v_{fx}$
$36805 = 3490v_{fx}$
$10.5\ m/s = v_{fx}$

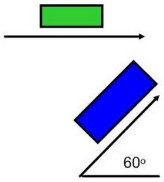
$y - direction$
$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$ and $v'_{1y} = v'_{2y} = v_{fy}$ (because they stick together)
$0.0 + (1325)(27) = 3490v_{fy}$
$35775 = 3490v_{fy}$
$10.25\ m/s = v_{fy}$

$|\vec{v}_f| = \sqrt{v_{fx}^2 + v_{fy}^2}$
$|\vec{v}_f| = \sqrt{215.3} = 14.7\ m/s$
$\theta = \tan^{-1}|\frac{v_{fy}}{v_{fx}}| = \tan^1|\frac{10.25}{10.5}|$
$\theta = 44°$

$\vec{v}_f = 14.7\ m/s\ [E\ 44°\ N]$

Example 2

A 1200 kg car is moving east at 30.0 m/s and collides with a 3600 kg car moving at 20.0 m/s in a direction E60.0^{o}N. The vehicles interlock and move off together. Find their common velocity.



x-direction
$v_{1x} = 30\ m/s$
$v_{2x} = 20\cos 60° = 10\ m/s$
$m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$
$v'_{1x} = v'_{2x} = v_{fx}$ stick together
$(1200)(30) + (3600)(10) = 4800v_{fx}$
$36000 + 36000 = 4800v_{fx}$
$15\ m/s = v_{fx}$

y-direction
$v_{1y} = 0.0\ m/s$
$v_{2y} = 20\sin 60° = 17.3\ m/s$
$m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$
$v'_{1y} = v'_{2y} = v_{fy}$ stick together
$(1200)(0) + (3600)(17.3) = 4800v_{fy}$
$62280 = 4800v_{fy}$

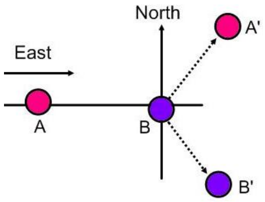
$13\ m/s = v_{fy}$

$|v_f| = \sqrt{v_{fx}^2 + v_{fy}^2}$
$|v_f| = \sqrt{(15)^2 + (13)^2}$
$|v_f| = 19.8\ m/s$
$\theta = \tan^{-1}|\frac{v_{fy}}{v_{fx}}| = \tan^{-1}|\frac{13}{15}|$
$\theta = 41^\circ$

$\vec{v}_f = 20\ m/s\ [E\ 41^\circ\ N]$

Example 3

A 6.0 kg object, call it A, moving at a velocity of 3.0 m/s east collides with a 6.0 kg object, B, at rest. After the ?collision, A moves off in a direction 40.0^o to the left of its original direction. B moves off in a direction 50.0^o to the right of A's original direction. What is the magnitude of the velocity of each object after the collision?



x-direction
$v_{Ax} = 3.0\ m/s \qquad v'_{Ax} = v'_A \cos 40^\circ$
$v_{Bx} = 0.0\ m/s \qquad v'_{Bx} = v'_B \cos 50^\circ$

$m_A v_{Ax} + m_B v_{Bx} = m_A v'_{Ax} + m_B v'_{Bx}$
$m_A = m_B$ so the masses divide out
$v_{Ax} + v_{Bx} = v'_{Ax} + v'_{Bx}$
$3.0 = v'_{Ax} + v'_{Bx}$
$3.0 = v'_A \cos 40^\circ + v'_B \cos 50^\circ$

y-direction
$v_{Ay} = 0.0\ m/s \qquad v'_{Ay} = v'_A \sin 40^\circ$
$v + By = 0.0\ m/s \qquad v'_{By} = -v'_B \sin 50^\circ$

$m_A v_{Ay} + m_B v_{By} = m_A v'_{Ay} + m_B v'_{By}$
$m_A = m_B$ so the masses divide out
$v_{Ay} + v_{By} = v'_{Ay} + v'_{By}$
$0.0 = v'_{Ay} + v'_{By}$
$0.0 = v'_A \sin 40^\circ + (-v'_B \sin 50^\circ)$
$0.0 = v'_A \sin 40^\circ - v'_B \sin 50^\circ$

We have two equations and two unknowns:
$3.0 = v'_A \cos 40^\circ + v'_B \cos 50^\circ$ Eq.①
$0.0 = v'_A \sin 40^\circ - v'_B \sin 50^\circ$ Eq. ②

Start by solving for v'_B in Eq. ②
$0.0 = v'_A \sin 40^\circ - v'_B \sin 50^\circ$
$v'_B \sin 50^\circ = v'_A \sin 40^\circ$
$v'_B = \frac{v'_A \sin 40^\circ}{\sin 50^\circ}$
$v'_B = 0.8391 v'_A \leftarrow$ Substitute into Eq. ①

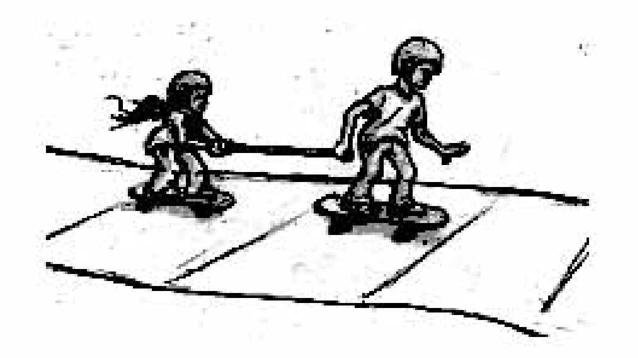
$3.0 = v'_A \cos 40^\circ + v'_B \cos 50^\circ$
$3.0 = v'_A \cos 40^\circ + (0.8391 v'_A) \cos 50^\circ$
$3.0 = 0.766 v'_A + 0.5394 v'_A$
$3.0 = 1.305 v'_A$
$2.2\ m/s = v'_A \leftarrow$ Substitute into Eq. ②

$v'_B = 0.8391 v'_A$
$v'_B = 0.8391(2.2)$
$v'_B = 1.9\ m/s$

Final answer is then: $v'_A = 2.2\ m/s$ and $v'_B = 1.9\ m/s$

Time for Practice

1. You find yourself in the middle of a frozen lake. There is no friction between your feet and the ice of the lake. You need to get home for dinner. Which strategy will work best?
    a. Press down harder with your shoes as you walk to shore.
    b. Take off your jacket. Then, throw it in the direction opposite to the shore.
    c. Wiggle your butt until you start to move in the direction of the shore.
    d. Call for help from the great Greek god Poseidon.
2. You and your sister are riding skateboards side by side at the same speed. You are holding one end of a rope and she is holding the other. Assume there is no friction between the wheels and the ground. If your sister lets go of the rope, how does your speed change?
    a. It stays the same.
    b. It doubles.
    c. It reduces by half.



3. You and your sister are riding skateboards (see Problem 3), but now she is riding behind you. You are holding one end of a meter stick and she is holding the other. At an agreed time, you push back on the stick hard enough to get her to stop. What happens to your speed? Choose one. (For the purposes of this problem pretend you and your sister weigh the same amount.)
    a. It stays the same.

   b. It doubles.
   c. It reduces by half.
4. An astronaut is using a drill to fix the gyroscopes on the Hubble telescope. Suddenly, she loses her footing and floats away from the telescope. What should she do to save herself?
5. A 5.00 kg firecracker explodes into two parts: one part has a mass of 3.00 kg and moves at a velocity of 25.0 m/s towards the west. The other part has a mass of 2.00 kg. What is the velocity of the second piece as a result of the explosion?
6. A firecracker lying on the ground explodes, breaking into two pieces. One piece has twice the mass of the other. What is the ratio of their speeds?
7. While driving in your pickup truck down Highway 280 between San Francisco and Palo Alto, an asteroid lands in your truck bed! Despite its 220 kg mass, the asteroid does not destroy your 1200 kg truck. In fact, it landed perfectly vertically. Before the asteroid hit, you were going 25 m/s. After it hit, how fast were you going?
8. An astronaut is 100 m away from her spaceship doing repairs with a 10.0 kg wrench. The astronaut#8217;s total mass is 90.0 kg and the ship has a mass of 1.00×10^4 kg. If she throws the wrench in the opposite direction of the spaceship at 10.0 m/s how long would it take for her to reach the ship?

Answers to Selected Problems

1. .
2. .
3. .
4. .
5. 37.5 m/s
6. $v_1 = 2v_2$
7. 21 m/s
8. a. 90 sec

7.6 Torque

Students will learn to calculate Torque in various situations.

Key Equations

$\vec{\tau} = rF_{\perp}$

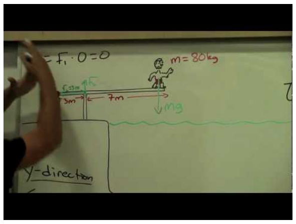
Individual torques are determined by multiplying the force applied by the *perpendicular* component of the moment arm.

To distiguish between the two rotational directions; Torque that is created from a counter-clockwise rotation is positive and from a clockwise rotation is negative.

Guidance

Torque is equal to the force acting on the object multiplied by the perpendicular distance from the application of the force to the rotational axis. Say you had a seesaw. It is easier to exert torque, get the seesaw to move, if you pushed on the board near the end rather than near the middle. It is the rotational version of Force.

Example 1

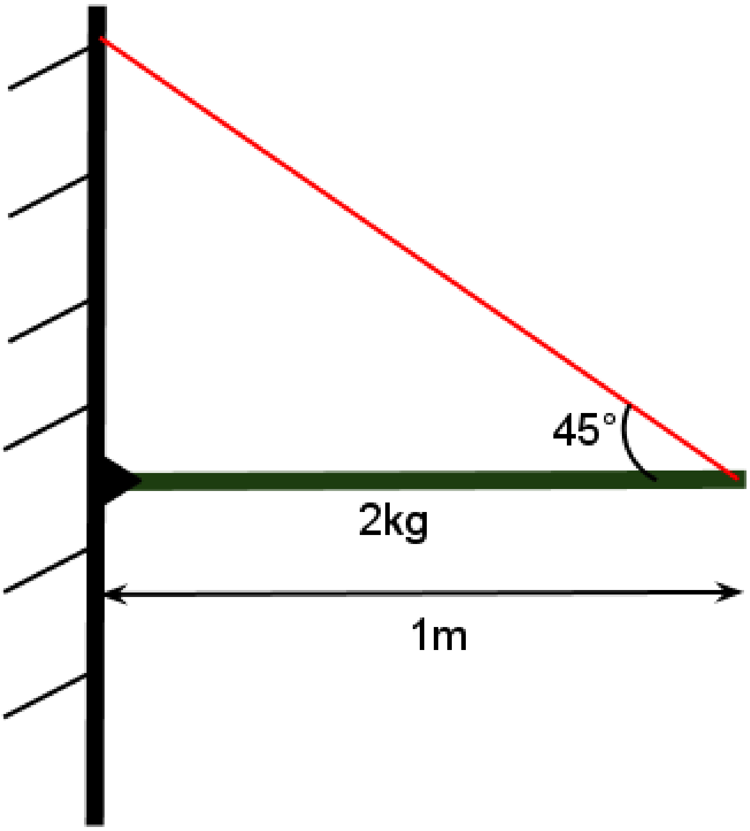


MEDIA

Click image to the left for more content.

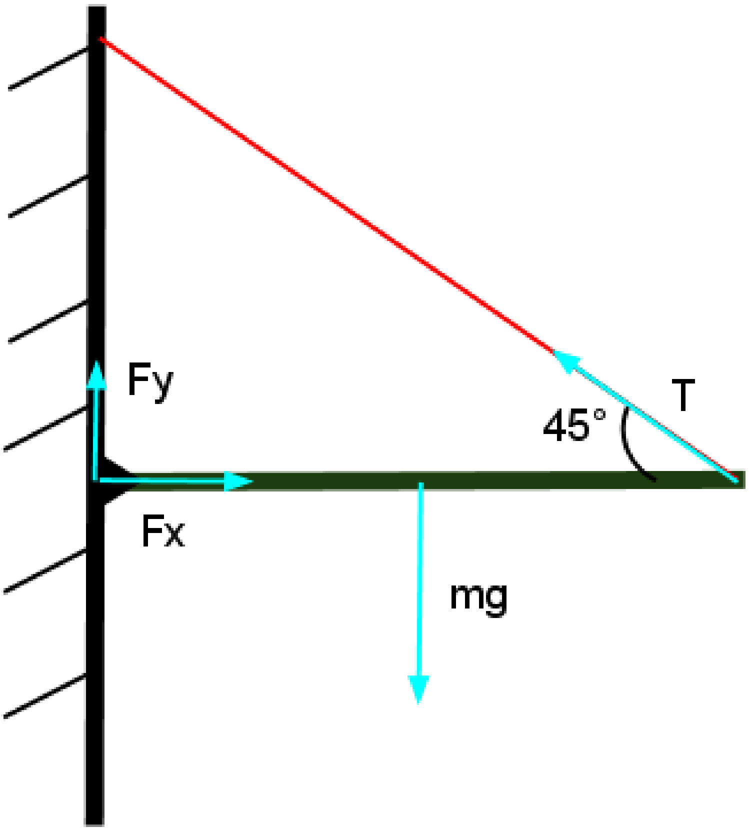
Example 2

A 1 m long 2 kg rod is attached to the side of a building with a hinge one one end. The rod is held level by a cable that makes an angle of 45 degrees with the rod and is also attached to the building above the rod. What is the tension in the cable? The situation is illustrated in a diagram below.



Solution

There will be multiple steps to this problem. First we'll make a free body diagram for the rod; making free body diagrams in torque problems is just as important as in Newton's Laws problems. In the diagram, the force of tension from the cable, the rod's weight, and the forces from the hinge are shown. For the purposes of calculating torque, an object's weight acts from it's center of mass (half way along the rod in this case). In the diagram below, the force on the rod from the hinge is already broken into it's components to make it easier to visualize.



Now, in order to find the tension in the cable, we'll sum the torques on the rod with the hinge as the axis of rotation. Notice that the sign of the torque due to tension in the cable and the torque due to the rod's weight will be opposite signs because they would cause the rod to rotate in opposite directions.

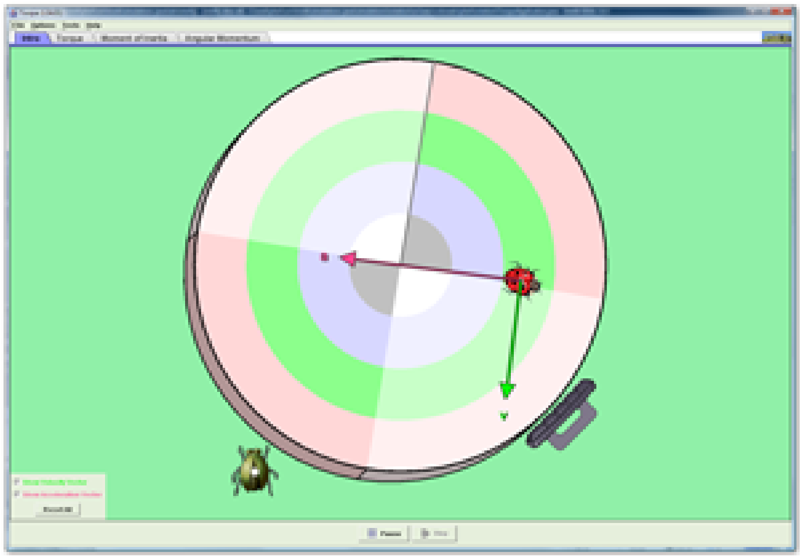
$$\begin{aligned}
\Sigma\tau &= 0\\
mg\frac{r}{2} - T\sin(\theta)r &= 0\\
T &= \frac{mgr}{2\sin(\theta)r}\\
T &= \frac{mg}{2\sin(\theta)}\\
T &= \frac{2\text{ kg} * 9.8\text{ m/s}^2}{2\sin(45)}\\
T &= 13.85\text{ N}
\end{aligned}$$

Watch this Explanation

MEDIA

Click image to the left for more content.

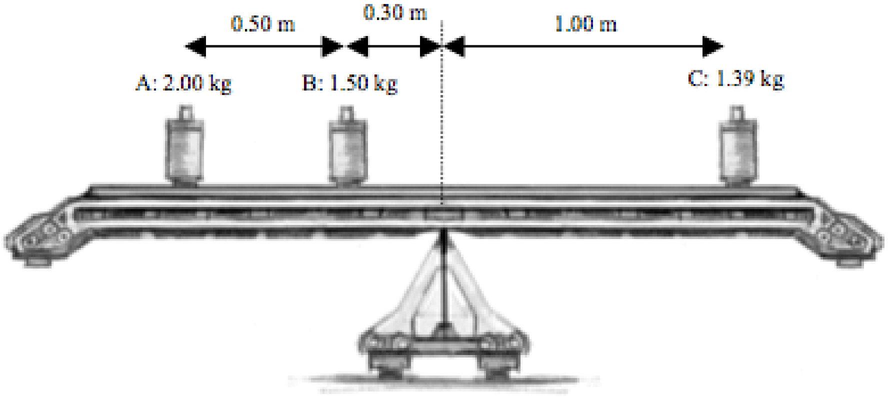
Simulation



Torque (PhETSimulation)

Time for Practice

1. Consider hitting someone with a Wiffle ball bat. Will it hurt them more if you grab the end or the middle of the bat when you swing it? Explain your thinking, but do so using the vocabulary of *moment of inertia* (treat the bat as a rod) and *torque* (in this case, torques caused by the contact forces the other person#8217;s head and the bat are exerting on each other).
2. A wooden plank is balanced on a pivot, as shown below. Weights are placed at various places on the plank.



Consider the torque on the plank caused by weight *A*.

a. What force, precisely, is responsible for this torque?

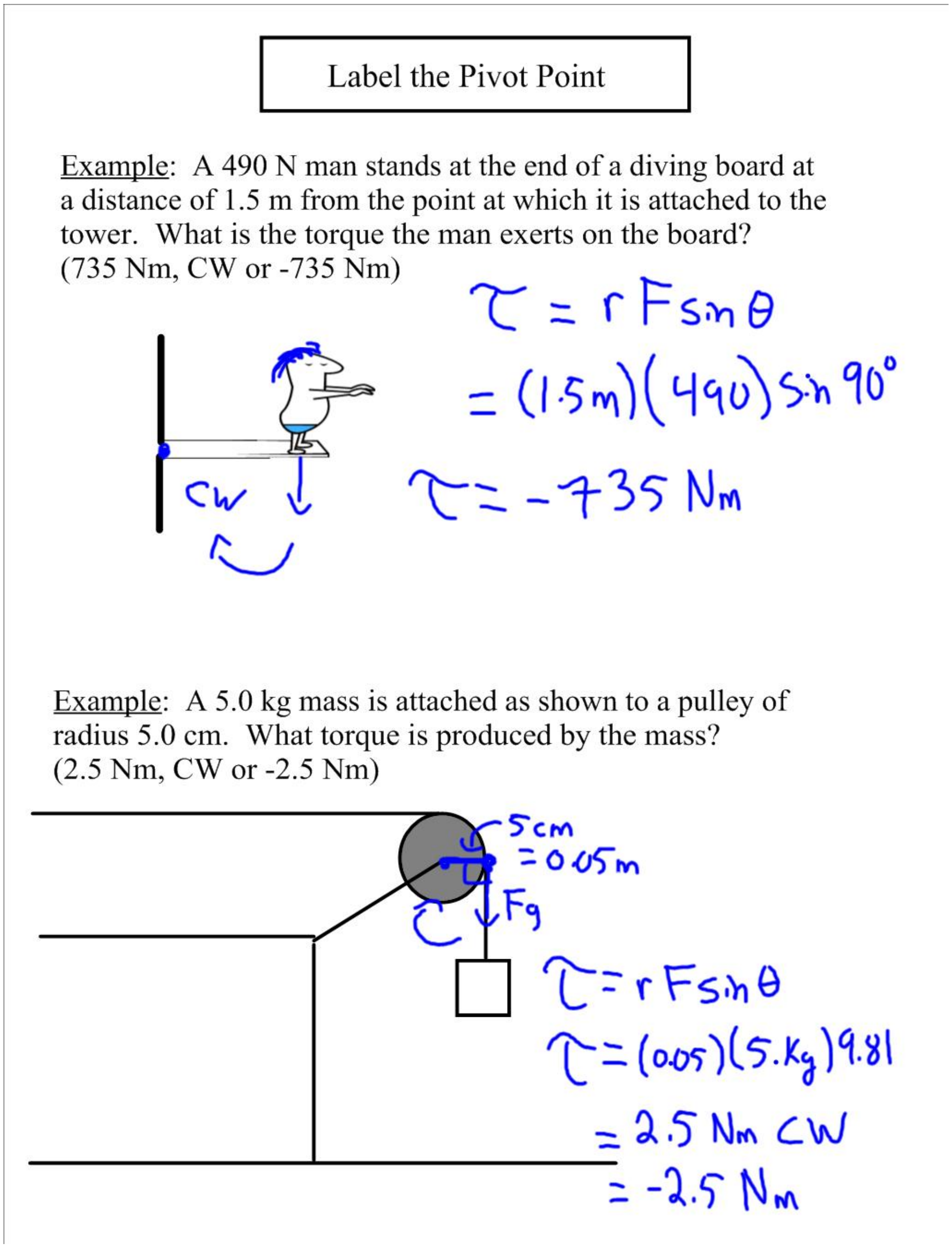
  b. What is the magnitude (value) of this force, in Newtons?
  c. What is the moment arm of the torque produced by weight A?
  d. What is the magnitude of this torque, in $N \cdot m$?
  e. Repeat parts (a #8211; d) for weights B and C.
  f. Calculate the net torque. Is the plank balanced? Explain.

3. There is a uniform rod of mass 2.0 kg of length 2.0 m. It has a mass of 2.6 kg at one end. It is attached to the ceiling .40 m from the end with the mass. The string comes in at a 53degree angle to the rod.
   a. Calculate the total torque on the rod.
   b. Determine its direction of rotation.
   c. Explain, but don#8217;t calculate, what happens to the angular acceleration as it rotates toward a vertical position.

Answers

1. At End. Discuss in class. Moment of inertia at the end $1/3$ ML^2 at the center $1/12$ ML^2 and torque, $\tau = I\alpha$ change the in the same way
2. a. weight b. 19.6 N c. plank#8217;s length (0.8m) left of the pivot d. 15.7 N m, e. Ba. weight, Bb. 14.7 N, Bc. plank#8217;s length (0.3m) left of the pivot, Bd. 4.4 N m, Ca. weight, Cb. 13.6 N, Cc. plank#8217;s length (1.00 m) right of the pivot, Cd. 13.6 N m, f) 6.5 N m CC, g) no, net torque doesn#8217;t equal zero
3. CCW

7.7 Torque Examples



Label the Pivot Point

Example: A 490 N man stands at the end of a diving board at a distance of 1.5 m from the point at which it is attached to the tower. What is the torque the man exerts on the board? (735 Nm, CW or -735 Nm)

$$\tau = rF\sin\theta$$

$$= (1.5m)(490)\sin 90^\circ$$

$$\tau = -735\ Nm$$

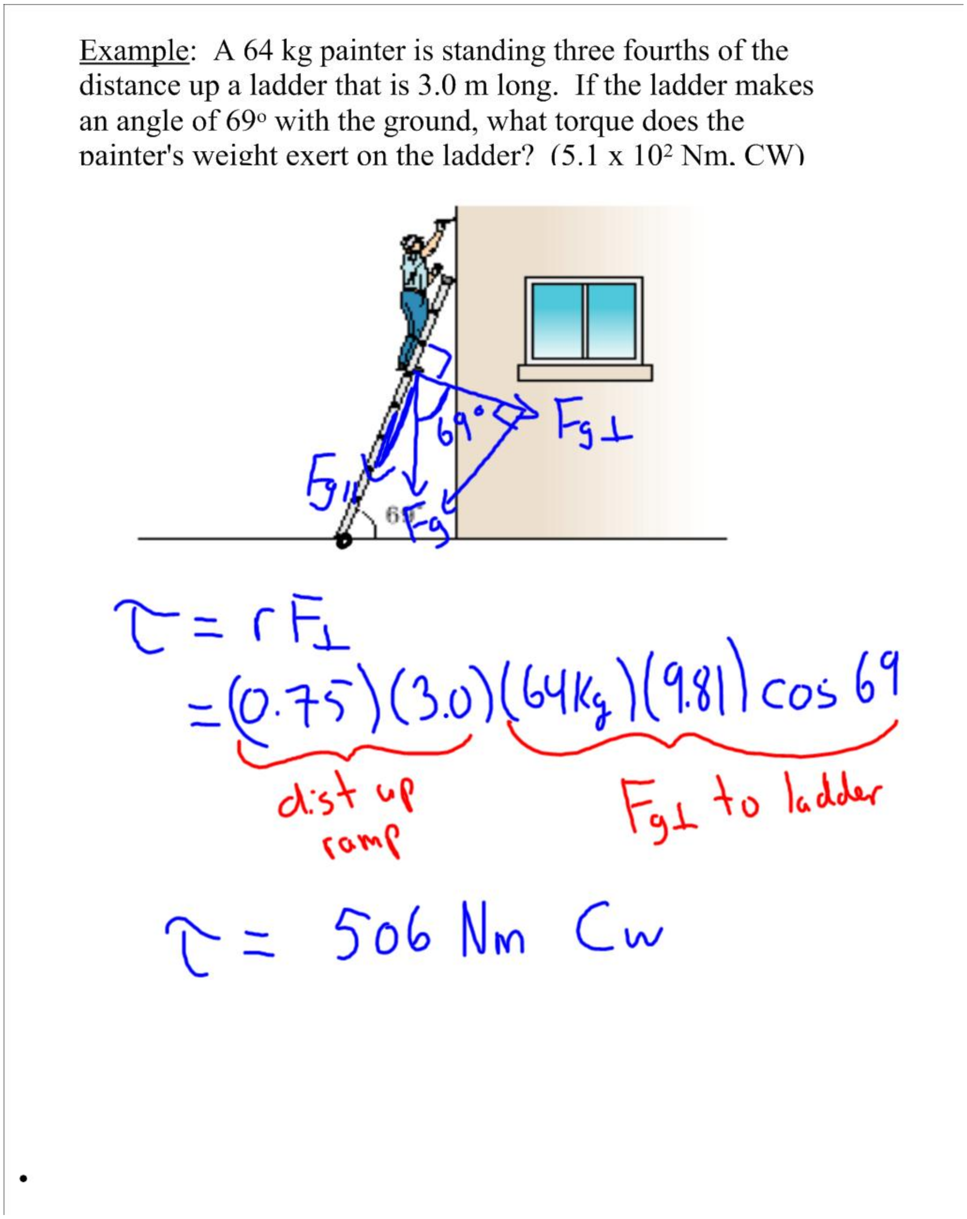
Example: A 5.0 kg mass is attached as shown to a pulley of radius 5.0 cm. What torque is produced by the mass? (2.5 Nm, CW or -2.5 Nm)

$$\tau = rF\sin\theta$$

$$\tau = (0.05)(5.kg)9.81$$

$$= 2.5\ Nm\ CW$$

$$= -2.5\ Nm$$



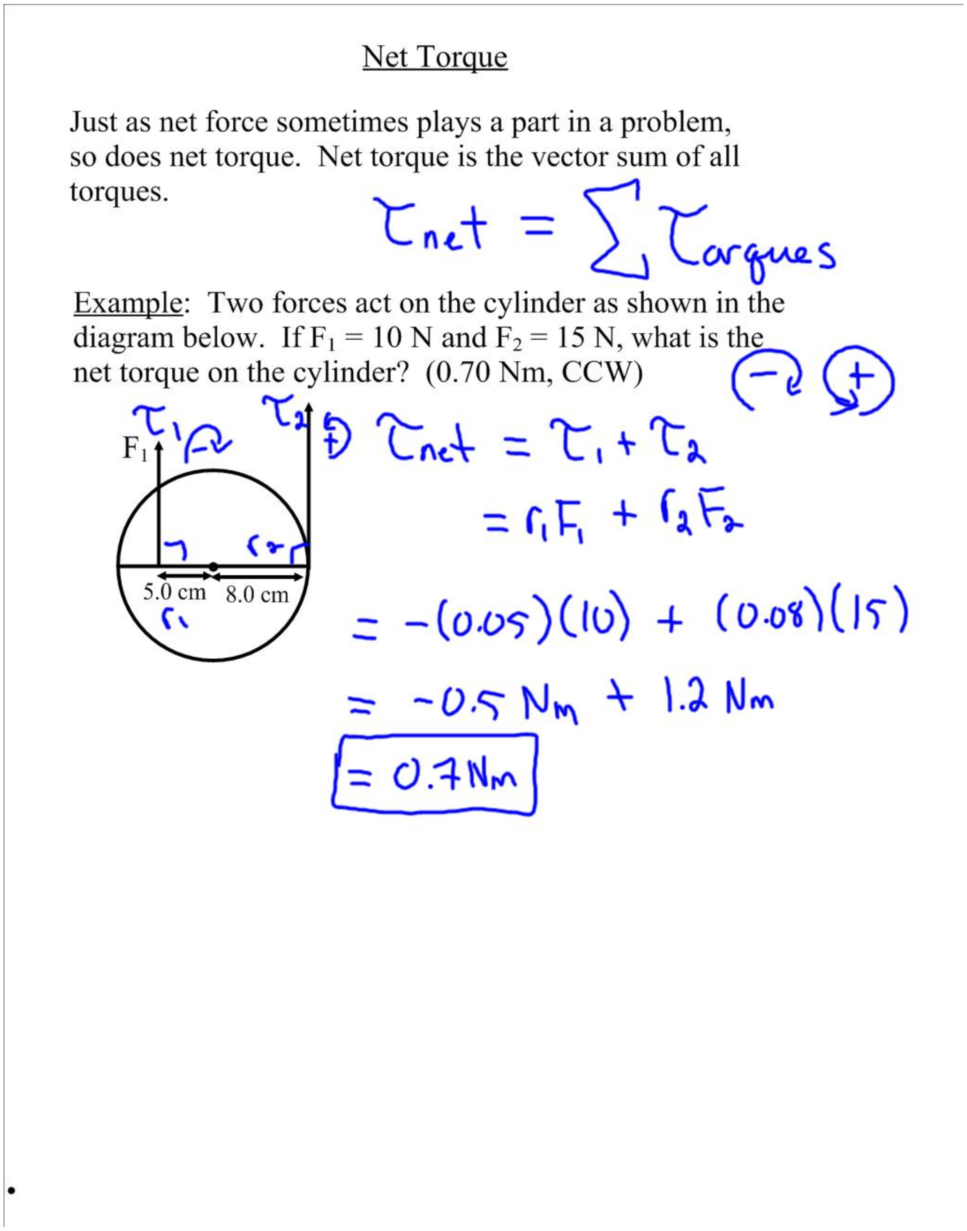
Example: A 64 kg painter is standing three fourths of the distance up a ladder that is 3.0 m long. If the ladder makes an angle of 69° with the ground, what torque does the painter's weight exert on the ladder? (5.1×10^2 Nm, CW)

$$\tau = r F_{\perp}$$

$$= \underbrace{(0.75)(3.0)}_{\text{dist up ramp}}\underbrace{(64\text{kg})(9.81)\cos 69}_{F_{g\perp}\text{ to ladder}}$$

$$\tau = 506 \text{ Nm Cw}$$

-



Net Torque

Just as net force sometimes plays a part in a problem, so does net torque. Net torque is the vector sum of all torques.

$$\tau_{net} = \sum \tau_{orques}$$

Example: Two forces act on the cylinder as shown in the diagram below. If $F_1 = 10$ N and $F_2 = 15$ N, what is the net torque on the cylinder? (0.70 Nm, CCW)

$(-\circlearrowright) \quad (+\circlearrowleft)$

$$(+)\ \tau_{net} = \tau_1 + \tau_2$$

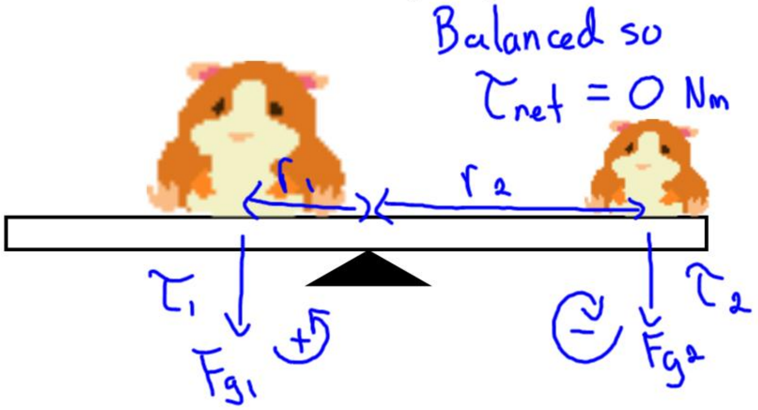
$$= r_1F_1 + r_2F_2$$

$$= -(0.05)(10) + (0.08)(15)$$

$$= -0.5\ Nm + 1.2\ Nm$$

$$\boxed{= 0.7 Nm}$$

Example: A massless board serves as a seesaw for two giant hamsters as shown below. One hamster has a mass of 30 kg and sits 2.5 m from the pivot point. At what distance from the pivot point must a 25 kg hamster place himself to balance the seesaw? (3.0 m)



Balanced so $\tau_{net} = 0$ Nm

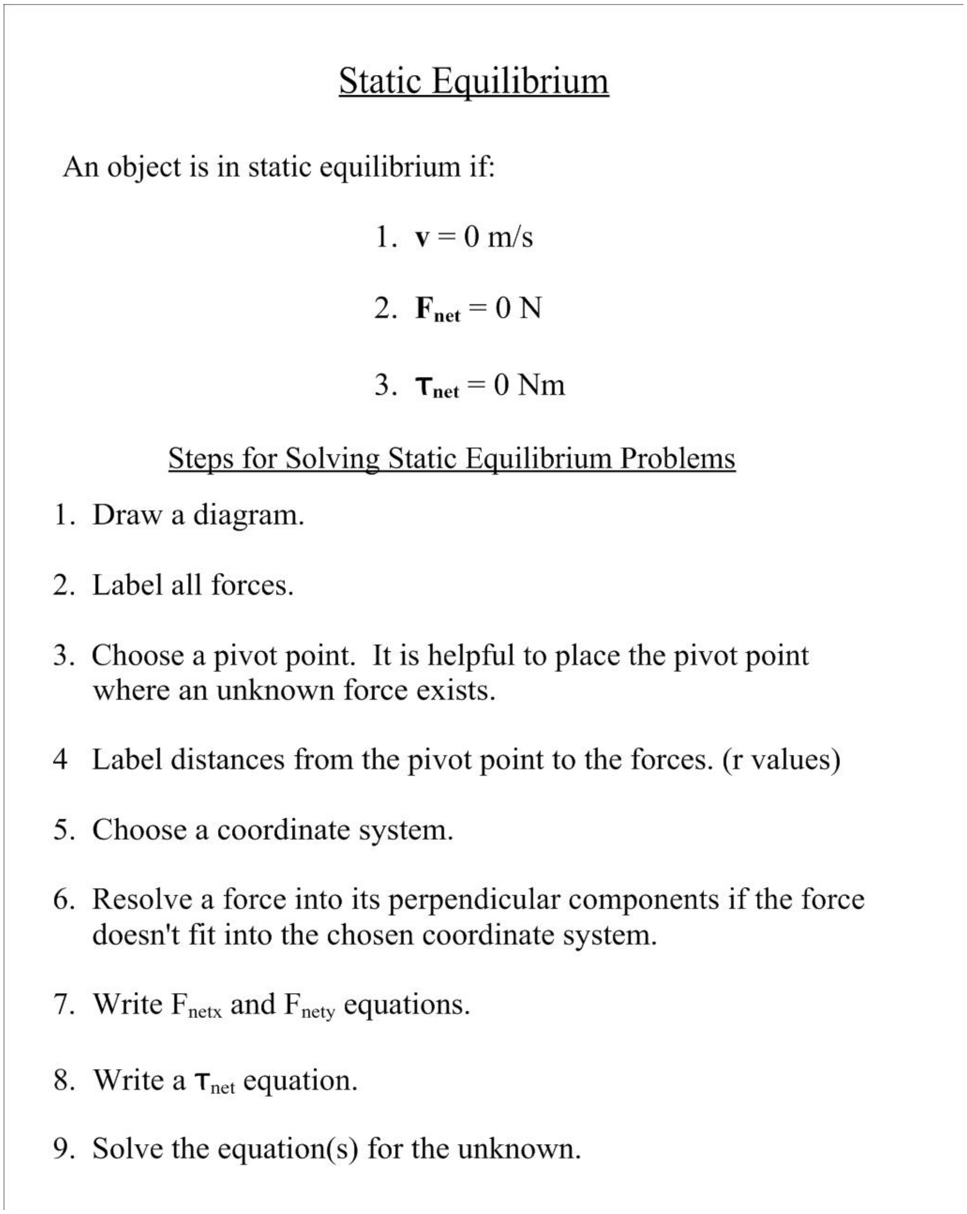
$$\tau_{net} = \tau_1 + \tau_2$$

$$0 = (2.5)(30\,kg)(9.81) - r_2(25)(9.81)$$

$$0 = 735.75 - 245\,r_2$$

$$\frac{-735.75}{-245.25} = r_2$$

$$\boxed{3.0\,m = r_2}$$



Static Equilibrium

An object is in static equilibrium if:

1. $\mathbf{v} = 0$ m/s
2. $\mathbf{F}_{\mathbf{net}} = 0$ N
3. $\boldsymbol{\tau}_{\mathbf{net}} = 0$ Nm

Steps for Solving Static Equilibrium Problems

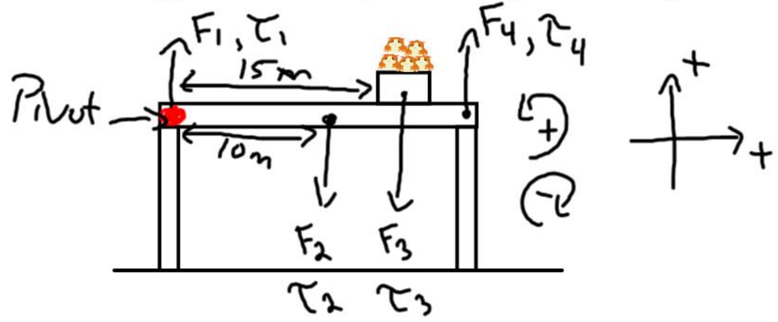
1. Draw a diagram.
2. Label all forces.
3. Choose a pivot point. It is helpful to place the pivot point where an unknown force exists.

4 Label distances from the pivot point to the forces. (r values)

5. Choose a coordinate system.
6. Resolve a force into its perpendicular components if the force doesn't fit into the chosen coordinate system.
7. Write F_{netx} and F_{nety} equations.
8. Write a τ_{net} equation.
9. Solve the equation(s) for the unknown.

* If a solid object has mass, treat the object as if all its mass were concentrated at a point - the center of mass.

Example: A uniform 1500 kg beam, 20.0 m long, supports a 15000 kg box of hamsters 5.0 m from the right support column. Calculate the magnitude of the forces on the beam exerted by each of the vertical support columns. (1.2 x 10^5 N, 4.2 x 10^4 N)



$$F_{net_y} = F_1 + F_2 + F_3 + F_4$$

$$0 = F_1 - (1500)(9.81) - (15000)(9.81) + F_4$$

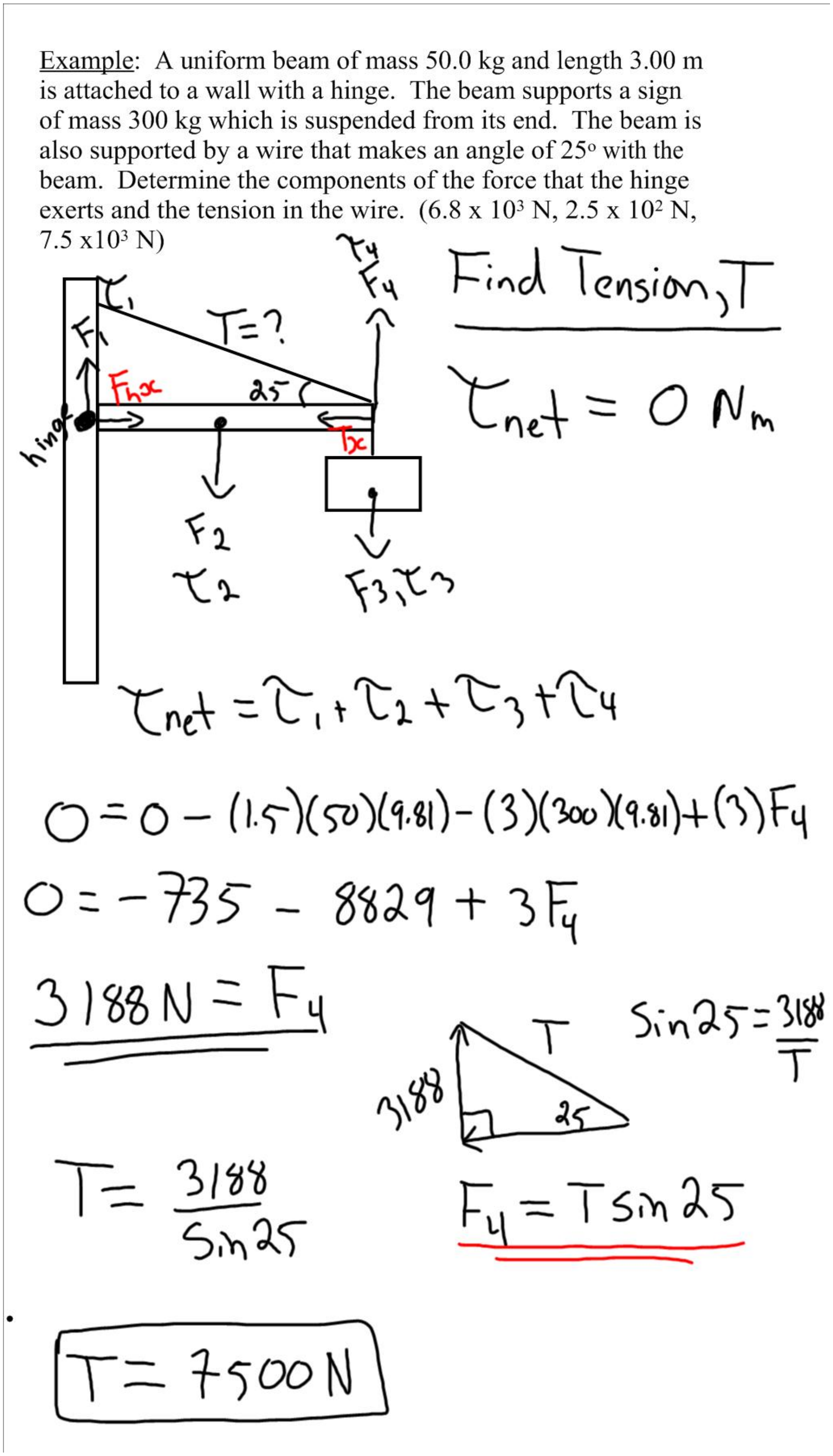
$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = r_1F_1 + r_2F_2 + r_3F_3 + r_4F_4$$

$$0 = 0 - (10)F_2 - (15)F_3 + 20F_4$$

$$0 = -147000 - 2{,}207{,}000 + 20F_4$$

$$F_4 = \underline{120000\ \text{N}}$$



Example: A uniform beam of mass 50.0 kg and length 3.00 m is attached to a wall with a hinge. The beam supports a sign of mass 300 kg which is suspended from its end. The beam is also supported by a wire that makes an angle of 25° with the beam. Determine the components of the force that the hinge exerts and the tension in the wire. (6.8×10^3 N, 2.5×10^2 N, 7.5×10^3 N)

Find Tension, T

$\tau_{net} = 0$ Nm

$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$

$0 = 0 - (1.5)(50)(9.81) - (3)(300)(9.81) + (3)F_4$

$0 = -735 - 8829 + 3F_4$

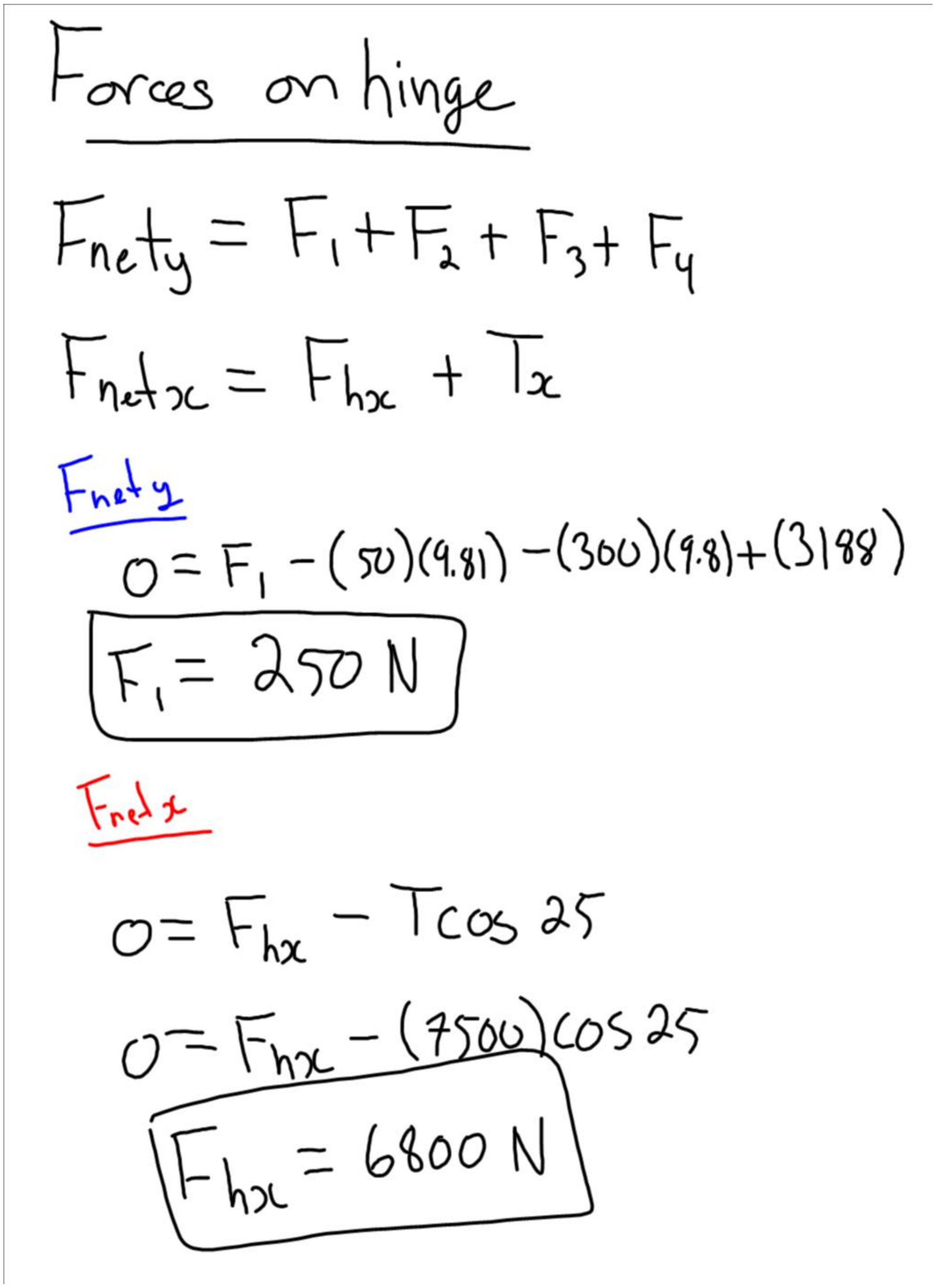
$3188\text{ N} = F_4$

$\sin 25 = \frac{3188}{T}$

$T = \frac{3188}{\sin 25}$

$F_4 = T \sin 25$

$\boxed{T = 7500\text{ N}}$



Forces on hinge

$$F_{net_y} = F_1 + F_2 + F_3 + F_4$$

$$F_{net_x} = F_{hx} + T_x$$

$F_{net\,y}$

$$0 = F_1 - (50)(9.81) - (300)(9.8) + (3188)$$

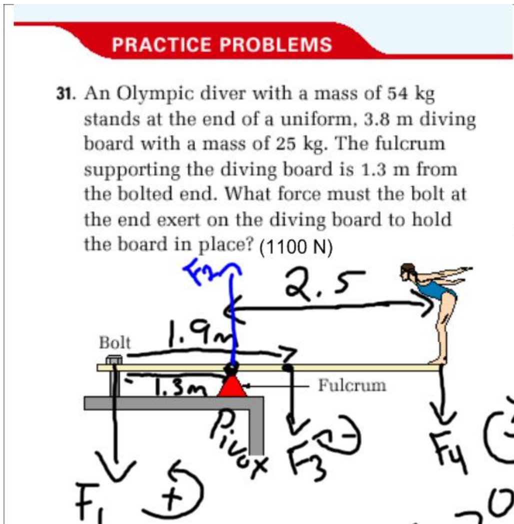
$$\boxed{F_1 = 250\text{ N}}$$

$F_{net\,x}$

$$0 = F_{hx} - T\cos 25$$

$$0 = F_{hx} - (7500)\cos 25$$

$$\boxed{F_{hx} = 6800\text{ N}}$$



PRACTICE PROBLEMS

31. An Olympic diver with a mass of 54 kg stands at the end of a uniform, 3.8 m diving board with a mass of 25 kg. The fulcrum supporting the diving board is 1.3 m from the bolted end. What force must the bolt at the end exert on the diving board to hold the board in place? (1100 N)

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = r_1F_1 + \overset{\to 0}{\cancel{r_2F_2}} + r_3F_3 + r_4F_4$$

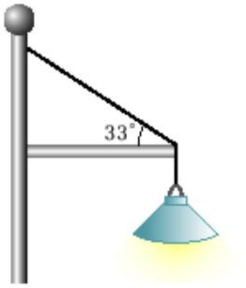
$$0 = +(1.3)F_1 - (0.6)(25)(9.81) - 2.5(54)(9.81)$$

$$0 = 1.3F_1 - 147\ Nm - 1324\ Nm$$

$$F_1 = 1132\ Nm$$

-

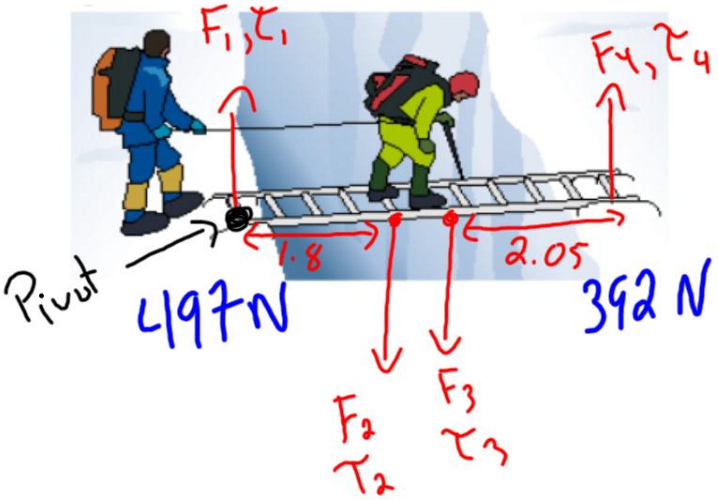
33. A 24 kg light fixture is hanging from a uniform, 3.5 kg horizontal beam that is 1.6 m long. A supporting cable makes an angle of 33° with the beam.



(a) Find the tension in the cable.

-

27. Mountain climbers have placed a 3.6 kg uniform ladder across an icy crevasse. The edges of the crevasse are 4.1 m apart. The first climber starts to cross the crevasse on the ladder and reaches a point 1.8 m from the edge. The mass of the climber and her gear is 87 kg. With what force is the ice on each side of the crevasse pushing up on the ladder?



$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = 0 - (1.8)(87)(9.81) - (2.05)(3.6)(9.8) + (4.1)F_4$$

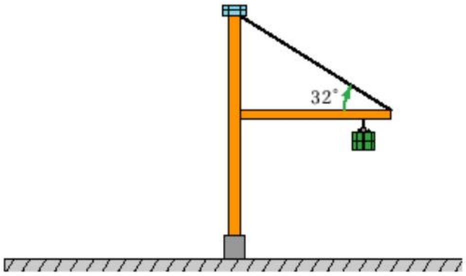
$$\boxed{F_4 = 392\text{ N}}$$

$$F_{net} = F_1 + F_2 + F_3 + F_4$$

$$0 = F_1 - (87)(9.81) - (3.6)(9.81) + 392$$

$$\boxed{F = 497\text{N}}$$

28. A crane with a movable pulley system on a horizontal arm is moving a large container. The 355 kg container is hanging from a cable that is 6.15 m out on the 7.50 m arm. The arm has a mass of 345 kg. A cable that is attached to its end makes an angle 32.0° with the horizontal arm.



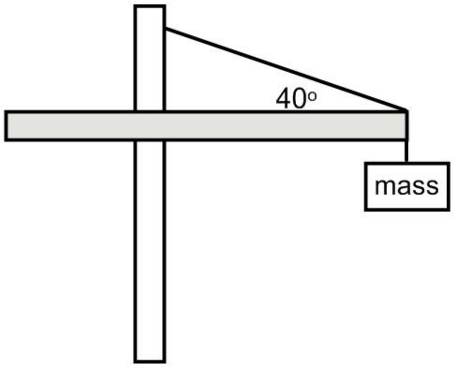
(a) What is the tension in the cable supporting the arm? (8582 N)

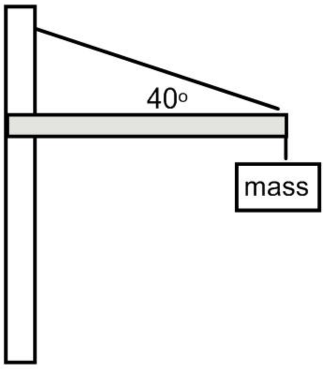
A construction crane is designed such that part of the boom acts as a counterweight. The boom is constructed of uniform material with a linear density of 25 kg/m. The left side of the crane is 10 m long and the right side is 15 m.

a) If the mass at the right end is 300 kg what is the tension in the cable? (T = 6200 N)

b) What is the tension in the cable if there was no left side of the boom? (T = 7400 N)

c) Suppose each cable can support a tension of 12000 N. What is the maximum mass that each crane can support? (Left: 680 kg; Right: 600 kg)





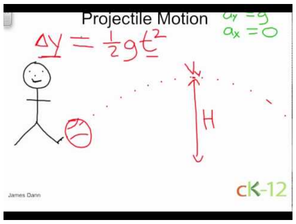
7.8 Projectile Motion

The aim here is to understand and explain the parabolic motion of a thrown object, known as projectile motion. Motion in one direction is unrelated to motion in other perpendicular directions. Once the object has been thrown, the only acceleration is in the y (up/down) direction due to gravity. The x (right/left) direction velocity remains unchanged.

Guidance

- In projectile motion, the horizontal displacement of an object from its starting point is called its *range.*
- Vertical (y) speed is zero only at the highest point of a thrown object's flight.
- Since in the absence of air resistance there is no acceleration in the horizontal direction, this component of velocity does not change over time. This is a counter-intuitive notion for many. (Air resistance will cause velocity to decrease slightly or significantly depending on the object. But this factor is ignored for the time being.)
- Motion in the vertical direction must include the acceleration due to gravity, and therefore the velocity in the vertical direction changes over time.
- The shape of the path of an object undergoing projectile motion in two dimensions is a parabola.

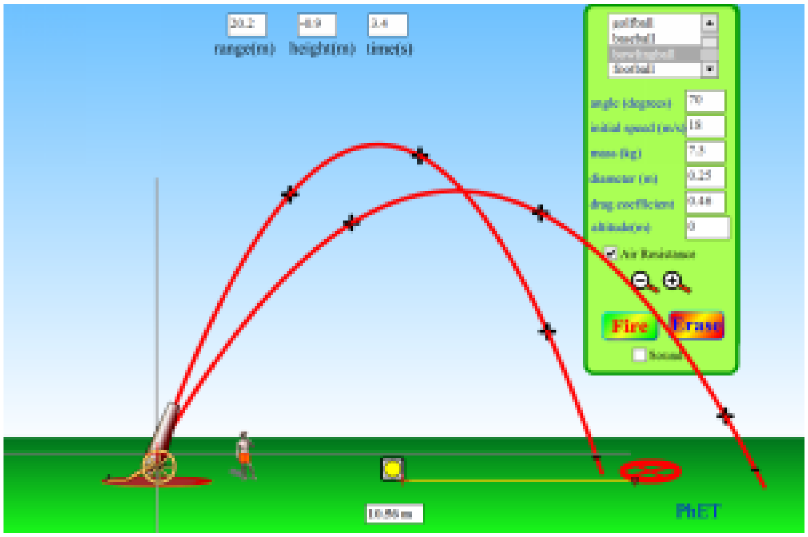
Watch this Explanation



MEDIA

Click image to the left for more content.

Simulation



Projectile Motion (PhET Simulation)

Examples

1. A projectile is fired horizontally from a height of 44.1 m at a speed of 50.0 m/s.

(a) How long after it was fired did the projectile hit the ground?

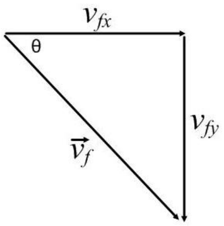
$\theta = 0^\circ$
$\Delta y = -44.1\ m \quad$ (down so it is negative)
$v_{ox} = 50\ m/s$
$v_{oy} = 0.0\ m/s$
$t = ?$

$\Delta y = v_{oy}t + \frac{1}{2}at^2$
$-44.1 = (0)t + \frac{1}{2}(-9.81)t^2$
$-44.1 = -4.9t^2$
$9.0 = t^2$
$\pm\sqrt{9.0} = t$
$\pm 3.0 = t \leftarrow$ Only solutions that are physically possible are valid so we reject the -3 answer.
$+3.0 = t$

(b) How far forward did the projectile travel?

$x = ?$
$v_{ox} = \frac{\Delta x}{\Delta t}$
$50 = \frac{\Delta x}{3.0}$
$150\ m = \Delta x$

2. A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff that is 6.4 m high. Calculatte the velocity of the stone as it hits the ground.



$v_{fx} = 5.0\ m/s$
$\Delta y = -6.4\ m$
$v_{yo} = 0.0\ m/s$
$v_{yf} = ?$
$\vec{v}_f = ?$

$v_{yf}^2 = v_{yo}^2 + 2a\Delta y$
$v_{yf}^2 = 0.0 + 2(-9.81)(-6.4)$
$v_{yf}^2 = 125.6\ m^2$
$v_{yf} = \pm\sqrt{125.6\ m^2}$
$v_{yf} = -11\ m/s$ (downward so negative)

$|v_f| = \sqrt{v_{xf}^2 + v_{yf}^2}$
$|v_f| = \sqrt{(5.0)^2 + (11)^2}$
$|v_f| = 12.3\ m/s$

$\theta = \tan^{-1}|\frac{v_{yf}}{v_{xf}}| = \tan^{-1}|\frac{11}{5}|$
$\theta = 66^\circ$

$\vec{v}_f = 12.3\ m/s\ 66^\circ$ down from the horizontal

3. A ball is released at an angle with a speed of 12 m/s and stays in the air for 2.0 seconds.

(a) Calculate the angle the ball made with the horizontal upon release.

$\Delta y = 0.0\ m$
$|\vec{v}| = 12\ m/sa = -9.81\ m/s^2$
$t = 2.0\ s$

$\Delta y = v_{yo}t + \frac{1}{2}at^2$
$0.0 = v_{yo}(2) + \frac{1}{2}(-9.81)(2.0)^2$
$0.0 = 2v_{yo} - 19.6$
$9.81\ m/s = v_{yo}$

$v_{yo} = v\sin\theta$
$9.81 = 12\sin\theta$
$\frac{9.81}{12} = \sin\theta$
$\sin^{-1}(\frac{9.81}{12}) = \theta$
$55^\circ = \theta$

(b) Calculate the horizontal distance travelled.

$x = v_x t$
$v_x = v\cos\theta$
$v_x = 12\cos 55^\circ = 6.88\ m/s$

$x = (6.88)(2) = 14\ m$

3. A golfer standing on a fairway hits a golf ball to a green that is elevated 5.50 m above the point where she is standing. If the ball leaves the club with a velocity of 46.0 m/s, 35^o to the ground, calculate the time the ball spends in the air.

$\Delta y = 5.50\ m$
$\vec{v} = 46\ m/s\ 35^\circ$ to the ground
$v_x = 46\cos 35^\circ = 37.7\ m/s$
$v_{yo} = 46\sin 35^\circ = 26.4\ m/s$
$t = ?$

$\Delta y = v_{yo}t + \frac{1}{2}at^2$
$5.50 = 26.4t - 4.9t^2 \leftarrow$ quadratic formula required
$4.9t^2 - 26.4t + 5.50 = 0 \leftarrow$ Rearrange to equal zero and ensure quadratic term is positive

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$t = \frac{-(-26.4) \pm \sqrt{(-26.4)^2 - 4(4.9)(5.5)}}{(2)(4.9)}$$

$$t = \frac{26.4 \pm \sqrt{589.16}}{9.8}$$

$$t = \frac{26.4 \pm 24.3}{9.8}$$

$t = 0.22\ s$ and $5.17\ s$
$t = 5.17\ s \leftarrow$ Ball is coming down

Time for Practice

1. Determine which of the following is in projectile motion. Remember that #8220;projectile motion#8221; means that gravity is the only means of acceleration for the object.
   a. A jet airplane during takeoff.
   b. A baseball during a Barry Bonds home run.
   c. A spacecraft just after all the rockets turn off in Earth orbit.
   d. A basketball thrown towards a basket.
   e. A bullet shot out of a gun.
   f. An inter-continental ballistic missile.
   g. A package dropped out of an airplane as it ascends upward with constant speed.
2. Decide if each of the statements below is True or False. Then, explain your reasoning.
   a. At a projectile#8217;s highest point, its velocity is zero.
   b. At a projectile#8217;s highest point, its acceleration is zero.
   c. The rate of change of the x position is changing with time along the projectile path.
   d. The rate of change of the y position is changing with time along the projectile path.
   e. Suppose that after 2 s, an object has traveled 2 m in the horizontal direction. If the object is in projectile motion, it must travel 2 m in the vertical direction as well.

   f. Suppose a hunter fires his gun. Suppose as well that as the bullet flies out horizontally and undergoes projectile motion, the shell for the bullet falls directly downward. Then, the shell hits the ground before the bullet.

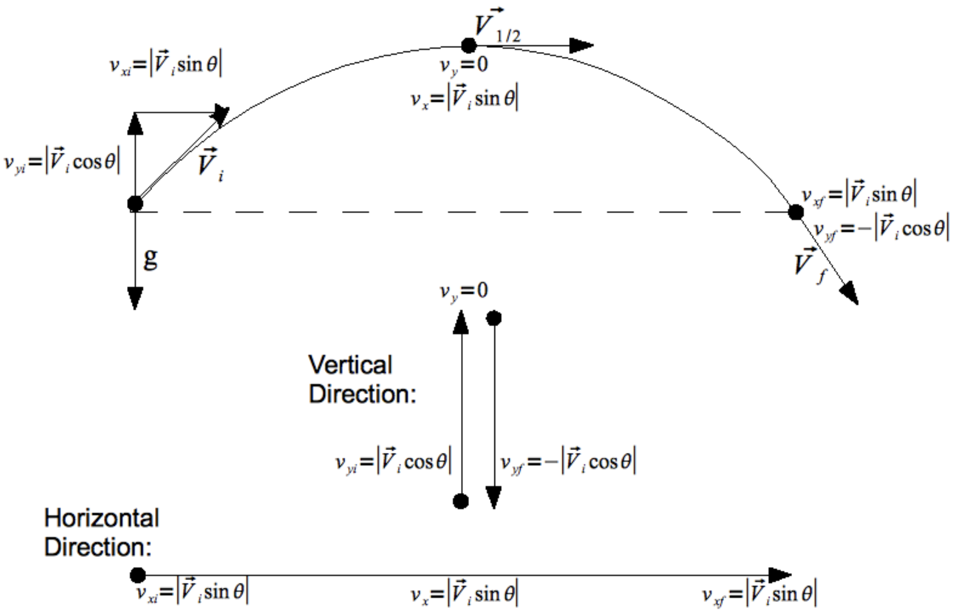
3. Imagine the path of a soccer ball in projectile motion. Which of the following is true at the highest point in its flight?
   a. $v_x = 0, v_y = 0, a_x = 0, a_y = 0.$
   b. $v_x > 0, v_y = 0, a_x = 0, a_y = 0.$
   c. $v_x = 0, v_y = 0, a_x = 0, a_y = -9.8\text{ m/s}^2.$
   d. $v_x > 0, v_y = 0, a_x = 0, a_y = -9.8\text{ m/s}^2.$
4. A hunter with an air blaster gun is preparing to shoot at a monkey hanging from a tree. He is pointing his gun directly at the monkey. The monkey#8217;s got to think quickly! What is the monkey#8217;s best chance to avoid being smacked by the rubber ball?
   a. The monkey should stay right where he is: the bullet will pass beneath him due to gravity.
   b. The monkey should let go when the hunter fires. Since the gun is pointing right at him, he can avoid getting hit by falling to the ground.
   c. The monkey should stay right where he is: the bullet will sail above him since its vertical velocity increases by 9.8 m/s every second of flight.
   d. The monkey should let go when the hunter fires. He will fall faster than the bullet due to his greater mass, and it will fly over his head.
5. You are riding your bike in a straight line with a speed of 10 m/s. You accidentally drop your calculator out of your backpack from a height of 2.0 mabove the ground. When it hits the ground, where is the calculator in relation to the position of your backpack? (Neglect air resistance.)
   a. You and your backpack are 6.3 m ahead of the calculator.
   b. You and your backpack are directly above the calculator.
   c. You and your backpack are 6.3 m behind the calculator.
   d. None of the above.

7.9 Projectile Motion Problem Solving

Students will learn how to use the equations of motion in two dimensions in order to solve problems for projectiles. It is necessary to understand how to break a vector into its x and y components.

Key Equations

Break the Initial Velocity Vector into its Components



Apply the Kinematics Equations

^2 = v_0y^2 - 2g (Δy)
$a_y = -g = -9.8\text{m/s}^2 \approx -10\text{m/s}^2 a_x = 0$

Guidance

- To work these problems, separate the #8220;Big Three#8221; equations into two sets: one for the vertical direction, and one for the horizontal. Keep them separate.
- The only variable that can go into both sets of equations is time; use time to communicate between the x and y components of the object’s motion.

Example 1

CSI discovers a car at the bottom of a 72 m cliff. How fast was the car going if it landed 22m horizontally from the cliff#8217;s edge? (Note that the cliff is flat, i.e. the car came off the cliff horizontally).

Question: $v = ?\ [m/s]$

Given: $h = \Delta y = 72\ m$

$d = \Delta x = 22\ m$

$g = 10.0\ m/s^2$

Equation: $h = v_{iy}t + \frac{1}{2}gt^2$ and $d = v_{ix}t$

Plug n#8217; Chug: Step 1: Calculate the time required for the car to freefall from a height of 72 m.

$h = v_{iy}t + \frac{1}{2}gt^2$ but since $v_{iy} = 0$, the equation simplifies to $h = \frac{1}{2}gt^2$ rearranging for the unknown variable, t, yields

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(72\ m)}{10.0\ m/s^2}} = 3.79\ s$

Step 2: Solve for initial velocity:

$v_{ix} = \frac{d}{t} = \frac{22\ m}{3.79\ s} = 5.80\ m/s$

Answer:

$$\boxed{\mathbf{5.80\ m/s}}$$

Example 2

Question: A ball of mass m is moving horizontally with a speed of v_i off a cliff of height h. How much time does it take the ball to travel from the edge of the cliff to the ground? Express your answer in terms of g (acceleration due to gravity) and h (height of the cliff).

Solution: Since we are solving or how long it takes for the ball to reach ground, any motion in the x direction is not pertinent. To make this problem a little simpler, we will define down as the positive direction and the top of the cliff to be

$$y = 0$$

. In this solution we will use the equation

$$y(t) = y_o + v_{oy}t + \frac{1}{2}gt^2$$

.

$y(t) = y_o + v_{oy}t + \frac{1}{2}gt^2$ start with the equation

$h = y_o + v_{oy}t + \frac{1}{2}gt^2$ substitute h for $y(t)$ because that's the position of the ball when it hits the ground after time t

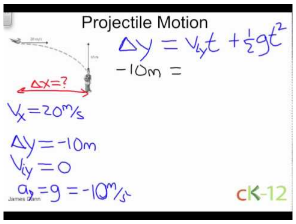
$h = 0 + v_{oy} + \frac{1}{2}gt^2$ substitute 0 for $y_o$because the ball starts at the top of the cliff

$h = 0 + 0 + \frac{1}{2}gt^2$ substitute 0 for v_{oy} becauese the ball starts with no vertical component to it's velocity

$h = \frac{1}{2}gt^2$ simplify the equation

$t = \sqrt{\frac{2h}{g}}$ solve for t

Watch this Explanation

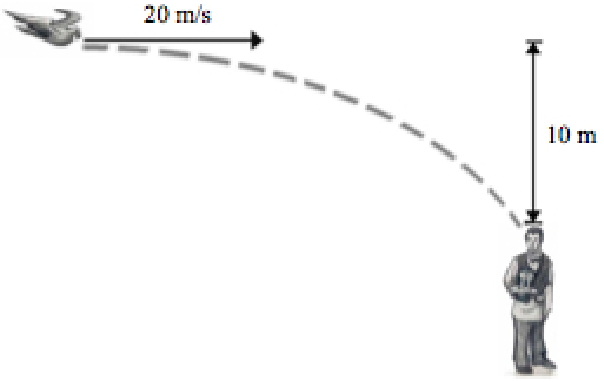


MEDIA

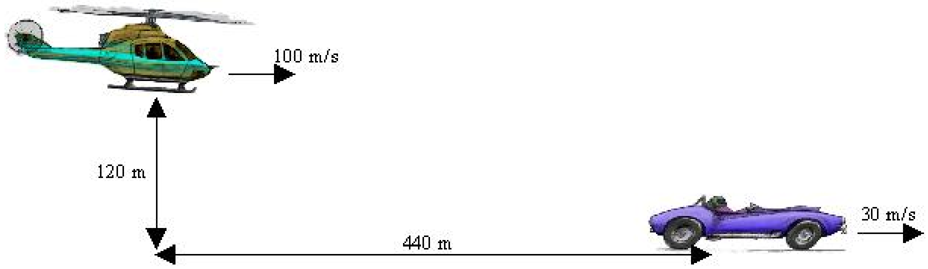
Click image to the left for more content.

Time for Practice

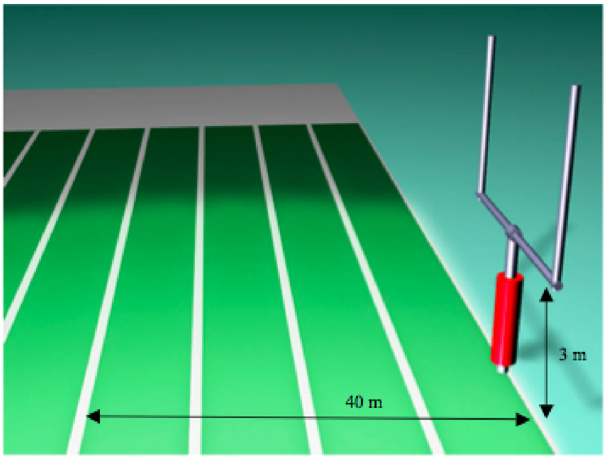
1. A stone is thrown horizontally at a speed of 8.0 m/s from the edge of a cliff 80 m in height. How far from the base of the cliff will the stone strike the ground?
2. A toy truck moves off the edge of a table that is 1.25 m high and lands 0.40 mfrom the base of the table.
    a. How much time passed between the moment the car left the table and the moment it hit the floor?
    b. What was the horizontal velocity of the car when it hit the ground?
3. A hawk in level flight 135 m above the ground drops the fish it caught. If the hawk#8217;s horizontal speed is 20.0 m/s, how far ahead of the drop point will the fish land?
4. A pistol is fired horizontally toward a target 120 m away, but at the same height. The bullet#8217;s velocity is 200 m/s. How long does it take the bullet to get to the target? How far below the target does the bullet hit?
5. A bird, traveling at 20 m/s, wants to hit a waiter 10 m below with his dropping (see image). In order to hit the waiter, the bird must release his dropping some distance before he is directly overhead. What is this distance?



6. Joe Nedney of the *San Francisco 49ers* kicked a field goal with an initial velocity of 20 m/s at an angle of 60°.
    a. How long is the ball in the air? *Hint:* you may assume that the ball lands at same height as it starts at.
    b. What are the range and maximum height of the ball?
7. A racquetball thrown from the ground at an angle of 45° and with a speed of 22.5 m/s lands exactly 2.5 s later on the top of a nearby building. Calculate the horizontal distance it traveled and the height of the building.
8. Donovan McNabb throws a football. He throws it with an initial velocity of 30 m/s at an angle of 25°. How much time passes until the ball travels 35 m horizontally? What is the height of the ball after 0.5 seconds? (Assume that, when thrown, the ball is 2 m above the ground.)
9. Pablo Sandoval throws a baseball with a horizontal component of velocity of 25 m/s. After 2 seconds, the ball is 40 m above the release point. Calculate the horizontal distance it has traveled by this time, its initial vertical component of velocity, and its initial angle of projection. Also, is the ball on the way up or the way down at this moment in time?
10. Barry Bonds hits a 125 m(450′) home run that lands in the stands at an altitude 30 m above its starting altitude. Assuming that the ball left the bat at an angle of 45° from the horizontal, calculate how long the ball was in the air.
11. A golfer can drive a ball with an initial speed of 40.0 m/s. If the tee and the green are separated by 100 m, but are on the same level, at what angle should the ball be driven? (*Hint:* you should use $2\cos(x)\sin(x) = \sin(2x)$ at some point.)
12. How long will it take a bullet fired from a cliff at an initial velocity of 700 m/s, at an angle 30° below the horizontal, to reach the ground 200 m below?
13. A diver in Hawaii is jumping off a cliff 45 m high, but she notices that there is an outcropping of rocks 7 m out at the base. So, she must clear a horizontal distance of 7 m during the dive in order to survive. Assuming the diver jumps horizontally, what is his/her minimum push-off speed?
14. If Monte Ellis can jump 1.0 m high on Earth, how high can he jump on the moon assuming same initial velocity that he had on Earth (where gravity is 1/6 that of Earth#8217;s gravity)?
15. James Bond is trying to jump from a helicopter into a speeding Corvette to capture the bad guy. The car is going 30.0 m/s and the helicopter is flying completely horizontally at 100 m/s. The helicopter is 120 m above the car and 440 m behind the car. How long must James Bond wait to jump in order to safely make it into the car?



16. A field goal kicker lines up to kick a 44 yard (40 m) field goal. He kicks it with an initial velocity of 22 m/s at an angle of 55°. The field goal posts are 3 meters high.



   a. Does he make the field goal?
   b. What is the ball#8217;s velocity and direction of motion just as it reaches the field goal post (*i.e.,* after it has traveled 40 m in the horizontal direction)?

17. In a football game a punter kicks the ball a horizontal distance of 43 yards (39 m). On TV, they track the hang time, which reads 3.9 seconds. From this information, calculate the angle and speed at which the ball was kicked. *(Note for non-football watchers: the projectile starts and lands at the same height. It goes* 43 *yards horizontally in a time of* 3.9 *seconds)*

Answers to Selected Problems

1. 32 m
2. a. 0.5 s b. 0.8 m/s
3. 104 m
4. $t = 0.60 \text{ s}, 1.8 \text{ m}$ below target
5. 28 m.
6. a. 3.5 s. b. $35 \text{ m}; 15 \text{ m}$
7. $40 \text{ m}; 8.5 \text{ m}$
8. 1.3 seconds, 7.1 meters
9. $50 \text{ m}; v_{0y} = 30 \text{ m/s}; 50^0$; on the way up
10. 4.4 s
11. 19°
12. 0.5 s
13. 2.3 m/s
14. 6 m
15. 1.4 seconds
16. a. yes b. 14 m/s @ 23 degrees from horizontal
17. 22 m/s @ 62 degrees

7.10 References

1. CK-12 Foundation. . CCSA
2. CK-12 Foundation. . CCSA
3. CK-12 Foundation. . CCSA
4. CK-12 Foundation. . CCSA

CHAPTER 8

Circular Motion and Universal Gravitation

Chapter Outline

8.1 Uniform Circular Motion

Students will learn that in circular motion there is always an acceleration (and hence a force) that points to the center of the circle defined by the objects motion. This force changes the direction of the velocity vector of the object but not the speed. Students will also learn how to calculate that speed using the period of motion and the distance of its path (circumference of the circle it traces out).

Key Equations

$v = \frac{2\pi r}{T}$

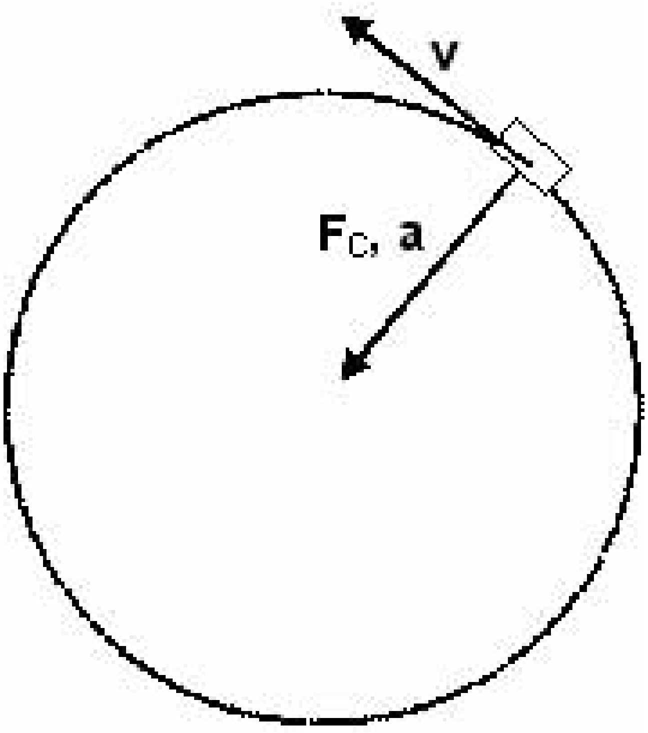
If a particle travels a distance $2\pi r$ in an amount of time T, then its speed is distance over time or $\frac{2\pi r}{T}$

The Earth-Sun distance is about $1.5 \times 10^{11}\ m$ The Earth-Moon distance is about $3.84 \times 10^{8}\ m$

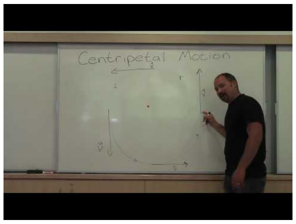
Guidance

- An orbital period, T, is the time it takes to make one complete rotation.
- If a particle travels a distance $2\pi r$ in an amount of time T, then its speed is distance over time or $2\pi r/T$.
- An object moving in a circle has an instantaneous velocity vector *tangential* to the circle of its path. The force and acceleration vectors point to the center of the circle.
- Net force and acceleration *always* have the same direction.
- Centripetal acceleration is just the acceleration provided by centripetal forces.

Example 1



Watch this Explanation

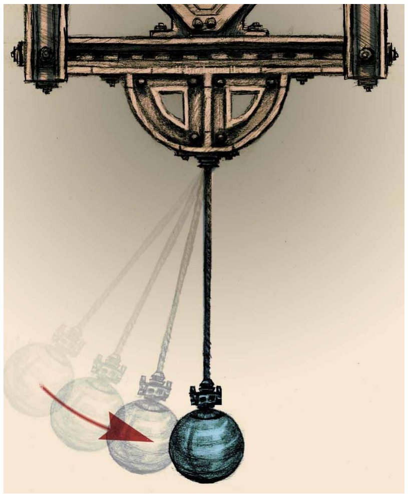


MEDIA

Click image to the left for more content.

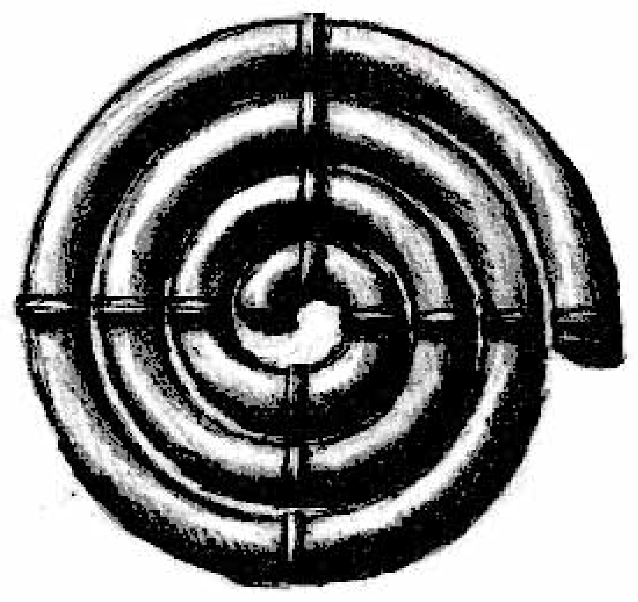
Time for Practice

1. When you make a right turn at constant speed in your car what is the force that causes *you* (not the car) to change the direction of *your*velocity? Choose the best possible answer.
   a. Friction between your butt and the seat
   b. Inertia
   c. Air resistance
   d. Tension
   e. All of the above
   f. None of the above
2. A pendulum consisting of a rope with a ball attached at the end is swinging back and forth. As it swings downward to the right the ball is released at its lowest point. Decide which way the ball attached at the end of the string will go at the moment it is released.



a. Straight upwards
b. Straight downwards
c. Directly right
d. Directly left
e. It will stop

3. A ball is spiraling outward in the tube shown to the right. Which way will the ball go after it leaves the tube?



a. Towards the top of the page
b. Towards the bottom of the page
c. Continue spiraling outward in the clockwise direction
d. Continue in a circle with the radius equal to that of the spiral as it leaves the tube
e. None of the above

4. Explain using Newton#8217;s Second Law why an object moving in a circle must experience a net force towards the center of the circle.
5. Using the known distance Earth is from the sun, calculate the speed that Earth is moving through space in relation to the sun.
6. Using the known distance the moon is from Eath, calculate the speed that the moon is moving through space in relation to Earth.

Answers

1. .
2. .
3. .
4. .
5. 30,000 m/s
6. 1,020 m/s

8.2 Angular Speed

Students will learn the difference between angular speed (ω) and tangential speed (v) and how to calculate both.

Key Equations

$\omega = 2\pi/T = 2\pi f$; Relationship between period and angular frequency.

$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{rt} = \frac{v}{r}$

$v = \omega r$

Guidance

- When something rotates in a circle, it moves through a *position angle* θ that runs from 0 to 2π radians and starts over again at 0. The physical distance it moves is called the *path length.* If the radius of the circle is larger, the path length traveled is longer. According to the arc length formula $s = r\theta$, the path length Δs traveled by something at radius r through an angle θ is:

$$\Delta s = r\Delta\theta \text{ [1]}$$

- Just like the linear velocity is the rate of change of distance, angular velocity, usually called ω, is the rate of change of θ. The direction of angular velocity is either clockwise or counterclockwise. Analogously, the rate of change of ω is the angular acceleration α.
- For an object moving in a circle, the objects tangential speed is directly proportional to the distance it is from the rotation axis. the tangential speed (as shown in the key equations) equals this distance multiplied by the angular speed (in radians/sec).

Example 1

Question: A Merry Go Round is rotating once every 4 seconds. If you are on a horse that is 15 m from the rotation axis, how fast are you moving (i.e. what is your tangential speed)?

Answer:

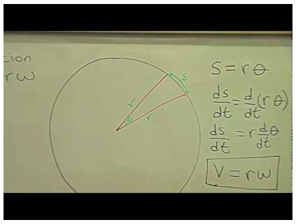
$v = r\omega$

Now we need to convert the angular speed to units of radians per second.

$\omega = \frac{1}{4}\frac{\cancel{rotations}}{second} * \frac{2\pi}{1}\frac{radians}{\cancel{rotation}} = \frac{\pi}{2}\frac{radians}{second}$

$v = 15m \times \frac{\pi}{2} = \frac{15}{2}\pi \approx 23.6\frac{radians}{second}$

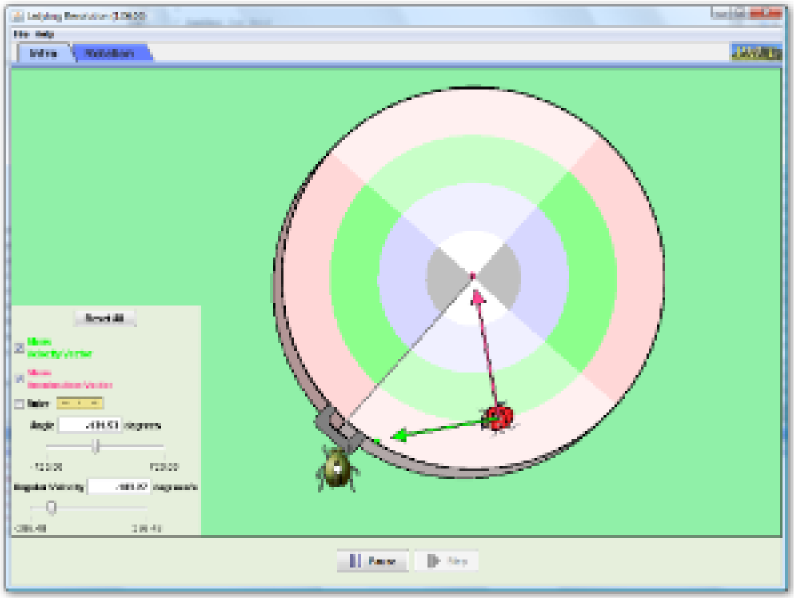
Watch this Explanation



MEDIA

Click image to the left for more content.

Simulation



Ladybug Rotation(PhET Simulation)

Time for Practice

1. You are riding your bicycle and going 8 m/s. Your bicycle wheel is 0.25 m in radius.
   a. What is its angular speed in radians per second?
   b. What is the angular speed in rotations per minute?
2. The angular speed of a record player is 33 rotations per minute. It has a diameter of 12 inches.
   a. What is the angular speed in radians per second?
   b. What is the tangential speed of the outer most part of the record?
   c. What is its tangential speed halfway out on the record?

Answers

1. a. 32 rads/s b. 306 rot/min
2. a.3.5 rads/s b. 0.53 m/s c. 0.26 m/s

8.3 Centripetal Acceleration

Students will learn what centripetal acceleration is, where it applies and how to calculate it. Students will also learn when a force is acting as a centripetal force and how to apply it.

Key Equations

Centripetal Force

$$F_C = \frac{mv^2}{r} \begin{cases} m & \text{mass (in kilograms, kg)} \\ v & \text{speed (in meters per second, m/s)} \\ r & \text{radius of circle} \end{cases}$$

Centripetal Acceleration

$$a_C = \frac{v^2}{r} \begin{cases} v & \text{speed (in meters per second, m/s)} \\ r & \text{radius of circle} \end{cases}$$

Guidance

If a mass m is traveling with velocity $\vec{v}$ and experiences a centripetal — always perpendicular — force $\vec{F}_c$, it will travel in a circle of radius

$$r = \frac{mv^2}{|\vec{F}|} \text{ [1]}$$

Alternatively, to keep this mass moving at this velocity in a circle of this radius, one needs to apply a centripetal force of

$$\vec{F}_c = \frac{mv^2}{r} \text{ [2]}$$

By Newton's Second Law, this is equivalent to a centripetal acceleration of:

$$\vec{F}_c = \cancel{m}\vec{a}_c = \cancel{m}\frac{v^2}{r} \text{ [3]}$$

Example 1

If you are 4m from the center of a Merry-Go-Round that is rotating at 1 revolution every 2 seconds, what is your centripetal acceleration?

Solution

First we need to find your tangential velocity. We can do this using the given angular velocity.

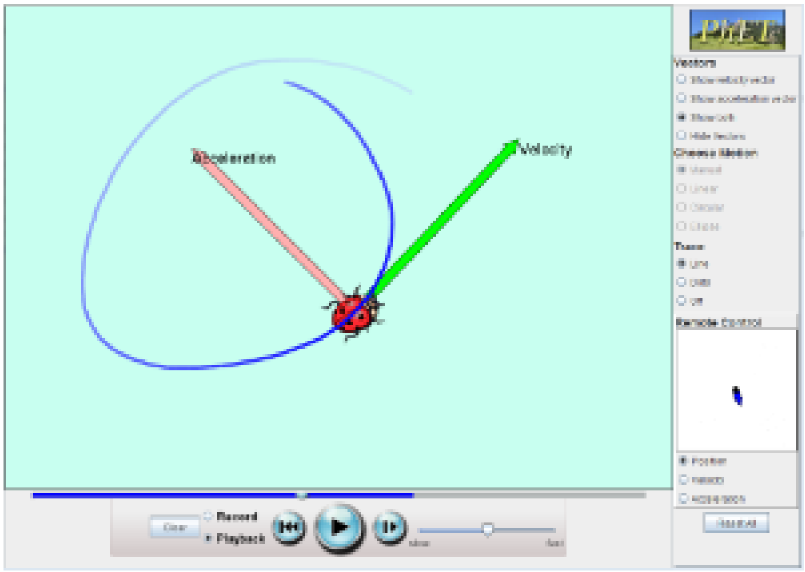
$$\omega = \frac{2\pi \text{ rad}}{2 \text{ s}}$$
$$\omega = \pi \text{ rad/s}$$
$$\omega = \frac{v}{r}$$
$$v = \omega r$$
$$v = \pi \text{ rad/s} * 4 \text{ m}$$
$$v = 4\pi \text{ m/s}$$

Now we can find your centripetal acceleration using the radius of your rotation.

$$a_c = \frac{v^2}{r}$$
$$a_c = \frac{(4\pi \text{ m/s})^2}{4 \text{ m}}$$
$$a_c = 4\pi^2 \text{ m/s}^2$$

Watch this explanation

Simulation



Ladybug Motion in 2D(PhET Simulation)

Time for Practice

1. A 6000 kg roller coaster goes around a loop of radius 30m at 6 m/s. What is the centripetal acceleration?

2. 

3. For the Gravitron ride above, assume it has a radius of 18 m and a centripetal acceleration of 32 m/s^2. Assume a person is in the graviton with 180 cm height and 80 kg of mass. What is the speed it is spinning at? Note you may not need all the information here to solve the problem.

Answers

1. 1.2 m/s^2
2. 24 m/s

8.4 Centripetal Force Problems

Students will learn how to analyze and solve Centripetal Force type problems.

Key Equations

Centripetal Force

$$F_C = \frac{mv^2}{r} \begin{cases} m & \text{mass (in kilograms, kg)} \\ v & \text{speed (in meters per second, m/s)} \\ r & \text{radius of circle} \end{cases}$$

$v = \sqrt{rg\mu_s}$ Unbanked Turn

$v = \sqrt{rg\tan\theta}$ Banked Turn

Guidance

Any force can be a centripetal force. Centripetal force is the umbrella term given to any force that is acting perpendicular to the motion of an object and thus causing it to go in a circle. Keep the following in mind when solving centripetal force type problems:

- The speed of the object remains constant. The centripetal force is changing the direction but not the speed of the object.
- Although the object 'feels' an outward pull, this is not a true force, but merely the objects inertia. Remember, Newton's first law maintains that the natural state of an object is to go in a straight line at constant speed. Thus, when you make a right turn in your car and the basketball in the back seat flies to the left, that is because the car is moving right and the basketball is maintaining it's position and thus from your point of view moves to the left. Your point of view in this case is different from reality because you are in a rotating reference frame.

Applications

- To find the maximum speed that a car can take a corner on a flat road without skidding out, set the force of friction equal to the centripetal force.

- To find the tension in the rope of a swinging pendulum, remember that it is the *sum* of the tension and gravity that produces a net upward centripetal force. A common mistake is just setting the centripetal force equal to the tension.

- To find the speed of a planet or satellite in an orbit, set the force of gravity equal to the centripetal force.

- Banked turns (the road is not flat, but rather at an angle) allow for a greater speed before skidding out. In this case the normal force aids the force of friction in creating a larger centripetal force than that which can be

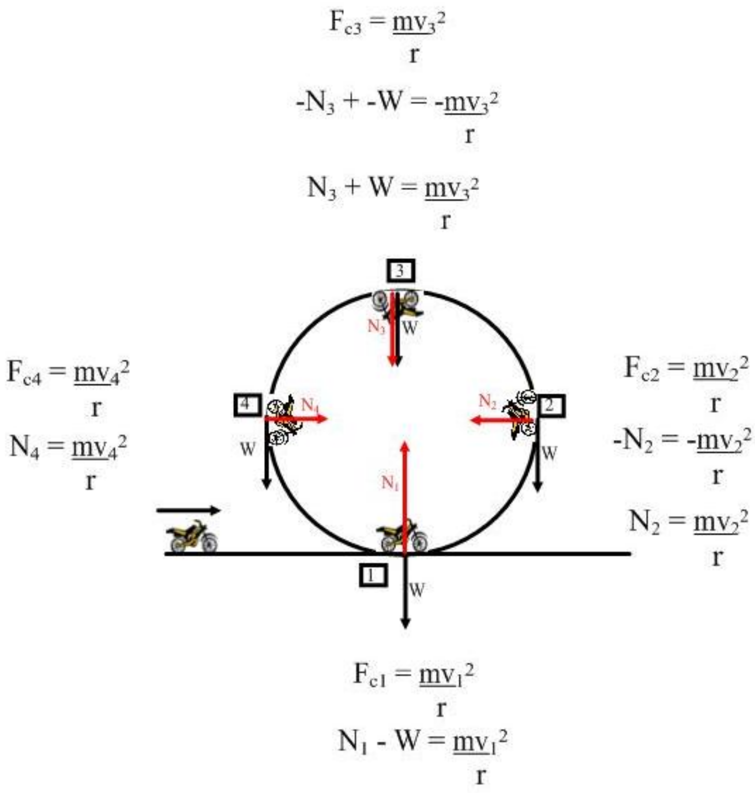
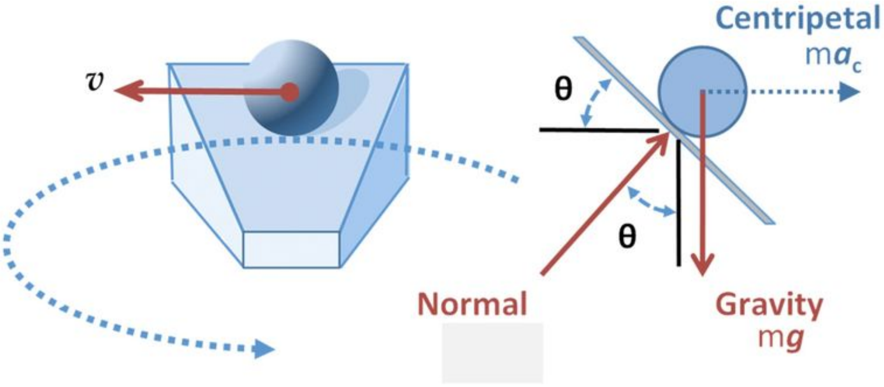


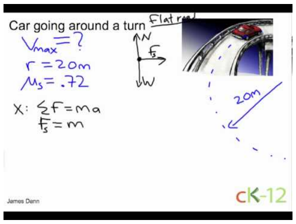
FIGURE 8.1

Force analysis of an object in a loop.

obtained by friction alone on a flat road. It#8217;s not as simple as adding them together, one must consider the components of these two forces in the direction to the center of the circle.



Watch this Explanation



MEDIA

Click image to the left for more content.

Example 1 - Object in a Loop

A bucket of water is swung in a vertical circle with a radius of 0.75 m. Calculate the minimum speed the bucket must have at the top of the loop to continue on its circular path.

$$N + W = \frac{mv^2}{r}$$
$$N = 0 \quad @ \quad v_{min}$$
$$0 + W = \frac{mv^2}{r}$$
$$mg = \frac{mv^2}{r}$$
$$gr = v^2$$
$$\sqrt{gr} = v$$
$$\sqrt{(9.81)(0.75)} = v$$
$$2.7\ m/s = v$$

Example 2 - Object on a String

A string requires a 135 N force in order to break it. A 2.00 kg mass is tied to the string and whirled in a vertical circle with a radius of 1.10 m. Calculate the maximum speed that this mass can be whirled without breaking the string.

$$T - W = \frac{mv^2}{r}$$
$$135 - (2.00)(9.81) = \frac{2v^2}{1.10}$$
$$115.4 = 1.82v^2$$
$$63.4 = v^2$$
$$7.96\ m/s = v$$

Example 3 - Unbanked Turn

If the maximum speed at which a car can safely navigate an unbanked turn of radius 50.0 m is 21.0 m/s, what is the coefficient of static friction?

$$v = \sqrt{rg\mu_s}$$
$$21.0 = \sqrt{(50.0)(9.81)\mu_s}$$
$$441 = 490\mu_s$$
$$0.900 = \mu_s$$

Example 4 - Banked Turn

The turns in a track have a maximum radius at the top of 316 m and are banked steeply. If a car travels at a maximum speed of 43 m/s for skidding, what is the angle of the curve with respect to the horizontal? Assume the turn is frictionless.

$$v = \sqrt{rg\tan\theta}$$
$$43 = \sqrt{(316)(9.81)\tan\theta}$$
$$1849 = 3100\tan\theta$$
$$\tan^{-1}\left(\frac{1849}{3100}\right) = \theta$$
$$31^\circ = \theta$$

Time for Practice

1. A 700kg car makes a turn going at 30 m/s with radius of curvature of 120m. What is the force of friction between the car#8217;s tires and the road?
2. An object of mass 10 kg is in a circular orbit of radius 10 m at a velocity of 10 m/s.
   a. Calculate the centripetal force (in N) required to maintain this orbit.
   b. What is the acceleration of this object?
3. Suppose you are spinning a child around in a circle by her arms. The radius of her orbit around you is 1 meter. Her speed is 1 m/s. Her mass is 25 kg.
   a. What is the magnitude and direction of tension in your arms?
   b. In her arms?
4. A racecar is traveling at a speed of 80.0 m/s on a circular racetrack of radius 450 m.
   a. What is its centripetal acceleration in $\mathrm{m/s^2}$?
   b. What is the centripetal force on the racecar if its mass is 500 kg?
   c. What provides the necessary centripetal force in this case?
5. The radius of the Earth is 6380 km. Calculate the velocity of a person standing at the equator due to the Earth#8217;s 24 hour rotation. Calculate the centripetal acceleration of this person and express it as a fraction of the acceleration g due to gravity. Is there any danger of #8220;flying off#8221;?
6. Neutron stars are the corpses of stars left over after supernova explosions. They are the size of a small city, but can spin several times per second. (Try to imagine this in your head.) Consider a neutron star of radius 10 km that spins with a period of 0.8seconds. Imagine a person is standing at the equator of this neutron star.
   a. Calculate the centripetal acceleration of this person and express it as a multiple of the acceleration g due to gravity (on Earth).
   b. Now, find the minimum acceleration due to gravity that the neutron star must have in order to keep the person from flying off.

Answers to Selected Problems

1. 5250 N
2. a. 100 N b. 10 $\mathrm{m/s^2}$
3. a. 25 N towards her b. 25 N towards you
4. a. 14.2 $\mathrm{m/s^2}$ b. 7.1×10^3 N c. friction between the tires and the road
5. .0034g
6. a. $6.3 \times 10^4 g\ \mathrm{m/s^2}$ b. The same as a.

8.5 Universal Law of Gravity

Students will learn to use Newton's Universal Law of Gravity equation to solve problems. They will also learn how the acceleration of gravity on Earth is calculated and gain an understanding of gravitational fields.

Key Equations

- $F_G = \frac{Gm_1m_2}{r^2}$; the force of gravity between an object with mass m_1 and another object of mass m_2 and a distance between them of r.
- $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$; the universal constant of gravity
- $g = \frac{Gm}{r^2}$; gravitational field strength or gravitational acceleration of a planet with mass m and radius r. Note that this is not really a separate equation but comes from Newton#8217;s second law and the law of universal gravitation.
- Some data needed for the problems:

  The radius of Earth is $6.4 \times 10^6\ m$ The mass of Earth is about $6.0 \times 10^{24}\ kg$ The mass of Sun is about $2.0 \times 10^{30}\ kg$ The Earth-Sun distance is about $1.5 \times 10^{11}\ m$ The Earth-Moon distance is about $3.8 \times 10^8\ m$

Guidance

- When using the Universal Law of Gravity formula and the constant G above, make sure to use units of meters and kilograms.
- The direction of the force of gravity is in a straight line between two objects. It is always attractive.
- Newton invented calculus in order to prove that for a spherical object (like Earth) one can assume all of its mass is at the center of the sphere (thus in his formula, one can use the radius of Earth for the distance between a falling rock and Earth).
- Newton's Laws apply to all forces; but when he developed them only one was known: gravity. Newton's major insight — and one of the greatest in the history of science — was that the same force that causes objects to fall when released is also responsible for keeping the planets in orbit.

Universal Gravity

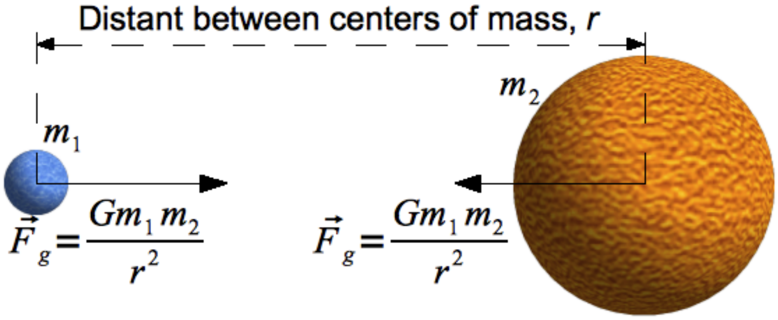
Any two objects in the universe, with masses m_1 and m_2 with their centers of mass at a distance r apart will experience a force of mutual attraction along the line joining their centers of mass equal to:

$$\vec{F}_G = \frac{Gm_1m_2}{r^2} \qquad \text{Universal Gravitation,}$$

where G is the Gravitational constant:

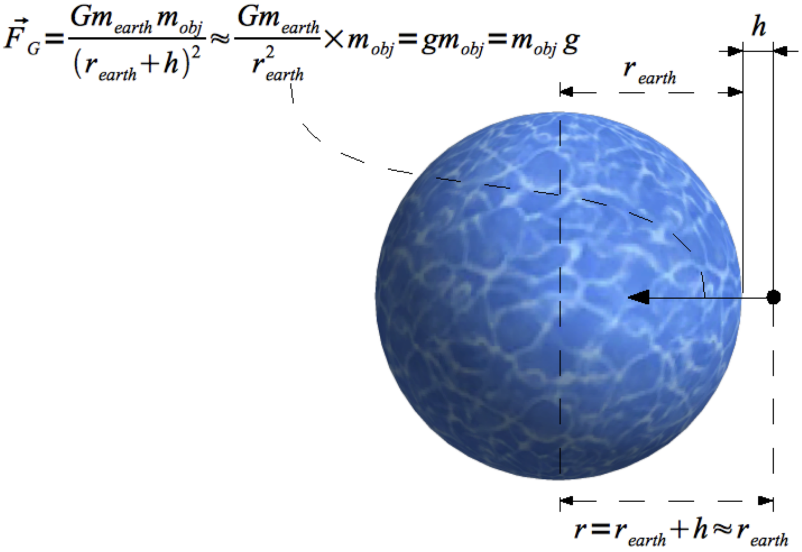
$$G = 6.67300 \times 10^{-11} \mathrm{m^3kg^{-1}s^{-2}}$$

Here is an illustration of this law for two objects, for instance the earth and the sun:



Gravity on the Earth's Surface

On the surface of a planet — such as earth — the r in formula [3] is very close to the radius of the planet, since a planet's center of mass is — usually — at its center. It also does not vary by much: for instance, the earth's radius is about 6,000 km, while the heights we consider for this book are on the order of at most a few kilometers — so we can say that for objects near the surface of the earth, the r in formula [3] is constant and equal to the earth's radius. This allows us to say that gravity is more or less constant on the surface of the earth. Here's an illustration:



For any object a height h above the surface of the earth, the force of gravity may be expressed as:

$$\vec{F}_G = \frac{Gm_{earth} m_{obj}}{(r_{earth} + h)^2}$$

Now we make the approximation that

$$r_{earth} + h \approx r_{earth}$$

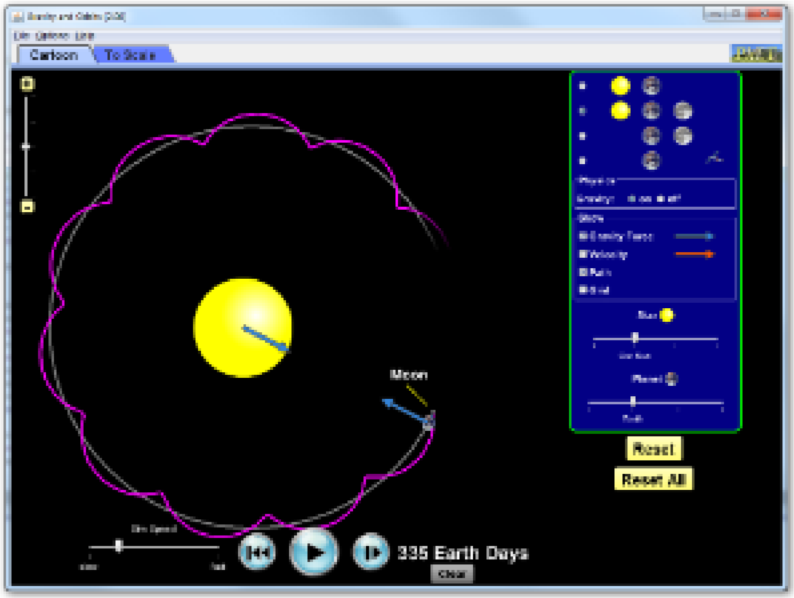
then, we can rewrite the force of gravity equation as

$$\vec{F}_G = \underbrace{\frac{Gm_{earth}}{r^2_{earth}}}_{\vec{g}_{earth}} \times m_{obj} = m_{obj} \times \vec{g}$$

We can do this because the quantity in braces only has constants; we can combine them and call their product g. Remember, *this is an approximation that holds **only** when the r in formula [3] is more or less constant.*

We call the quantity mg an object's **weight**. Unlike an object's mass, an object's weight can change and depends on the gravitational force it experiences. In fact, an object's weight is the magnitude of the gravitational force on it. To find the weight of an object on another planet, star, or moon, use the appropriate values in the formula for the force of gravity.

Simulation



Gravity and Orbits (PhETSimulation)

Time for Practice

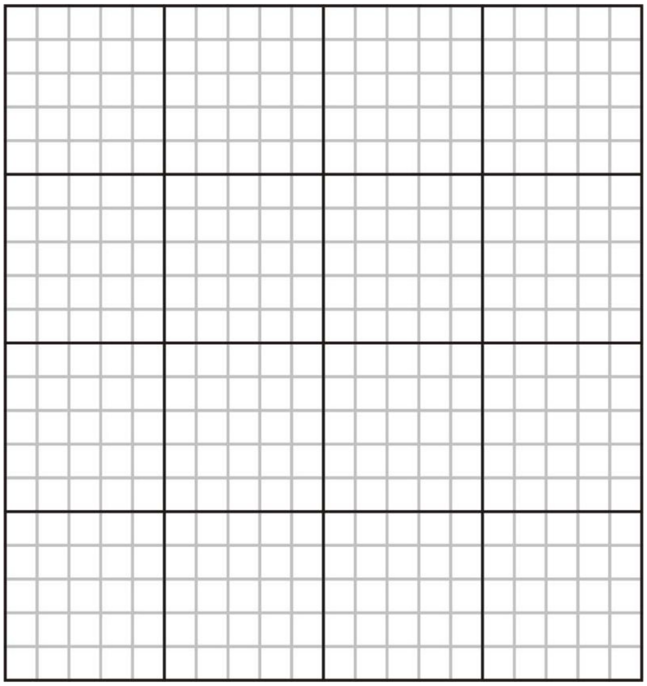
1. Which is greater #8211; the gravitational force that the Sun exerts on the moon, or the force the Earth exerts on the moon? Does the moon orbit the Earth or the Sun? Explain.
2. Use Newton#8217;s Law of Universal Gravitation to explain why even though Jupiter has 300 times the mass of the earth, on the #8220;surface#8221; of Jupiter you#8217;d weigh only 3 times what you weigh on earth. What other factor has to be considered here?
3. Prove g is *approximately* 10 m/s^2on Earth by following these steps:
    a. Calculate the force of gravity between a falling object (for example an apple) and that of Earth. Use the symbol m_o to represent the mass of the falling object.
    b. Now divide that force by the object#8217;s mass to find the acceleration g of the object.
    c. Calculate the force of gravity between the Sun and the Earth. (sun mass $= 2.0 \times 10^{30}$ kg; average distance from sun to earth = 150 million km)
4. Calculate the gravitational force that your pencil or pen pulls on you. Use the center of your chest as the center of mass (and thus the mark for the distance measurement) and estimate all masses and distances.
    a. If there were no other forces present, what would your acceleration be towards your pencil? Is this a large or small acceleration?
    b. Why, in fact, doesn#8217;t your pencil accelerate towards you?

5. Mo and Jo have been traveling through the galaxy for eons when they arrive at the planet Remulak. Wanting to measure the gravitational field strength of the planet they drop Mo#8217;s lava lamp from the top deck of their spacecraft, collecting the velocity-time data shown below.

TABLE 8.1:

velocity (m/s)	time (s)
0	0
3.4	1.0
7.0	2.0
9.8	3.0
14.0	4.0
17.1	5.0

(a) Plot a velocity-time graph using the axes above. Put numbers, labels and units on your axes. Then draw a best-fit line (use a ruler) and use that line to find the gravitational field strength of Remulak. Explain below how you did that.



(b) Mo and Jo go exploring and drop a rock into a deep canyon #8211; it hits the ground in 8.4 s. How deep is the canyon?

(c) If the rock has a mass of 25 g and makes a hole in the ground 1.3 cm deep, what force does the ground exert to bring it to a stop?

(d) Mo and Jo observe the shadows of their lava lamps at different positions on the planet and determine (a la Eratosthenes, the Greek astronomer, around 200 B.C.) that the radius of Remulak is 4500 km. Use that and your result for g to find the mass of Remulak.

Answer to 5

5a. $\sim 3.4\ m/s^2$

5b. $\sim 120\ m$

5c. $\sim 800\ N$

5d. $\sim 10^{23}\ kg$

8.6 Gravity and Space Problems

Key Equations

$v = \frac{2\pi r}{T}$

for a particle travels a distance $2\pi r$ in an amount of time T

$\vec{F}_G = \frac{Gm_1m_2}{r^2}$

$\vec{a}_c = \frac{v^2}{r}$

Guidance

When can Gravity Act as a Centripetal Force?

We saw last chapter that the force of Gravity causes an attraction between two objects of mass m_1 and m_2 at a distance r of

$$\vec{F}_G = \frac{Gm_1m_2}{r^2}. \text{ [4]}$$

By Newton's Third Law, both objects experience the force: equal in magnitude and opposite in direction, and both will move as an effect of it. If one of the objects is much lighter than the other (like the earth is to the sun, or a satellite is to earth) we can approximate the situation by saying that the heavier mass (the sun) does not move, since its acceleration will be far smaller due to its large mass. Then, if the lighter mass remains at a relatively constant absolute distance from the heavier one (remember, centripetal force needs to be constant in magnitude), we can say that the lighter mass experiences an *effectively* centripetal force and thus has a centripetal acceleration.

Math of Centripetal Gravity

Gravity is not always a centripetal force. This is a really important point. It only acts as a centripetal force when conditions approximate those listed above — very much like it isn't constant near the surface of the earth, but very close to it.

If gravity provides centripetal force and acceleration, we can set [2] equal to [4]. It's important to remember that in [2] m refers to the lighter mass, since that is the one traveling. Then,

$$\frac{G\cancel{m_1}m_2}{\cancelto{r}{r^2}} = \frac{\cancel{m_1}v^2}{\cancel{r}}$$

So, the relationship between velocity and radius for a circular orbit of a light object around an heavy mass (note the mass of the lighter object cancels) is:

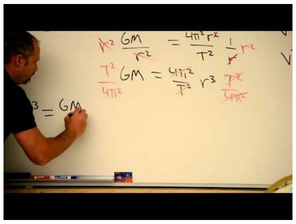
$$Gm_2 = {v_{\text{orb}}}^2 r_{\text{orb}}$$

Geosynchronous orbit

This is the orbit where a satellite completes one orbit of the Earth every 24 hours, staying above the same spot (longitude) on Earth. This is a very important orbit for spy satellites and TV satellites among others. You force the speed of the satellite to be a value such that the satellite makes one rotation every 24 hours.

Example 1

Watch this Explanation

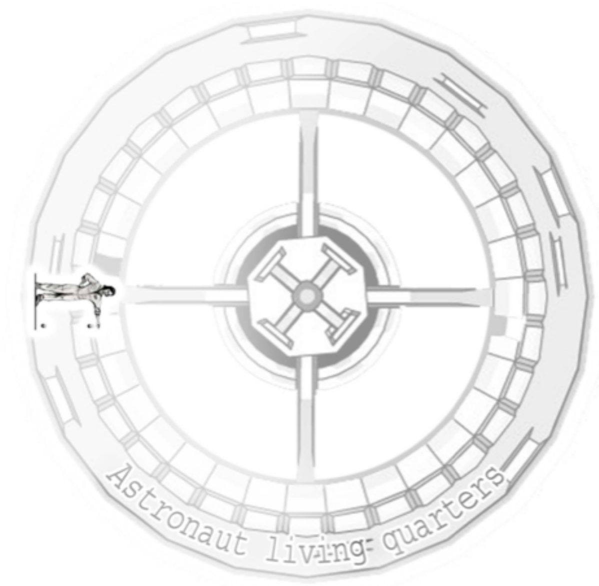


MEDIA

Click image to the left for more content.

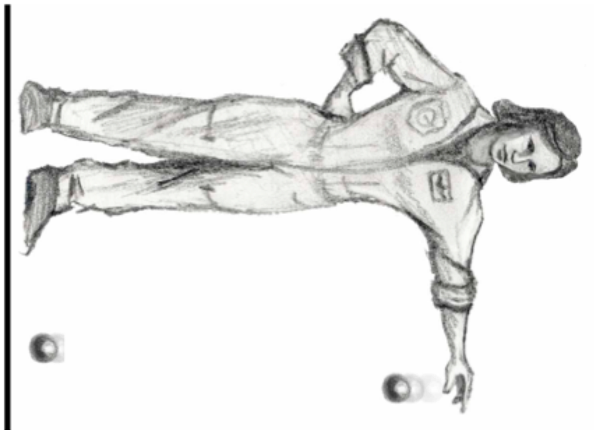
Time for Practice

1. A digital TV satellite is placed in geosynchronous orbit around Earth, so it is always in the same spot in the sky.
    a. Using the fact that the satellite will have the same period of revolution as Earth, calculate the radius of its orbit.
    b. What is the ratio of the radius of this orbit to the radius of the Earth?
    c. Draw a sketch, to scale, of the Earth and the orbit of this digital TV satellite.
    d. If the mass of the satellite were to double, would the radius of the satellite#8217;s orbit be larger, smaller, or the same? Why?
2. A top secret spy satellite is designed to orbit the Earth twice each day (*i.e.,* twice as fast as the Earth#8217;s rotation). What is the height of this orbit above the Earth#8217;s surface?
3. Two stars with masses 3.00×10^{31} kg and 7.00×10^{30} kg are orbiting each other under the influence of each other#8217;s gravity. We want to send a satellite in between them to study their behavior. However, the satellite needs to be at a point where the gravitational forces from the two stars are equal. The distance between the two stars is 2.0×10^{10} m. Find the distance from the more massive star to where the satellite should be placed. (*Hint:* Distance from the satellite to one of the stars is the variable.)
4. Calculate the mass of the Earth using *only: (i)* Newton#8217;s Universal Law of Gravity; *(ii)* the Moon-Earth distance (Appendix *B*); and *(iii)* the fact that it takes the Moon 27 days to orbit the Earth.



5.

6. A space station was established far from the gravitational field of Earth. Extended stays in zero gravity are not healthy for human beings. Thus, for the comfort of the astronauts, the station is rotated so that the astronauts *feel* there is an internal gravity. The rotation speed is such that the *apparent* acceleration of gravity is 9.8 $\mathrm{m/s^2}$. The direction of rotation is counter-clockwise.
    a. If the radius of the station is 80 m, what is its rotational speed, v?
    b. Draw vectors representing the astronaut#8217;s velocity and acceleration.
    c. Draw a free body diagram for the astronaut.
    d. Is the astronaut exerting a force on the space station? If so, calculate its magnitude. Her mass $m = 65$ kg.
    e. The astronaut drops a ball, which *appears* to accelerate to the #8216;floor,#8217; (see picture) at 9.8 $\mathrm{m/s^2}$.
        a. Draw the velocity and acceleration vectors for the ball while it is in the air.
        b. What force(s) are acting on the ball while it is in the air?
        c. Draw the acceleration and velocity vectors after the ball hits the floor and comes to rest.
        d. What force(s) act on the ball after it hits the ground?



Answers to Selected Problems

1. a. 4.23×10^7m b. 6.6 R_e d. The same, the radius is independent of mass
2. 1.9×10^7m
3. You get two answers for r, one is outside of the two stars one is between them, that#8217;s the one you want, 1.32×10^{10}m from the larger star.

4. .
5. a. $v = 28$ m/s b. $v-$down, $a-$right c. $f-$right d. Yes, 640N

8.7 References

1. CK-12 Foundation. . CCSA

CHAPTER 9 Work, Energy, and Power

Chapter Outline

9.1 Springs

Students will learn to calculate periods, frequencies, etc. of spring systems in harmonic motion.

Key Equations

$T = \frac{1}{f}$; Period is the inverse of frequency

$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$; Period of mass m on a spring with constant k

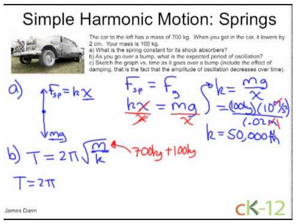
$F_s = -kx$; the force of a spring equals the spring constant multiplied by the amount the spring is stretched or compressed from its equilibrium point. The negative sign indicates it is a restoring force (i.e. direction of the force is opposite its displacement from equilibrium position.

$E_e = \frac{1}{2}kx^2$; the potential energy of a spring is equal to one half times the spring constant times the distance squared that it is stretched or compressed from equilibrium

Guidance

- The oscillating object does not lose any energy in SHM. Friction is assumed to be zero.
- In harmonic motion there is always a *restorative force,* which attempts to *restore* the oscillating object to its equilibrium position. The restorative force changes during an oscillation and depends on the position of the object. In a spring the force is given by Hooke#8217;s Law: $F = -kx$
- The period, T, is the amount of time needed for the harmonic motion to repeat itself, or for the object to go one full cycle. In SHM, T is the time it takes the object to return to its exact starting point and starting direction.
- The frequency, f, is the number of cycles an object goes through in 1 second. Frequency is measured in Hertz (Hz). 1 $Hz = 1$ cycle per sec.
- The amplitude, A, is the distance from the *equilibrium* (or center) *point* of motion to either its lowest or highest point (*end points*). The amplitude, therefore, is half of the total distance covered by the oscillating object. The amplitude can vary in harmonic motion, but is constant in SHM.

Example 1



Simple Harmonic Motion: Springs

The car to the left has a mass of 700 kg. When you get in the car, it lowers by 2 cm. Your mass is 100 kg.
a) What is the spring constant for its shock absorbers?
b) As you go over a bump, what is the expected period of oscillation?
c) Sketch the graph vs. time as it goes over a bump (include the effect of damping, that is the fact that the amplitude of oscillation decreases over time).

a) $F_{sp} = kx$ mg

$F_{sp} = F_g$

$\frac{k\cancel{x}}{\cancel{x}} = \frac{mg}{x}$

$k = \frac{mg}{x} = \frac{(100kg)(10m/s^2)}{(.02m)}$

$k = 50{,}000 \frac{N}{m}$

b) $T = 2\pi\sqrt{\frac{m}{k}}$ ← 700kg + 100kg

$T = 2\pi$

James Dann

MEDIA

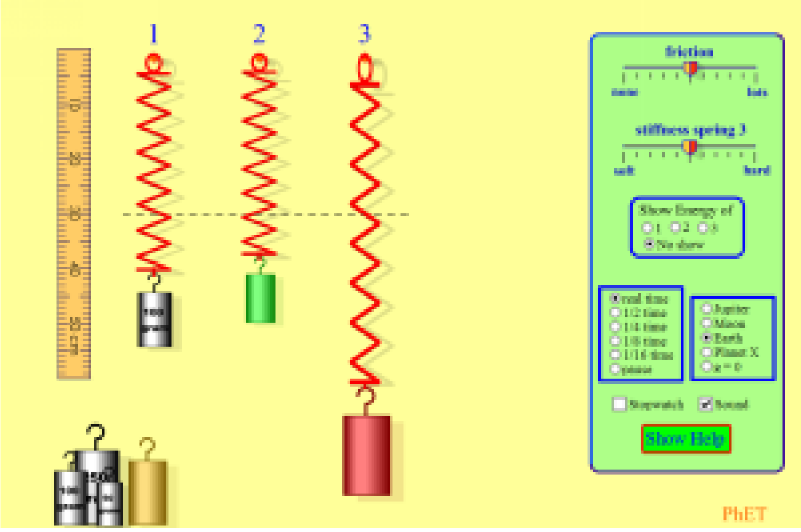
Click image to the left for more content.

Watch this Explanation

MEDIA

Click image to the left for more content.

Simulation



Mass& Springs (PhETSimulation)

Time for Practice

1. A rope can be considered as a spring with a very high spring constant k,so high, in fact, that you don#8217;t notice the rope stretch at all before it #8220;pulls back.#8221;
   a. What is the k of a rope that stretches by 1 mm when a 100 kg weight hangs from it?
   b. If a boy of 50 kg hangs from the rope, how far will it stretch?
   c. If the boy kicks himself up a bit, and then is bouncing up and down ever so slightly, what is his frequency of oscillation? Would he notice this oscillation? If so, how? If not, why not?
2. If a 5.0 kg mass attached to a spring oscillates 4.0 times every second, what is the spring constant k of the spring?
3. A horizontal spring attached to the wall is attached to a block of wood on the other end. All this is sitting on a frictionless surface. The spring is compressed 0.3 m. Due to the compression there is 5.0 J of energy stored in the spring. The spring is then released. The block of wood experiences a maximum speed of 25 m/s.
   a. Find the value of the spring constant.
   b. Find the mass of the block of wood.
   c. What is the equation that describes the position of the mass?
   d. What is the equation that describes the speed of the mass?
   e. Draw three complete cycles of the block#8217;s oscillatory motion on an x vs. t graph.

4. A spider of 0.5 g walks to the middle of her web. The web sinks by 1.0 mmdue to her weight. You may assume the mass of the web is negligible.
    a. If a small burst of wind sets her in motion, with what frequency will she oscillate?
    b. How many times will she go up and down in one s? In 20 s?
    c. How long is each cycle?
    d. Draw the x vs t graph of three cycles, assuming the spider is at its highest point in the cycle at $t = 0$ s.

Answers to Selected Problems

1. a. 9.8×10^5 N/m b. 0.5 mm c. 22 Hz
2. 3.2×10^3 N/m
3. a. 110 N/m d. $v(t) = (25)\cos(83\text{t})$
4. a. 16 Hz b. 16 complete cycles but 32 times up and down, 315 complete cycles but 630 times up and down c. 0.063 s

Investigation

1. Your task: Match the period of the circular motion system with that of the spring system. You are only allowed to change the velocity involved in the circular motion system. Consider the effective distance between the block and the pivot to be to be fixed at 1m. The spring constant(13.5N/m) is also fixed. You should view the charts to check whether you have succeeded. Instructions: To alter the velocity, simply click on the Select Tool, and select the pivot . The Position tab below will allow you to numerically adjust the rotational speed using the Motor field. To view the graphs of their respective motion in order to determine if they are in sync, click on Chart tab below.

2.

MEDIA
Click image to the left for more content.

3. Now the mass on the spring has been replaced by a mass that is twice the rotating mass. Also, the distance between the rotating mass and the pivot has been changed to 1.5 m. What velocity will keep the period the same now?

4.

MEDIA
Click image to the left for more content.

9.2 Kinetic Energy

Students will learn about kinetic energy, how and when to apply it and how to use kinetic energy

Key Equations

Kinetic energy

$$K = \frac{1}{2}mv^2 \begin{cases} m & \text{mass (in kilograms, kg)} \\ v & \text{speed (in meters per second, m/s)} \end{cases}$$

Guidance

The energy of motion is kinetic energy, KE. Whenever an object is in motion it has kinetic energy. The faster it is going, the more energy it has.

Example 1

You are using a sling to throw a small stone. If the sling is .5 m long and you are spinning it at 15 rad/s, how high would the rock go if you throw it straight up?

Solution

We'll start by setting the kinetic energy of rock to it's gravitational potential energy at it's maximum height and then solving for the rock's height.

$$\begin{aligned} KE_i &= PE_f \\ \frac{1}{2}mv^2 &= mgh \\ h &= \frac{v^2}{2g} \end{aligned}$$

We still don't know the rock's linear velocity, but we do know the sling's angular velocity and radius so we can put those into the equation instead.

$$h = \frac{(\omega r)^2}{2g}$$
$$h = \frac{(15\ \text{rad/s} * .5\ \text{m})^2}{2 * 9.8\ \text{m/s}^2}$$
$$h = 2.9\ \text{m}$$

Time for Practice

1. A bomb with 8 x 104 J of potential energy explodes. Assume 20% of its potential energy is converted to kinetic energy of the metal pieces flying outward (shrapnel).
   a. What is the total kinetic energy of the shrapnel?
   b. Assume the average mass of the shrapnel is 0.4 kg and that there are 200 pieces. What is the average speed of one piece?
2. A 1500 kg car starts at rest and speeds up to 3.0 m/s.
   a. What is the gain in kinetic energy?
   b. We define efficiency as the ratio of output energy (in this case kinetic energy) to input energy. If this car#8217;s efficiency is 0.30, how much input energy was provided by the gasoline?
   c. If 0.15 gallons were used up in the process, what is the energy content of the gasoline in Joules per gallon?
3. An airplane with mass $200,000$ kg is traveling with a speed of 268 m/s.
   (a) What is the kinetic energy of the plane at this speed?

   A wind picks up, which causes the plane to lose 1.20×10^8 J per second.

   (a) How fast is the plane going after 25.0 seconds?

Answers to Selected Problems

1. a. $1.2 \times 10^6 J$ b. $20 m/s$
2. a. $6750J$ b. 2.25×10^5 J c. 1.5×10^5 J/gallon of gas
3. a. 7.18×10^9 J

9.3 Potential Energy

Students are to gain a basic understanding of potential energy and how to calculate it before diving into energy conservation.

Key Equations

Gravitational potential energy

$$U_g = mgh \begin{cases} h & \text{height above the ground in meters(m)} \\ g & \text{acceleration due to gravity, 9.8 m/s}^2 \\ U_g & \text{Potential energy of gravity (in Joules; 1 J} = \text{1kg m}^2/\text{s}^2\text{)} \end{cases}$$

Spring potential energy

$$U_{sp} = \tfrac{1}{2}k\Delta x^2 \begin{cases} k & \text{spring constant measured in Newtons(N) per meters(m)} \\ x & \text{amount spring is displaced from resting position} \\ U_{sp} & \text{potential energy of a spring (in Joules; 1 J} = \text{1kg m}^2/\text{s}^2\text{)} \end{cases}$$

Guidance

Example 1

A 2 kg block of wood is suspended 5m above a spring of spring constant 3000 N/m. When the block is dropped on the spring, how far will the spring be compressed from it's equilibrium position.

Solution

We can solve for the distance the spring will be compressed using conservation of energy. In this problem, the gravitational potential energy of the block will be turned into spring potential energy.

$$\begin{aligned} U_g &= U_{sp} && \text{start with by setting the gravitational potential energy equal to the spring potential energy} \\ mgh &= \frac{1}{2}k\Delta x^2 && \text{substitute the equations for gravitational and spring potential energy} \\ \Delta x &= \sqrt{\frac{2mgh}{k}} && \text{solve for } \Delta x \\ \Delta x &= \sqrt{\frac{2 * 2\text{ kg} * 9.8\text{ m/s}^2 * 5\text{ m}}{3000\text{ N/m}}} && \text{plug in the known values} \\ \Delta x &= .26\text{ m} \end{aligned}$$

Time for Practice

1. If you lift a 30 kg weight 0.5 meters, how much Potential energy has it gained?
2. A spring has a spring constant k equal to 200 N/m. If it is stretched 0.4 m, what is its potential energy?
3. A 12 kg box is resting on a table 1.5 m off the ground. It is then lifted up to 3.2 m off the ground. What is its increase in potential energy?

Answers

(using g = 10 m/s^2)

1. 150 J
2. 16 J
3. 204 J

9.4 Work

Students will learn that work is simply the transfer of energy into or out of a system. Students will learn how to calculate the work done and how to incorporate it into energy conservation.

Key Equations

$W = F_{||}d$; Work is equal to the distance an object moves multiplied by the component of the force in the direction that object is moving.

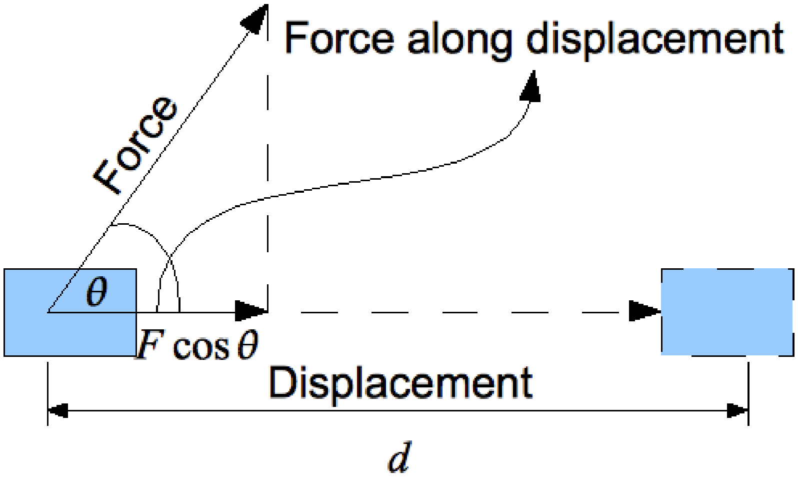
$W =$ work (in Joules; work is just energy being transferred)

Guidance

When an object moves in the direction of an applied force, we say that the force does **work** on the object. Note that the force may be slowing the object down, speeding it up, maintaining its velocity — any number of things. In all cases, the net work done is given by this formula:

$$W = \vec{F} \cdot \vec{d} = \vec{F} \cdot \Delta\vec{x} \qquad \text{Work is the dot product of force and displacement.}$$

In other words, if an object has traveled a distance d under force $\vec{F}$, the work done on it will equal to d multiplied by the component of $\vec{F}$ along the object's path. Consider the following example of a block moving horizontally with a force applied at some angle:



Here the net work done on the object by the force will be $Fd\cos\theta$.

Example 1

A 1kg ball has been attached to a 2m string and is at rest on a frictionless surface. If you exert a constant force of 10N on the string and pull the ball over the course of 5m and then begin spinning the ball in a circle, after 3 revolutions, what is the total amount of work you have done on the ball?

Solution

Since the centripetal force you exert on the ball in order to make it spin is perpendicular to the ball's path, you do not work on the ball while spinning it in a circle. Therefore, the only work you do on the ball is when you are pulling it in a straight line.

$$\begin{aligned}W &= Fd\\ W &= 10\text{N} * 5\text{m}\\ W &= 50\text{J}\end{aligned}$$

Example 2

A block of mass 5kg is sliding down a ramp inclined at 45 degrees. If the coefficient of kinetic friction between the ramp and the block is 0.3, how much work does the force of friction do as the block slides 3m down to the bottom of the ramp.

Solution

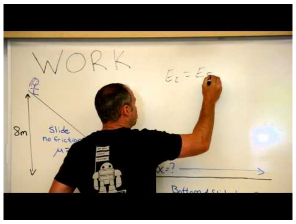
In order to find the work done by friction, we first want to find out the magnitude of the force of friction.

$$\begin{aligned}f &= \mu_k N && \text{start with the equation for the force of friction}\\ f &= \mu_k mg\cos(30) && \text{substitute the y-component of the weight of the block for the normal force}\end{aligned}$$

Now that we have the magnitude of the force of friction, we can plug that into the equation for work.

$$\begin{aligned}W &= Fd\\ W &= \mu_k mgcos(45)\\ W &= 0.3 * 5\text{kg} * 9.8\text{m/s}^2 * \cos(45)\\ W &= 10.4J\end{aligned}$$

Watch this Explanation

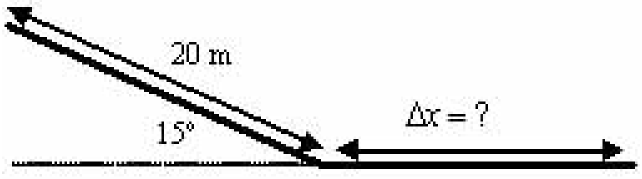


MEDIA

Click image to the left for more content.

Time for Practice

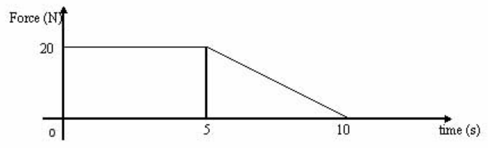
1. You slide down a hill on top of a big ice block as shown in the diagram. Your speed at the top of the hill is zero. The coefficient of kinetic friction on the slide down the hill is zero $(\mu_k = 0)$. The coefficient of kinetic friction on the level part just beneath the hill is $0.1(\mu_k = 0.1)$.



   a. What is your speed just as you reach the bottom of the hill?
   b. How far will you slide before you come to a stop?

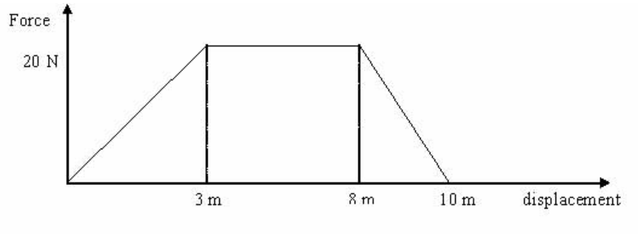
2. Marciel is at rest on his skateboard (total mass 50 kg) until he catches a ball traveling with a speed of 50 m/s. The baseball has a mass of 2 kg. What percent of the original kinetic energy is transferred into heat, sound, deformation of the baseball, and other non-mechanical forms when the collision occurs?

3. Investigating a traffic collision, you determine that a fast-moving car (mass 600 kg) hit and stuck to a second car (mass 800 kg), which was initially at rest. The two cars slid a distance of 30.0 m over rough pavement with a coefficient of friction of 0.60 before coming to a halt. What was the speed of the first car? Was the driver above the posted 60 MPH speed limit?
4. Force is applied in the direction of motion to a 15.0 kg cart on a frictionless surface. The motion is along a straight line and when $t = 0$, then $x = 0$ and $v = 0$. (The displacement and velocity of the cart are initially zero.) Look at the following graph:



  a. What is the change in momentum during the first 5 sec?
  b. What is the change in velocity during the first 10 sec?
  c. What is the acceleration at 4 sec?
  d. What is the total work done on the cart by the force from $0-10$ sec?
  e. What is the displacement after 5 sec?

5. Force is applied in the direction of motion to a 4.00 kg cart on a frictionless surface. The motion is along a straight line and when $t=0, v=0$ and $x=0$. look at the following graph:



  a. What is the acceleration of the cart when the displacement is 4 m?
  b. What work was done on the cart between $x=3$ m and $x=8$ m?
  c. What is the total work done on the cart between $0-10$ m?
  d. What is the speed of the cart at $x=10$ m?
  e. What is the impulse given the cart by the force from $1-10$ m?
  f. What is the speed at $x=8$ m?
  g. How much time elapsed from when the cart was at $x=8$ to when it got to $x=10$ m?

Answers to Selected Problems

1. a. 10 m/s b. 52 m
2. 96%
3. 43.8 m/s
4. .
5. .

9.5 Power and Efficiency

Students will learn how to calculate power and efficiency. Students will also learn the true meaning of both.

Key Equations

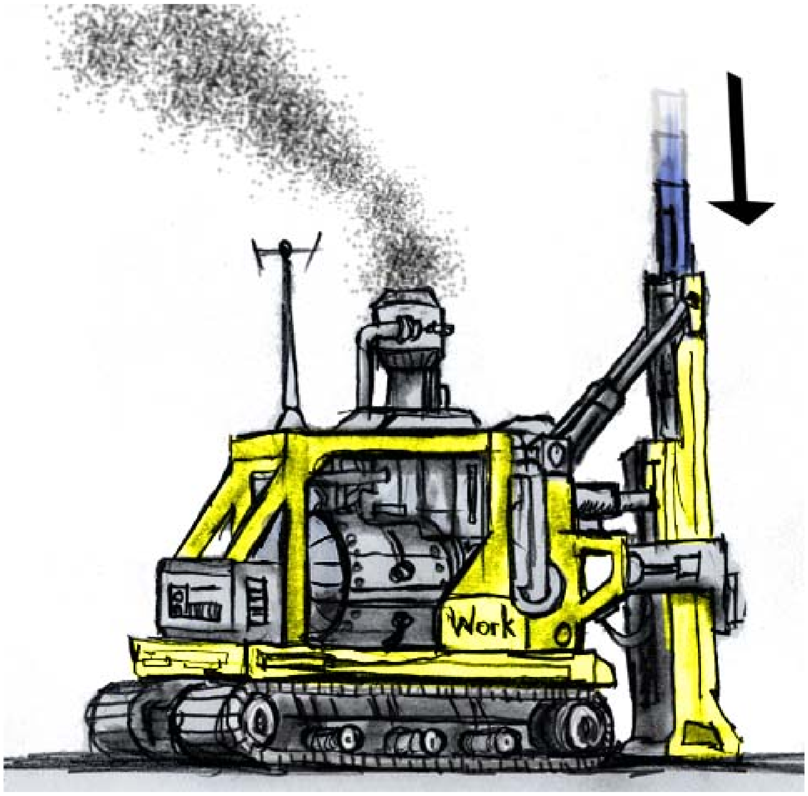
$P = \frac{W}{\Delta t}$; Power is equal to the energy released per second and has units of Watts (1 Watt = J/s).

$\text{Eff} = \frac{P_{out}}{P_{in}}$; The efficiency equals the output power divided by the input power

Guidance

Power is the rate at which energy is being transfered. Power tells you how many Joules per second of energy is being used to drive something. Thus power is simply the change in energy divided by the time. Since work is by definition the transfer of energy (in or out of a system), power is also equal to the work divided by the time. Efficiency tells you how efficient something is and gives a number between 0 and 1. If the efficiency is equal to 1, then the machine is perfectly efficient (that is all the power used to drive it goes to the out put of the machine with no energy losses). If the efficiency is zero, then all of the input power is lost in the machine and the machine can not output any energy.

Example 1



Question: A pile driver lifts a 500 kg mass a vertical distance of 20 m in 1.1 sec. It uses 225 kW of supplied power to do this.

a) How much work was done by the pile driver?

b) How much power was used in actually lifting the mass?

c) What is the efficiency of the machine? (This is the ratio of power used to power supplied.)

d) The mass is dropped on a pile and falls 20 m. If it loses 40,000 J on the way down to the ground due to air resistance, what is its speed when it hits the pile?

Answer:

a) We will use the equation for work (which gives us the amount of energy transfered) and plug in the known values to get the amount of work done by the pile driver.

$$W = Fd = mgd = 500\text{kg} \times 9.8\text{m/s}^2 \times 20\text{m} = 9.8 \times 10^4\text{J}$$

b) We will use the power equation and plug in the known values and then convert to kW at the end.

$$P = \frac{W}{\Delta t} = \frac{9.8 \times 10^4\text{J}}{1.1\text{s}} = 89000\text{W} \times \frac{1\text{kW}}{1000\text{W}} = 89\text{kW}$$

c) Efficiency is defined as the Power out divided by the Power in. Thus, this is simply a division problem.

$$Eff = \frac{\text{power used}}{\text{power supplied}} = \frac{89\text{kW}}{225\text{kW}} = .40$$

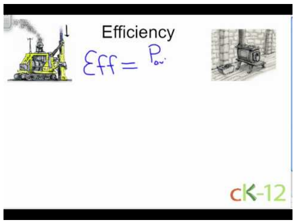
d) We have already solved for the amount of energy the mass has after the pile driver performs work on it (it has 9.8×10^4J). If on the way down it loses 40000J due to air resistance, then it effectively has

$$98000\text{J} - 40000\text{J} = 58000\text{J}$$

of energy. So we will set the kinetic energy equation equal to the total energy and solve for v. This will give us the velocity of the mass when it hits the ground because right before the mass hits the ground, all of the potential energy will have been converted into kinetic energy.

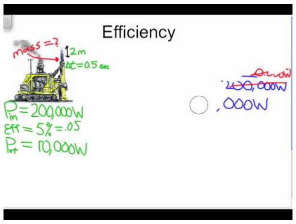
$$58000\text{J} = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{58000\text{J} \times 2}{m}} = \sqrt{\frac{58000\text{J} \times 2}{500\text{kg}}} = 15.2\text{m/s}$$

Watch this Explanation



MEDIA

Click image to the left for more content.



MEDIA

Click image to the left for more content.

Time for Practice

1. Before a run, you eat an apple with 1,000,000 Joules of binding energy.
    a. 550,000 Joules of binding energy are wasted during digestion. How much remains?
    b. Some 95% of the remaining energy is used for the basic processes in your body (which is why you can warm a bed at night!). How much is available for running?
    c. Let#8217;s say that, when you run, you lose 25% of your energy overcoming friction and air resistance. How much is available for conversion to kinetic energy?
    d. Let#8217;s say your mass is 75 kg. What could be your top speed under these idealized circumstances?
    e. But only 10% of the available energy goes to KE, another 50% goes into heat exhaust from your body. Now you come upon a hill if the remaining energy is converted to gravitational potential energy. How high do you climb before running out of energy completely?
2. A pile driver#8217;s motor expends 310,000 Joules of energy to lift a 5400 kg mass. The motor is rated at an efficiency of 0.13. How high is the mass lifted?
3. 15. A 1500 kg car starts at rest and speeds up to 3.0 m/s with a constant acceleration.
    a. What is the car#8217;s gain in kinetic energy?
    b. What power is exerted by the engine?
    c. We define efficiency as the ratio of output energy (in this case kinetic energy) to input energy. If this car#8217;s efficiency is 0.30, how much input energy was provided by the gasoline?
    d. If 0.00015 gallons were used up in the process, what is the energy content of the gasoline in Joules per gallon?
    e. Compare that energy to the food energy in a gallon of Coke, if a 12-oz can contains 150 Calories (food calories) and one gallon is 128 ounces.

Answers to Selected Problems

1. a. 450,000 J b. 22,500 J c. 5,625 J d. 21.2 m/s e. 9.18 m
2. 0.76 m
3. a. 6750 J b. 5.6 kW c. 22.5 kJ d. 150 MJ/gallon of gas e. 6.7 MJ/gallon Coke

9.6 Conservation of Mechanical Energy

Students will learn how to analyze and solve more complicated problems involving energy conservation.

Key Equations

$\sum E_{\text{initial}} = \sum E$

final ;$Thetotalenergydoesnotchangeinclosedsystems$

$\Delta E_T = 0\ J$; The total energy does not change in closed systems

$W_{nc} = \Delta E_T$; Work done by a non-conservative force

$E_k = \frac{1}{2}\,mv^2$; Kinetic energy

$E_g = mgh$; Potential energy of gravity

$E_e = \frac{1}{2}\,kx^2$; Potential energy of a spring

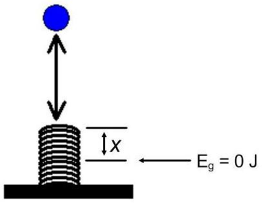
$W = F_x \Delta x = F_{\|} d$; Work is equal to the distance multiplied by the component of the force in the direction it is moving.

Guidance

The main thing to always keep prescient in your mind is that the total energy before must equal the total energy after. If some energy has transferred out of or into the system via work, you calculate that work done and include it in the energy sum equation. Generally work done by friction is listed on the 'after' side and work put into the system, via a jet pack for example, goes on the 'before' side. Another important point is that on turns or going over hills or in rollercoaster loops, one must include the centripetal motion equations -for example to insure that you have enough speed to make the loop.

Example - Falling on a Spring

A 100 kg elevator is moving downward at 3.00 m/s, when the cable snaps. The car falls 4.00 m onto a huge spring with a k-value of 8000 N/m. By how much will the spring be compressed when the car reaches zero velocity. Assume only conservative forces in the problem.



$$
\begin{aligned}
&\Delta E_T = 0\\
&\Delta E_k + \Delta E_g + \Delta E_e = 0\\
&(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2) + (mgh_f - mgh_o) + (\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_o^2) = 0\\
&(0 - \tfrac{1}{2}(100)(3)^2) + (0 - (100)(9.81)(4 + x)) + (\tfrac{1}{2}(8000)(x)^2 - 0) = 0\\
&-450 - 3924 - 981x + 4000x^2 = 0\\
&x^2 - 0.24525x - 1.0935 = 0\\
&x = \frac{-(-0.24525) \pm \sqrt{(-0.24525)^2 + 4(1)(1.0935)}}{2(1)}\\
&x = \frac{0.24525 \pm 2.102}{2}\\
&x = 1.17\ m \quad \text{or} \quad -0.9285\\
&x = 1.17\ m
\end{aligned}
$$

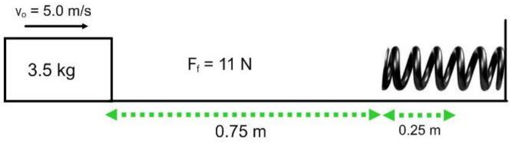
Example - Work and Friction

A 0.75 kg mass, initiall at rest, is pulled by an applied force of 8.5 N a distance of 12 m. If the final velocity is 9.5 m/s, calculate the force of friction.

m%2Fs%5C%5C%0Av_1%3D%20%5Ctext%7B%20speed%20if%20no%20friction%7D%5C%5C%20%5C%5C%0A%5Ctex

Example - Conservation of Mechanical Energy

A 3.5 kg block is 0.75 m from a spring (see diagram below) and initially moving with a velocity of 5.0 m/s. The force of friction is a constant 11 N and the block compresses the spring 0.25 m before changing direction.



a) Calculate the k-value of the spring.

epsck12_math%5C%5C%0AW_%7Bnc%7D%3D-F_fd%5C%5C%0A-F_fd%3D%5CDelta%20E_k%20%2B%20%5CDelta%

b) How fast will the mass be moving as it leaves the spring?

m%2Fs%3Dv_f

Example - Work and Circular Motion

A block starts from rest and is subjected to a force over a distance of 5.2 m. After the object is free of the above force, a 375 N force can turn the object through a 90^o corner with a radius of 0.75 m. Calculate the initial force.

$$W = F_o d = \Delta E_k$$
$$F_o d = E_{kf} - E_{ko}$$

$$F_o d = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_o^2$$

$$F_o d = \tfrac{1}{2}mv_f^2; \quad v_o = 0\ m/s$$

We need an expression for v_f

$$F_c = \frac{mv_f^2}{r}$$

$$v_f^2 = \frac{F_c r}{m}$$

$$F_o d = \tfrac{1}{2}m(\tfrac{F_c r}{m})$$

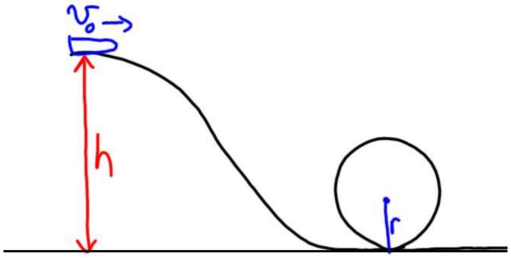
$$F_o d = \frac{F_c r}{2} \quad \text{masses divide out}$$

$$F_o(5.2) = \frac{(375)(0.75)}{2}$$

$$F_o = 27\ N$$

Example - Work and Loops

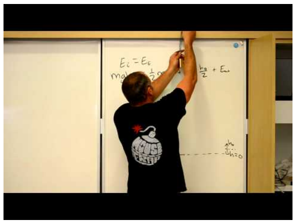
A frictionless roller coaster is made with one circular loop as shown below. If the car starts from rest, what is the minimum height of the roller coaster for the car to survive a 10 m radius loop?



Determine the minimum velocity for survive the loop:
$N+W = \frac{mv^2}{r}$; W is the weight and N = 0 N at minimum speed
$mg = \frac{mv^2}{r}$
$rg = v^2$
$v_f = \sqrt{rg} = \sqrt{(10)(9.81)} = 9.9\ m/s$
above is the final velocity used in our energy equations

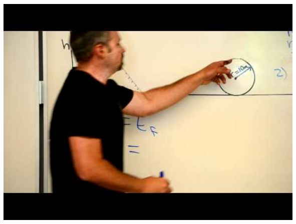
$\Delta E_T = 0$
$\Delta E_k + \Delta E_g = 0$
$(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2) + (mgh_f - mgh_o) = 0$
$v_o = 0\ m/s$ and the mass divides out:
$\frac{1}{2}v_f^2 + gh_f - gh_o = 0$
$\frac{1}{2}(9.9)^2 + (9.81)(20) - (9.81)h_o = 0$
$49.05 + 196.2 = 9.81h_o$
$25\ m = h_o$

Example



MEDIA
Click image to the left for more content.

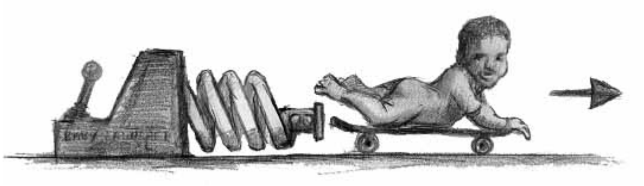
Watch this Explanation



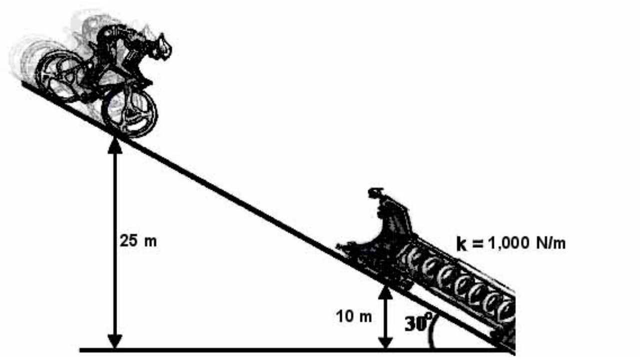
MEDIA
Click image to the left for more content.

Time for Practice

1. A rock with mass m is dropped from a cliff of height h. What is its speed when it gets to the bottom of the cliff?
   a. $\sqrt{mg}$
   b. $2gh$
   c. $\sqrt{2gh}$
   d. gh
   e. None of the above



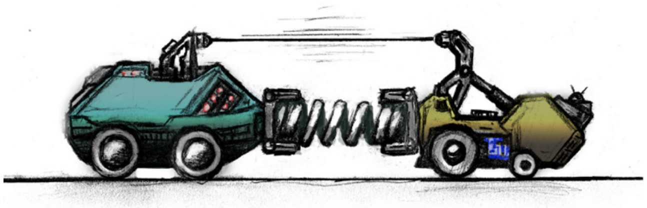
2. In the picture above, a 9.0 kg baby on a skateboard is about to be launched horizontally. The spring constant is 300 N/m and the spring is compressed 0.4 m. For the following questions, ignore the small energy loss due to the friction in the wheels of the skateboard and the rotational energy used up to make the wheels spin.
   a. What is the speed of the baby after the spring has reached its uncompressed length?
   b. After being launched, the baby encounters a hill 7 m high. Will the baby make it to the top? If so, what is his speed at the top? If not, how high does he make it?
   c. Are you finally convinced that your authors have lost their minds? Look at that picture!



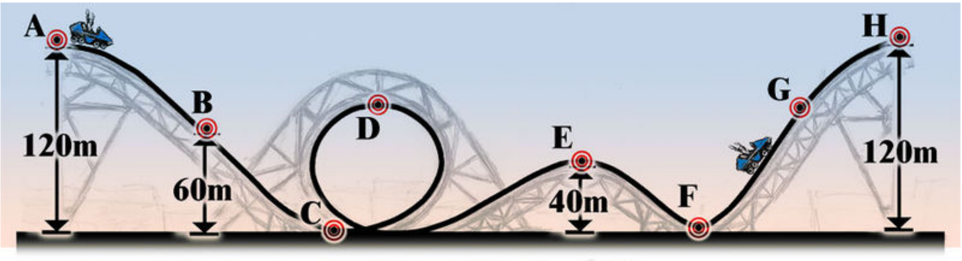
3. When the biker is at the top of the ramp shown above, he has a speed of 10 m/s and is at a height of 25 m. The bike and person have a total mass of 100 kg. He speeds into the contraption at the end of the ramp, which slows him to a stop.
   a. What is his initial total energy? (Hint: Set $U_g = 0$ at the very bottom of the ramp.)
   b. What is the length of the spring when it is maximally compressed by the biker? (Hint: The spring does *not* compress all the way to the ground so there is still some gravitational potential energy. It will help to draw some triangles.)
4. An elevator in an old apartment building in Switzerland has four huge springs at the bottom of the shaft to cushion its fall in case the cable breaks. The springs have an uncompressed height of about 1meter. Estimate the spring constant necessary to stop this elevator, following these steps:
   a. First, guesstimate the mass of the elevator with a few passengers inside.
   b. Now, estimate the height of a five-story building.
   c. Lastly, use conservation of energy to estimate the spring constant.
5. You are skiing down a hill. You start at rest at a height 120 m above the bottom. The slope has a 10.0° grade. Assume the total mass of skier and equipment is 75.0 kg.

a. Ignore all energy losses due to friction. What is your speed at the bottom?
b. If, however, you just make it to the bottom with zero speed what would be the average force of friction, including air resistance?

6. Two horrific contraptions on frictionless wheels are compressing a spring ($k = 400 \text{ N/m}$) by 0.5 m compared to its uncompressed (equilibrium) length. Each of the 500 kg vehicles is stationary and they are connected by a string. The string is cut! Find the speeds of the vehicles once they lose contact with the spring.



7. A roller coaster begins at rest 120 m above the ground, as shown. Assume no friction from the wheels and air, and that no energy is lost to heat, sound, and so on. The radius of the loop is 40 m.



a. If the height at point G is 76 m, then how fast is the coaster going at point G?
b. Does the coaster actually make it through the loop without falling? (Hint: You might review the material from centripetal motion lessons to answer this part.)

Answers to Selected Problems

1. .
2. a. 2.3 m/s c. No, the baby will not clear the hill.
3. a. 29,500 J b. Spring has maximum compressed length of 13 m
4. .
5. a. 48.5 m/s b. 128 N
6. 0.32 m/s each
7. a.29 m/s b. just barely, $a_C = 9.8 \text{ m/s}^2$

CHAPTER 10 Electrostatics and Electric Current

Chapter Outline

10.1 Electrostatics

Students will learn the inner workings of electrostatics: Why certain objects repulse, why certain objects attract and how to calculate the number of excess electrons or protons in an object.

Key Equations

$q = Ne$ Any object's charge is an integer multiple of an electron's charge.

Guidance

- Opposite charges attract and like charges repulse
- The electron (and proton) is the fundamental charge unit. The charge of an electron and proton is 1.6×10^{-19} C. One can determine the number of excess electrons (or protons if positive charge) by dividing the objects charge by the fundamental charge.
- Most objects are electrically neutral (equal numbers of electrons and protons) and that's why gravity dominates on a macro scale.

Example 1

Question If an object has +0.003 C of charge, how many excess protons does the object have?

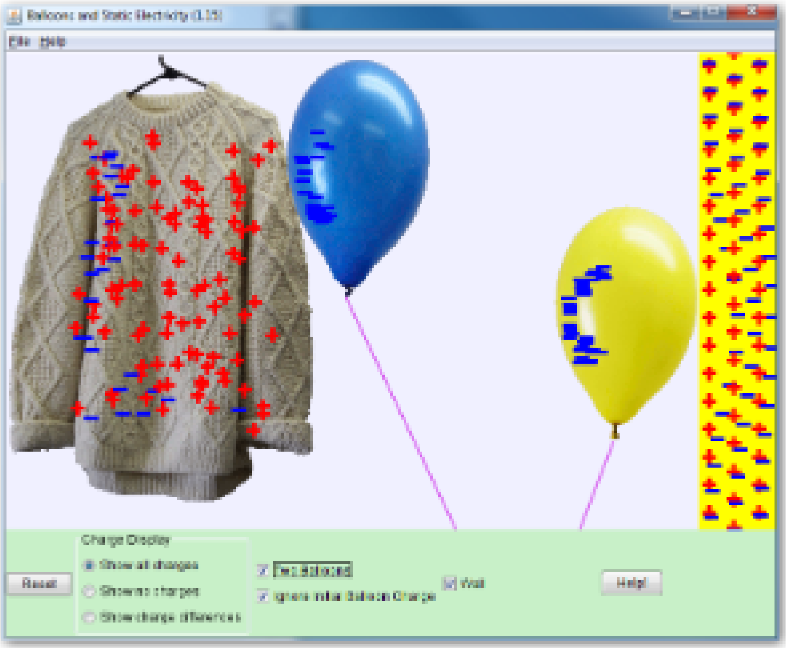
Answer $q = Ne$ $0.003\text{C} = N \times 1.6 \times 10^{-19}$ N = 1.875×10^{16} ; protons

Watch this Explanation

MEDIA

Click image to the left for more content.

Simulation



Balloons(PhET Simulation)

Time for Practice

1. After sliding your feet across the rug, you touch the sink faucet and get shocked. Explain what is happening.
2. What is the net charge of the universe? Of your toaster?
3. As you slide your feet along the carpet, you pick up a net charge of +4 mC. Which of the following is true?
   a. You have an excess of 2.5×10^{16} electrons
   b. You have an excess of 2.5×10^{19} electrons
   c. You have an excess of 2.5×10^{16} protons
   d. You have an excess of 2.5×10^{19} protons
4. You rub a glass rod with a piece of fur. If the rod now has a charge of $-0.6\ \mu C$, how many electrons have been added to the rod?
   a. 3.75×10^{18}
   b. 3.75×10^{12}
   c. 6000
   d. 6.00×10^{12}
   e. Not enough information

10.2 Coulomb's Law

Students will learn how to solve problems involving Coulomb's Law.

Key Equations

$$F = \frac{kq_1q_2}{r^2}\begin{cases} k & \text{The electric constant, } k = 8.987 \times 10^9 \text{ N} \cdot \text{m}^2\text{C}^{-2}. \\ q_1,\ q_2 & \text{Magnitude of the charges, units of Coulombs (C)} \\ r & \text{Distance between charges, m} \end{cases}$$

Guidance

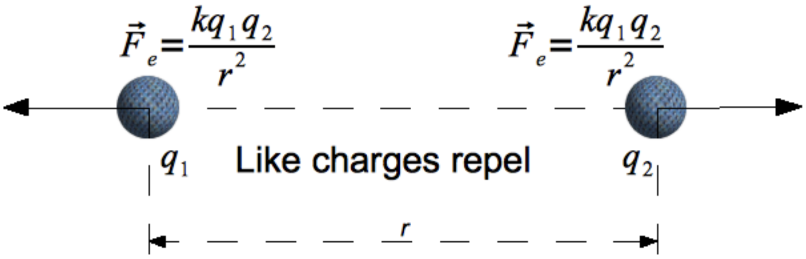
The Coulomb Force Law states that any two charged particles (q_1, q_2) — with charge measured in units of Coulombs — at a distance r from each other will experience a force of repulsion or attraction along the line joining them equal to:

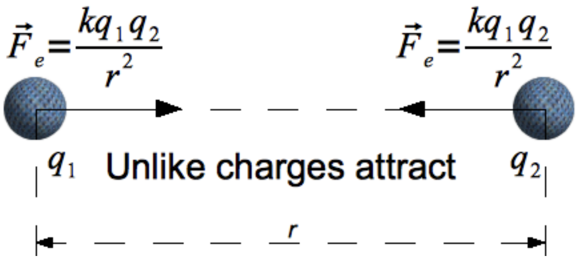
$$\vec{F}_e = \frac{kq_1q_2}{r^2} \qquad \text{The Coulomb Force [1]}$$

Where

$$k = 8.987 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \qquad \text{The Electric Constant}$$

This looks a lot like the Law of Universal Gravitation, which deals with attraction between objects with mass. The big difference is that while any two masses experience mutual *attraction*, two charges can either attract or repel each other, depending on whether the signs of their charges are alike:

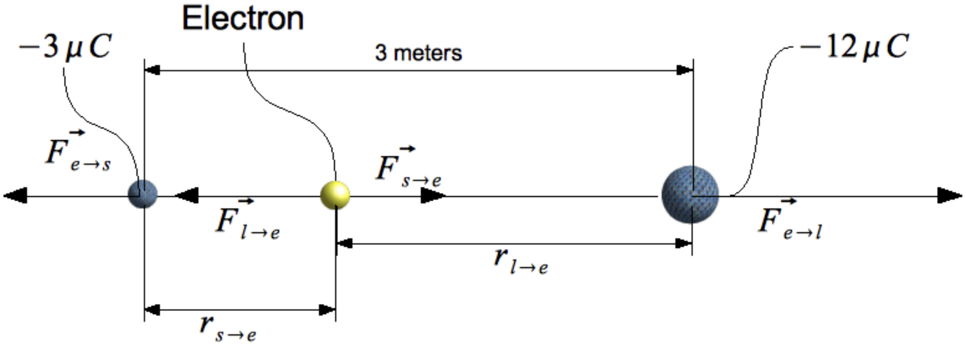




Like gravitational (and all other) forces, Coulomb forces add as vectors. Thus to find the force on a charge from an arrangement of charges, one needs to find the vector sum of the force from each charge in the arrangement.

Example 1

Question: Two negatively charged spheres (one with -12μC; the other with -3μC) are 3m apart. Where could you place an electron so that it will be suspended in space between them with a net force of zero (for this problem we will ignore the force of repulsion between the two charges because they are held in place)?



Answer: Consider the diagram above; here $r_{s\to e}$ is the distance between the electron and the small charge, while $\vec{F}_{s\to e}$ is the force the electron feels due to it. For the electron to be balanced in between the two charges, the forces of repulsion caused by the two charges on the electron would have to be balanced. To do this, we will set the equation for the force exerted by two charges on each other equal and solve for a distance ratio. We will denote the difference between the charges through the subscripts "s" for the smaller charge, "e" for the electron, and "l" for the larger charge.

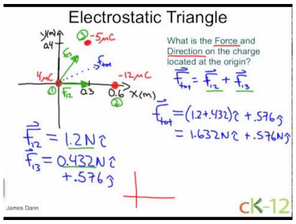
$$\frac{kq_sq_e}{r^2_{s\to e}} = \frac{kq_lq_e}{r^2_{e\to l}}$$

Now we can cancel. The charge of the electron cancels. The constant k also cancels. We can then replace the large and small charges with the numbers. This leaves us with the distances. We can then manipulate the equation to produce a ratio of the distances.

$$\frac{-3\mu\text{C}}{r^2_{s\to e}} = \frac{-12\mu\text{C}}{r^2_{e\to l}} \Rightarrow \frac{r^2_{s\to e}}{r^2_{e\to l}} = \frac{-12\mu\text{C}}{-3\mu\text{C}} \Rightarrow \frac{r_{s\to e}}{r_{e\to l}} = \sqrt{\frac{1\mu\text{C}}{4\mu\text{C}}} = \frac{1}{2}$$

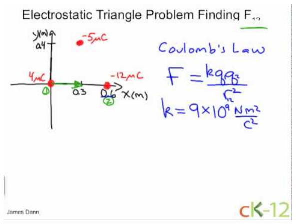
Given this ratio, we know that the electron is twice as far from the large charge (-12μC) as from the small charge (-3μC). Given that the distance between the small and large charges is 3m, we can determine that the electron must be located 2m away from the large charge and 1m away from the smaller charge.

Watch this Explanation



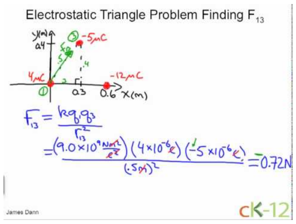
MEDIA

Click image to the left for more content.



MEDIA

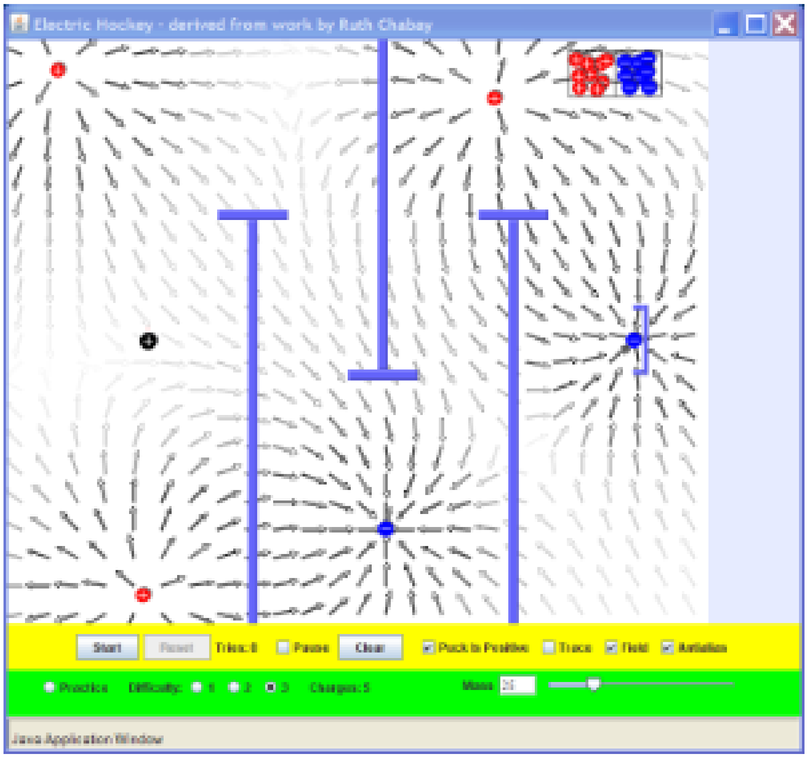
Click image to the left for more content.



MEDIA

Click image to the left for more content.

Simulation



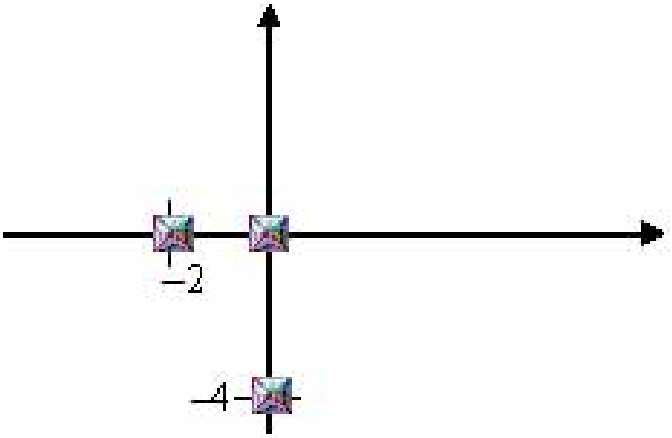
ElectricHockey (PhET Simulation)

Time for Practice

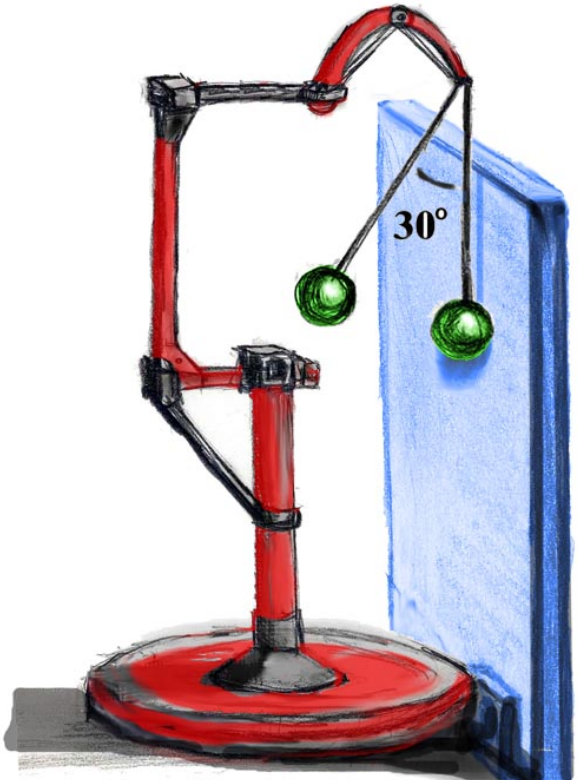
1. A suspended pith ball possessing $+10\ \mu$C of charge is placed 0.02 m away from a metal plate possessing $-6\ \mu$Cof charge.
   a. Are these objects attracted or repulsed?
   b. What is the force on the negatively charged object?

   c. What is the force on the positively charged object?

2. Consider the hydrogen atom. Does the electron orbit the proton due to the force of gravity or the electric force? Calculate both forces and compare them. (You may need to look up the properties of the hydrogen atom to complete this problem.)
3. Find the direction and magnitude of the force on the charge at the origin (see picture). The object at the origin has a charge of 8 μC, the object at coordinates $(-2\text{ m},\ 0)$ has a charge of 12 μC, and the object at coordinates $(0, -4\text{ m})$ has a charge of 44 μC. All distance units are in meters.



4. Two pith balls of equal and like charges are repulsed from each other as shown in the figure below. They both have a mass of 2 g and are separated by 30°. One is hanging freely from a 0.5 m string, while the other, also hanging from a 0.5 mstring, is stuck like putty to the wall.
   a. Draw the free body diagram for the hanging pith ball
   b. Find the distance between the leftmost pith ball and the wall (this will involve working a geometry problem)
   c. Find the tension in the string (Hint: use $y-$direction force balance)
   d. Find the amount of charge on the pith balls (Hint: use $x-$direction force balance)



Answers

1. a. attracted b. 1350 N c. 1350 N
2. $F_g = 1.0 \times 10^{-47}$ N and $F_e = 2.3 \times 10^{-8}$ N. The electric force is 39 orders of magnitudes bigger.
3. 0.293 N and at 42.5°
4. b. 0.25m c. $F_T = 0.022$ N d. 0.37μC

10.3 Electric Fields

Students will learn what an electric field is, how to draw electric field lines and how to calculate electric fields. They will also learn to apply the electric field in order to find the direction a charge would move upon entering the field and the force on it.

Key Equations

$E = \frac{kq}{r^2}$; Electric field due to charge q at a distance r from the charge.

$F = qE$; Force due to an electric field.

Guidance

Gravity and the Coulomb force have a nice property in common: they can be represented by **fields**. Fields are a kind of bookkeeping tool used to keep track of forces. Take the electromagnetic force between two charges given above:

$$\vec{F}_e = \frac{kq_1q_2}{r^2}$$

If we are interested in the acceleration of the first charge only — due to the force from the second charge — we can rewrite this force as the product of q_1 and $\frac{kq_2}{r^2}$. The first part of this product only depends on properties of the object we're interested in (the first charge), and the second part can be thought of as a property of the point in space where that object is.

In fact, the quantity $\frac{kq_2}{r^2}$ captures everything about the electromagnetic force on any object possible at a distance **r** from q_2. If we had replaced q_1 with a different charge, q_3, we would simply multiply q_3 by $\frac{kq_2}{r^2}$ to find the new force on the new charge. Such a quantity, $\frac{kq_2}{r^2}$ here, is referred to as the electric field from charge q_2 at that point: in this case, it is the electric field due to a single charge:

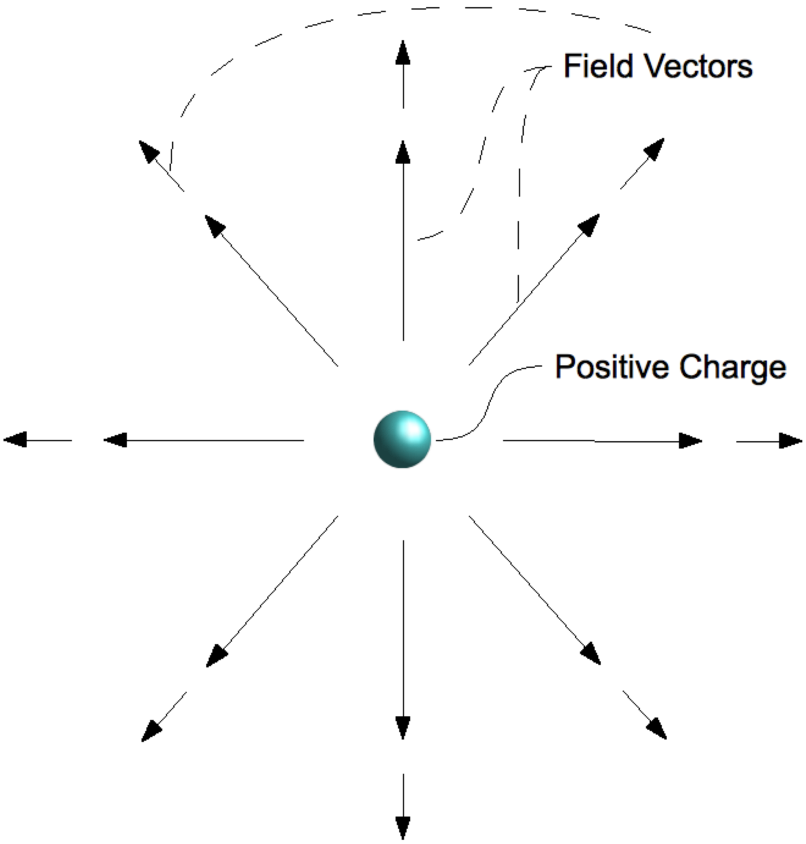
$$\vec{E}_f = \frac{kq}{r^2}$$

The electric field is a vector quantity, and points in the direction that a force felt by a positive charge at that point would. If we are given the electric field at some point, it is just a matter of multiplication — as illustrated above — to find the force any charge q_0 would feel at that point:

$$\underbrace{\vec{F}_e}_{\text{Force on charge } q_0} = \underbrace{\vec{E}_f}_{\text{Field}} \times \underbrace{q_0}_{\text{Charge}} \qquad \text{Force on charge } q_0 \text{ in an electric field}$$

Note that this is true for *all* electric fields, not just those from point charges. In general, the **electric field** at a point *is the force a* ***positive test charge of magnitude 1*** *would feel at that point.* Any other charge will feel a force along the same line (but possibly in the other direction) in proportion to its magnitude. In other words, the electric field can be thought of as "force per unit charge".

In the case given above, the field was due to a single charge. Such a field is shown in the figure below. Notice that this a field due to a positive charge, since the field arrows are pointing outward. The field produced by a point charge will be radially symmetric i.e., the strength of the field only depends on the distance, r, from the charge, not the direction; the lengths of the arrows represent the strength of the field.



Example 1

Question: Calculate the electric field a distance of 4.0mm away from a -2.0μC charge. Then, calculate the force on a -8.0μC charge placed at this point.

Answer: To calculate the electric field we will use the equation

$$E = \frac{kq}{r^2}$$

Before we solve for the electric field by plugging in the values, we convert all of the values to the same units.

$$4.0\text{mm} \times \frac{1\text{m}}{1000\text{mm}} = .004\text{m}$$
$$-2.0\mu\text{C} \times \frac{1\text{C}}{1000000\mu\text{C}} = -2.0 \times 10^{-6}\text{C}$$

Now that we have consistent units we can solve the problem.

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \text{Nm}^2/\text{C}^2 \times -2.0 \times 10^{-6}\text{C}}{(.004\text{m})^2} = -1.1 \times 10^9 \text{N/C}$$

To solve for the force at the point we will use the equation

$$F = Eq$$

. We already know all of the values so all we have to do is convert all of the values to the same units and then plug in the values.

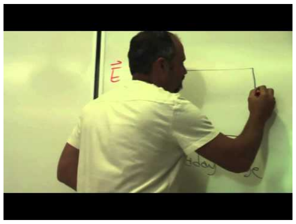
$$-8.0\mu\text{C} \times \frac{1\text{C}}{1000000\mu\text{C}} = -8.0 \times 10^{-6}\text{C}$$

$$F = Eq = -8.0 \times 10^{-6}\text{C} \times -1.1 \times 10^{9}\text{N/C} = 9000\text{N}$$

Watch this Explanation

MEDIA

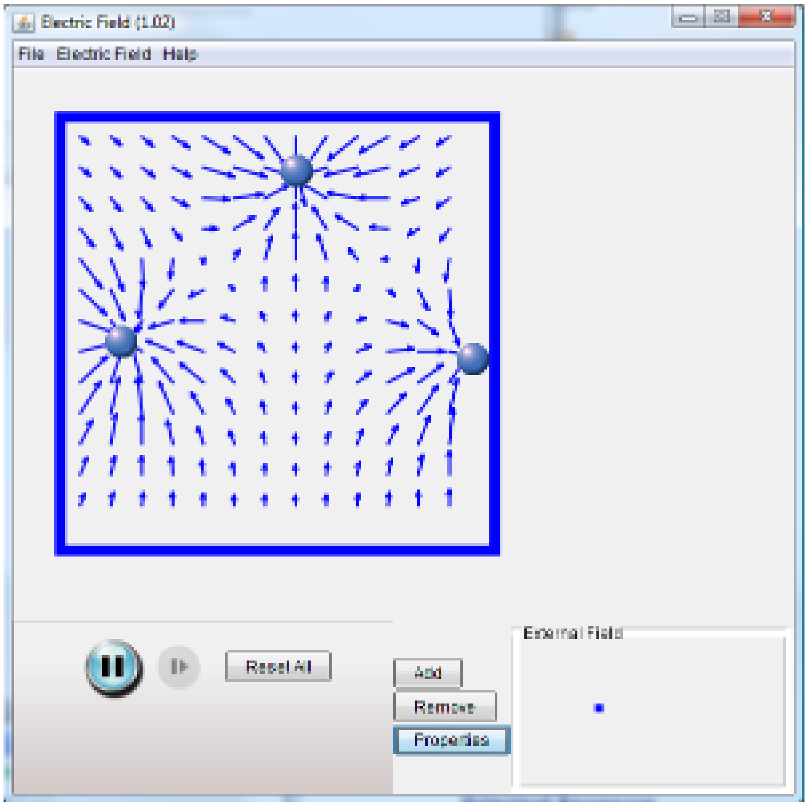
Click image to the left for more content.



MEDIA

Click image to the left for more content.

Simulation

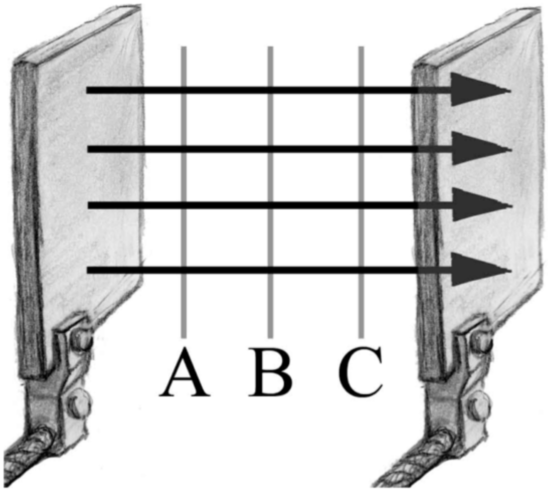


ElectricField of Dreams(PhET Simulation)

Time for Practice

1. What is the direction of the electric field if an electron initially at rest begins to move in the North direction as a result of the field?

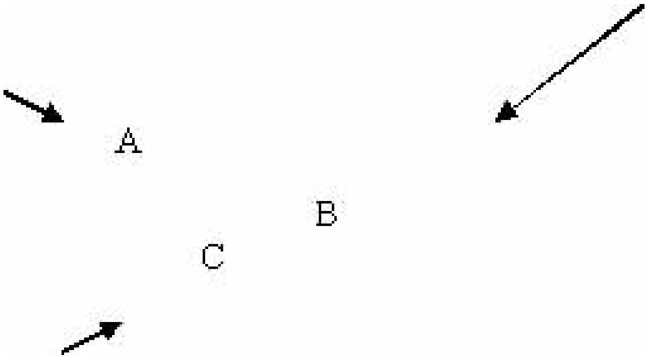
   a. North
   b. East
   c. West
   d. South
   e. Not enough information



2. Two metal plates have gained excess electrons in differing amounts through the application of rabbit fur. The arrows indicate the direction of the electric field which has resulted. Three electric potential lines, labeled A, B, and $C$are shown. Order them from the greatest electric potential to the least.

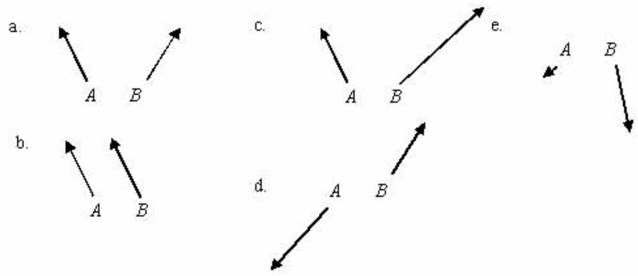
    a. A,B,C
    b. C,B,A
    c. B,A,C
    d. B,C,A
    e. $A = B = C\ldots$ they#8217;re all at the same potential

3. The three arrows shown here represent the magnitudes of the electric field and the directions at the tail end of each arrow. Consider the distribution of charges which would lead to this arrangement of electric fields. Which of the following is most likely to be the case here?



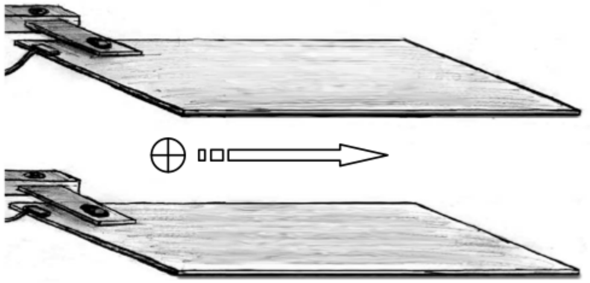
    a. A positive charge is located at point A
    b. A negative charge is located at point B
    c. A positive charge is located at point B and a negative charge is located at point C
    d. A positive charge is located at point A and a negative charge is located at point C
    e. Both answers a) and b) are possible

4. Particles A and B are both positively charged. The arrows shown indicate the direction of the *forces* acting on them due to an applied electric field (not shown in the picture). For each, draw in the electric field lines that would best match the observed force.



5. Calculate the electric field a distance of 4.0 mm away from a $-2.0\ \mu$C charge.
6. Copy the arrangement of charges below. Draw the electric field from the -2 C charge in one color and the electric field from the $+2$ C charge in a different color. Be sure to indicate the directions with arrows. Now take the individual electric field vectors, add them together, and draw the resultant vector. This is the electric field created by the two charges together.

7. A proton traveling to the right moves in between the two large plates. A vertical electric field, pointing downwards with magnitude 3.0 N/C, is produced by the plates.



   a. What is the direction of the force on the proton?
   b. Draw the electric field lines on the diagram.
   c. If the electric field is 3.0 N/C, what is the acceleration of the proton in the region of the plates?
   d. Pretend the force of gravity doesn#8217;t exist; then sketch the path of the proton.
   e. We take this whole setup to another planet. If the proton travels straight through the apparatus without deflecting, what is the acceleration of gravity on this planet?

8. A molecule shown by the square object shown below contains an excess of 100 electrons.

   a. What is the direction of the electric field at point A, 2.0×10^{-9} m away?
   b. What is the value of the electric field at point A?
   c. A molecule of charge $8.0\ \mu$C is placed at point A. What are the magnitude and direction of the force acting on this molecule?

Answers

1. .

2. .
3. .
4. .
5. 1.1×10^9 N/C
6. .
7. a. down b. Up $16c, 5.5 \times 10^{11}$ m/s^2 e. 2.9×10^8 m/s^2
8. a. Toward the object b. 3.6×10^4 N/C to the left c. 2.8×10^{-7} N

10.4 Voltage

Students will learn the concept of Voltage and how to apply it in energy conservation problems.

Key Equations

$E = \frac{-\Delta V}{\Delta x}$ Electric field vs electric potential.

$\Delta E_q = q\Delta V$ Change in potential energy due to travel through changing voltage.

$V = \frac{kq}{r}$ Electric potential of a single charge.

Guidance

Like gravity, the electric force can do work and has a potential energy associated with it. But like we use fields to keep track of electromagnetic forces, we use **electric potential**, or **voltage** to keep track of electric potential energy. So instead of looking for the potential energy of specific objects, we define it in terms of properties of the space where the objects are.

The ***electric potential difference****, or* ***voltage difference*** *(often just called voltage) between two points (A and B) in the presence of an electric field is defined as the work it would take to move a* ***positive test charge of magnitude 1*** *from the first point to the second against the electric force provided by the field.* For any other charge q, then, the relationship between potential difference and work will be:

$$\Delta V_{AB} = \frac{W_{AB}}{q} \quad \text{[4] Electric Potential}$$

Rearranging, we obtain:

$$\underbrace{W}_{\text{Work}} = \underbrace{\Delta V_{AB}}_{\text{Potential Difference}} \times \underbrace{q}_{\text{Charge}}$$

The potential of electric forces to do work corresponds to electric potential energy:

$$\Delta U_{E,AB} = q\Delta V_{AB} \quad \text{[5] Potential energy change due to voltage change}$$

The energy that the object gains or loses when traveling through a potential difference is supplied (or absorbed) by the electric field — there is nothing else there. Therefore, it follows that *electric fields contain energy*.

To summarize: just as an electric field denotes force per unit charge, so electric potential differences represent potential energy differences *per unit charge*. Voltage is by definition the electric potential energy per Coulomb. So it is the electrical potential energy value divided by the charge. Thus, voltage difference is the potential value for potential energy. A 12V battery can not produce energy without charge flowing (i.e. you must connect the two ends). Electric potential is measured in units of Volts (V) #8211; thus electric potential is often referred to as #8220;voltage.#8221; Electric potential is the source of the electric potential energy. You can read the electric potential lines (that is the voltage lines) just like you would a contour map while backpacking in the mountains. Positive charges move towards lower electric potential; negative charges move toward higher electric potential. Thus, positive charges go 'downhill' and negative charges go 'uphill'.

Example 1

You have a negative charge of unknown value and a positive charge of magnitude q_1 and mass m. After fixing the negative charge in place, you place the positive charge a distance r_i away from the negative charge and then release it. If the speed of the positive charge when it is a distance r_f away from the negative charge is v, what was the magnitude of the negative charge in terms of the given values?

Solution

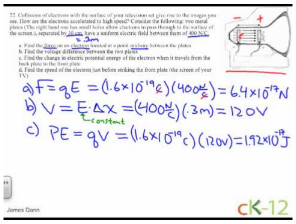
There are multiple ways to do this problem, we will solve it using conservation of energy and the change in voltage to determine the magnitude of the negative charge. When working through this problem, we'll call the positive charge q_1 and the negative charge q_2. To start we'll say that the charge had zero potential energy when it was .5m from the negative charge; this will help us as we work through the problem. Using this assertion, we will apply conservation of energy to the positive charge.

$$\begin{aligned}
\Delta E_q &= \Delta E_k && \text{start with conservation of energy}\\
q_1\Delta V &= \frac{1}{2}mv^2 && \text{substitute the equations for each energy term}\\
\Delta V &= \frac{mv^2}{2q_1} && \text{solve for V}
\end{aligned}$$

Now, since we know the voltage difference, we will express it using the equation for voltage at a certain distance from a point charge.

$$\begin{aligned}
\Delta V &= \frac{kq_2}{\Delta r} && \text{start with the equation for voltage at a certain distance}\\
\Delta V &= \frac{kq_2}{r_f} - \frac{kq_2}{r_i} && \text{express the change in radius in terms of the initial and final radius of the positive charge}\\
\Delta V &= kq_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right) && \text{factor the equation}\\
\frac{mv^2}{2q_1} &= kq_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right) && \text{substitute in the value from the first step}\\
q_2 &= \frac{mv^2}{2q_1k(\frac{1}{r_f} - \frac{1}{r_i})} && \text{solve for } q_2
\end{aligned}$$

Watch this Explanation



22. Collisions of electrons with the surface of your television set give rise to the images you see. How are the electrons accelerated to high speed? Consider the following: two metal plates (The right hand one has small holes allow electrons to pass through to the surface of the screen.), separated by 30 cm, have a uniform electric field between them of 400 N/C.

= .3m

a. Find the force on an electron located at a point midway between the plates
b. Find the voltage difference between the two plates
c. Find the change in electric potential energy of the electron when it travels from the back plate to the front plate
d. Find the speed of the electron just before striking the front plate (the screen of your TV)

a) $F = qE = (1.6\times10^{-19}\cancel{c})(400\frac{N}{\cancel{c}}) = 6.4\times10^{-17}N$

b) $V = E\cdot\Delta x = (400\frac{N}{c})(.3m) = 120V$

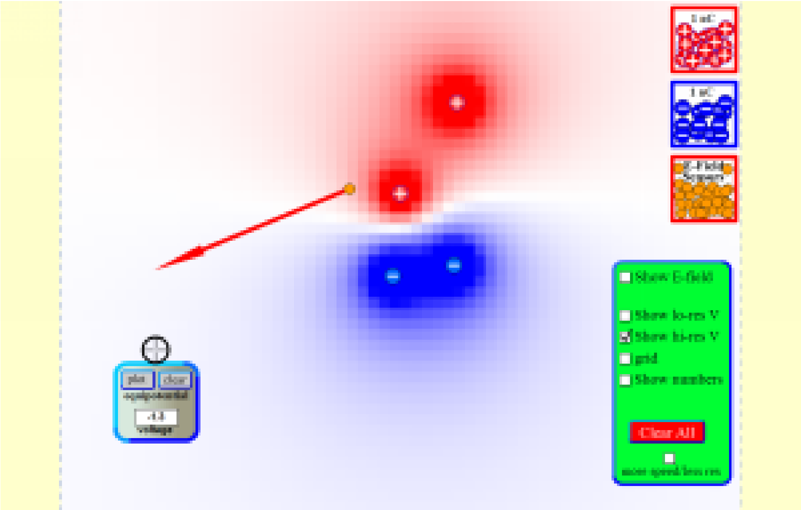
constant

c) $PE = qV = (1.6\times10^{-19}c)(120V) = 1.92\times10^{-17}J$

MEDIA

Click image to the left for more content.

Simulation



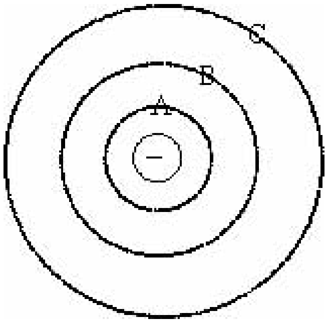
☐ Show E-field
☐ Show lo-res V
☑ Show hi-res V
☐ grid
☐ Show numbers
☐ more speed/less res

Charges and Fields (PhETSimulation)

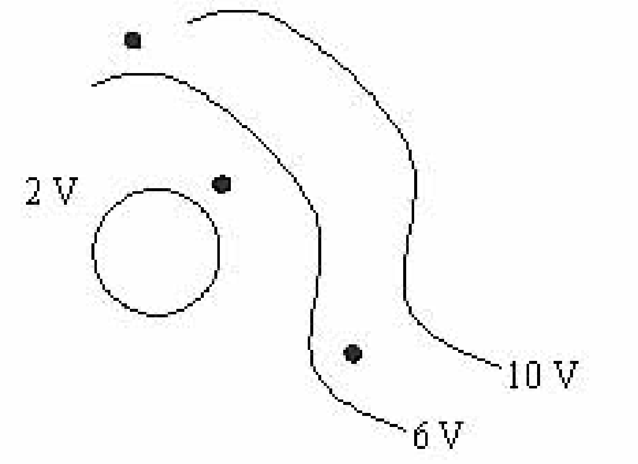
Time for Practice

1. The diagram to the right shows a negatively charged electron. Order the electric potential lines from greatest to least.

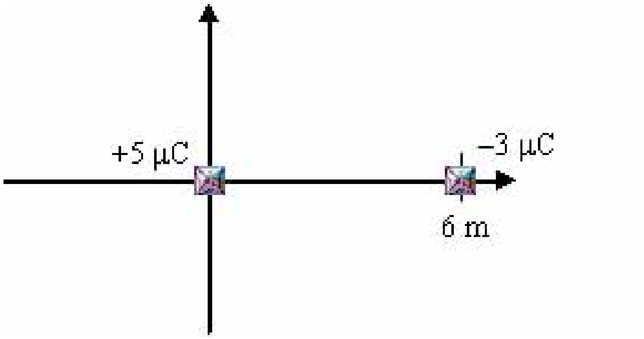
   a. A, B, C
   b. C, B, A
   c. B, A, C
   d. B, C, A
   e. $A = B = C \ldots$ they#8217;re all at the same electric potential



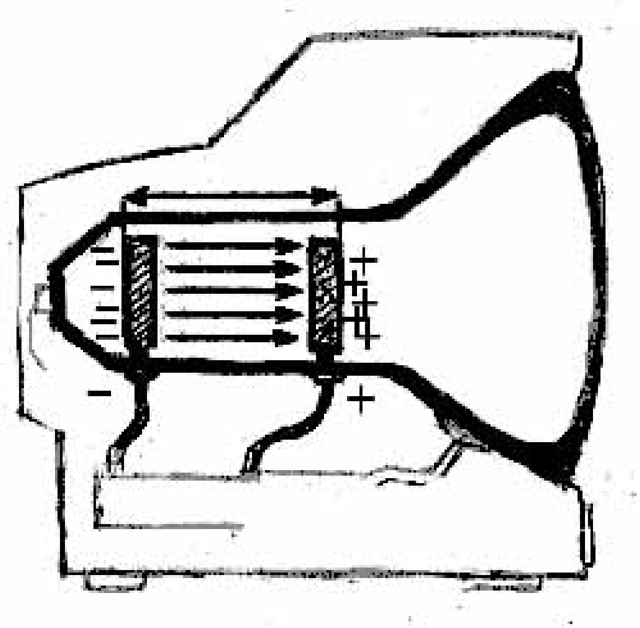
2. Below are the electric potential lines for a certain arrangement of charges. Draw the direction of the electric field for all the black dots.



3. A metal sphere with a net charge of $+5\ \mu C$ and a mass of 400 gis placed at the origin and held fixed there.
    a. Find the electric potential at the coordinate $(6\text{ m}, 0)$.
    b. If another metal sphere of $-3\ \mu C$ charge and mass of 20 g is placed at the coordinate $(6\text{ m}, 0)$ and left free to move, what will its speed be just before it collides with the metal sphere at the origin?



4. Collisions of electrons with the surface of your television set give rise to the images you see. How are the electrons accelerated to high speed? Consider the following: two metal plates (The right hand one has small holes allow electrons to pass through to the surface of the screen.), separated by 30 cm, have a uniform electric field between them of 400 N/C.



a. Find the force on an electron located at a point midway between the plates
b. Find the voltage difference between the two plates
c. Find the change in electric potential energy of the electron when it travels from the back plate to the front plate
d. Find the speed of the electron just before striking the front plate (the screen of your TV)

Answers

1. .
2. .
3. a. 7500V b. 1.5 m/s
4. a. 6.4×10^{-17} N b. 1300V c. 2.1×10^{-16} J d. 2.2×10^{7} m/s

10.5 Voltage and Current

Students will learn the concepts of voltage and current and the relationship between current and charge.

Key Equations

$I = \frac{\Delta q}{\Delta t}$; current is the rate at which charge passes by; the units of current are Amperes $(1\ A = 1\ C/s)$

Guidance

Conductors have an effectively infinite supply of charge, so when they are placed in an electric field, a **separation of charge** occurs. A battery with a potential drop across the ends creates such an electric field; when the ends are connected with a wire, charge will flow across it. The term given to the flow of charge is **electric current**, and it is measured in Amperes (A) — Coulombs per second. Current is analogous to a river of water, but instead of water flowing, charge does.

Current is the number of Coulomb's that flow by per second. Thus 1 Amp of current is equivalent to saying that 1 C of electric charge is passing every second (i.e. the rate of change of charge is 1 C/s).

Voltage is the electrical energy density (energy divided by charge) and differences in this density (voltage) cause electric current in the circuit. **Batteries** and **power supplies** often provide a voltage difference across the ends of a circuit, but other **voltage sources** exist. Using the water analogy that current is a river, then differences in voltage can be thought of as pipes coming out of a water dam at different heights. The lower the pipe along the dam wall, the larger the water pressure, thus the higher the voltage. If that pipe is connected then current will flow. Current will be larger for the pipe with the greatest pressure (i.e. the lowest pipe on the dam wall).

Example 1

Somehow, you are able to see the electrons passing through a wire as current flows. Over the course of 5 seconds, you count $1.5 * 10^{20}$ electrons pass a single point in the wire. How much charge passed you and the current in the wire?

Solution

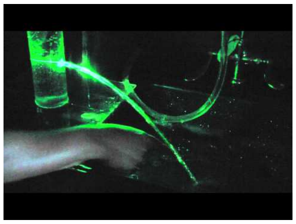
We can use the known charge of an electron to answer the first part of this problem.

$$q = 1.5 * 10^{20}\,\text{electrons} * 1.6 * 10^{-19}\,\text{C/electron}$$
$$q = 24\,\text{C}$$

Now we use the equation above to determine the current

$$I = \frac{\Delta q}{\Delta t}$$
$$I = \frac{24\,\text{C}}{5\,\text{s}}$$
$$I = 4.8\,\text{A}$$

Watch this Explanation

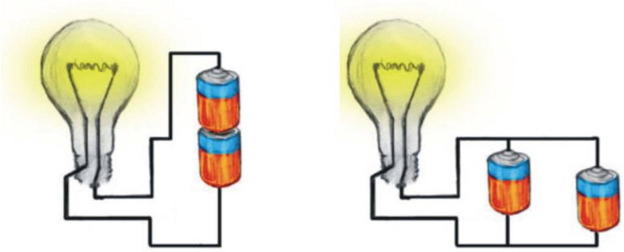


MEDIA

Click image to the left for more content.

Time for Practice

1. The current in a wire is 4.5 A.
   a. How many coulombs per second are going through the wire?
   b. How many electrons per second are going through the wire?
2. Which light bulb will shine brighter? Which light bulb will shine for a longer amount of time? Draw the schematic diagram for both situations. Note that the objects on the right are batteries, not resistors.



Answers to Selected Problems

1. a. 4.5C b. 2.8×10^{19} electrons
2. left = brighter, right = longer

10.6 Ohm's Law

Students will learn a basic understanding of electric circuits and how to apply Ohm's law.

Key Equations

Ohm#8217;s Law

$V = IR$; Voltage drop equals current multiplied by resistance.

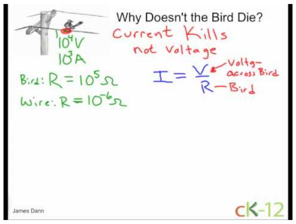
Power

$P = IV$; Power released is equal to the voltage multiplied by the current.

Guidance

- Ohm's Law is the main equation for electric circuits but it is often misused. In order to calculate the voltage drop across a light bulb use the formula: $V_{lightbulb} = I_{lightbulb}R_{lightbulb}$. For the *total* current flowing out of the power source, you need the *total* resistance of the circuit and the *total* voltage: $V_{total} = I_{total}R_{total}$.
- The equations used to calculate the **power** dissipated in a circuit is $P = IV$. As with Ohm#8217;s Law, one must be careful not to mix apples with oranges. If you want the power of the entire circuit, then you multiply the *total* voltage of the power source by the *total* current coming out of the power source. If you want the power dissipated (i.e. released) by a light bulb, then you multiply the *voltage drop* across the light bulb by the *current going through that light bulb.*
- Power is the rate that energy is released. The units for power are Watts (W), which equal Joules per second $[W] = [J]/[s]$. Therefore, a 60 W light bulb releases 60 Joules of energy every second.

Example 1



MEDIA

Click image to the left for more content.

Example 2

A small flash light uses a single AA battery which provides a voltage of 1.5 V. If the bulb has a resistance of 2 Ω, how much power is dissipated by the light bulb.

Solution

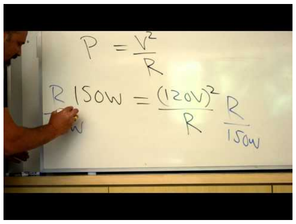
Since the light bulb is the only object in the circuit, we know the voltage drop across the light bulb is equal to that of the battery. Therefore, we can use Ohm's law to solve for the current in the resistor.

$$\begin{aligned} V &= IR \\ I &= \frac{V}{R} \\ I &= \frac{1.5\,\text{V}}{2\,\Omega} \\ I &= .75\,\text{A} \end{aligned}$$

Now we can determine the power of the bulb.

$$\begin{aligned} P &= IV \\ P &= .75\,\text{A} * 1.5\,\text{V} \\ P &= 1.13\,\text{W} \end{aligned}$$

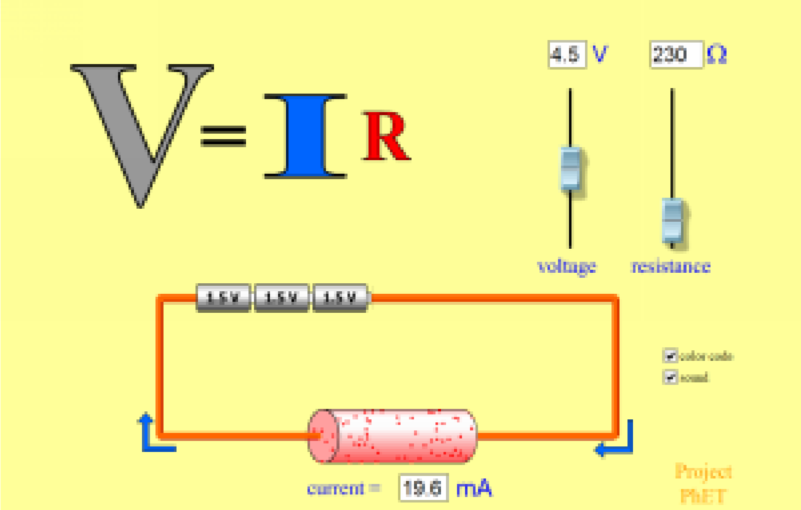
Watch this Explanation



MEDIA

Click image to the left for more content.

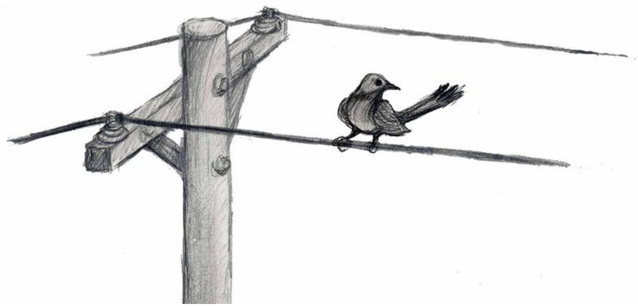
Simulation



Ohm's Law (PhET Simulation)

Time for Practice

1. A light bulb with resistance of 80 Ω is connected to a 9 Vbattery.
   a. What is the electric current going through it?
   b. What is the power (i.e. wattage) dissipated in this light bulb with the 9 V battery?
   c. How many electrons leave the battery every hour?
   d. How many Joules of energy leave the battery every hour?



2. 2. A bird is standing on an electric transmission line carrying 3000 A of current. A wire like this has about 3.0×10^{-5} Ω of resistance per meter. The bird#8217;s feet are 6 cm apart. The bird, itself, has a resistance of about 4×10^{5} Ω.
   a. What voltage does the bird feel?
   b. What current goes through the bird?
   c. What is the power dissipated by the bird?
   d. By how many Joules of energy does the bird heat up every hour?
3. 3. A 120 V, 75 Wlight bulb is shining in your room and you ask yourself#8230;
   a. What is the resistance of the light bulb?
   b. How bright would it shine with a 9 V battery (i.e. what is its power output)?
4. 4. Students measure an unknown resistor and list their results in the **Table** (below); based on their results, complete the following:
   a. Show a circuit diagram with the connections to the power supply, ammeter and voltmeter.
   b. Graph voltage vs. current; find the best-fit straight line.
   c. Use this line to determine the resistance.
   d. How confident can you be of the results?
   e. Use the graph to determine the current if the voltage were 13 V.

TABLE 10.1: **Student measurements of an unknown resistor**

Voltage (v)	Current (a)
15	.11
12	.08
10	.068
8	.052
6	.04
4	.025
2	.01

5. 5. A certain 48-V electric forklift can lift up to 7000 lb at a maximum rate of 76 ft/min.

  a. What is its power?
  b. What current must the battery produce to achieve this power?

Answers to Selected Problems

1. a. 0.11 A b. 1.0 W c. 2.5×10^{21} electrons d. 3636 W
2. a. 5.4 mV b. 1.4×10^{-8} A c. 7.3×10^{-11} W, not a lot d. 2.6×10^{-7} J
3. a. 192 Ω b. 0.42 W
4. .
5. a. 12300 W b. 256 A

10.7 Internal Resistance

Students will learn the difference between Emf and Voltage (i.e. ideal voltage and the output voltage) and how to calculate the internal resistance of a battery.

Key Equations

$V_{terminal} = Emf - Ir$

The terminal voltage (or 'output voltage') is equal to the emf (it's 'ideal voltage') minus the voltage drop across the internal resistance.

Guidance

A battery is a voltage source. A battery can be thought of as a perfect voltage source with a small resistor (called internal resistance) in series. The electric energy density produced by the chemistry of the battery is called **emf,** but the amount of voltage available from the battery is called **terminal voltage.** The terminal voltage equals the emf minus the voltage drop across the internal resistance (current of the external circuit times the internal resistance). In practice, if you short circuit a battery and measure its voltage you will see the voltage is less than what is marked on it and what it can produce when outputting smaller currents. The short circuit of the battery, makes it pump out a lot of current and then the voltage drop over the internal resistance gets large (**V = Ir**) which in turn reduces the terminal voltage.

Example 1

You have a battery with an EMF of 5 V and an unknown internal resistance. You hook the battery up to a circuit with one 3 Ω resistor and measure the current through the resistor to be 1.5 A. What is the internal resistance of the batter and how much power is the battery's resistance dissipating.

Solution

To start this problem we'll first find the terminal voltage of the battery using the information we know about the resistor. We know the voltage drop across the resistor must be equal to the terminal voltage because there the total change in voltage must be over the whole circuit.

$$\begin{aligned}
V &= IR\\
V &= 1.5\,\text{A} * 3\,\Omega\\
V &= 4.5\,\text{V}
\end{aligned}$$

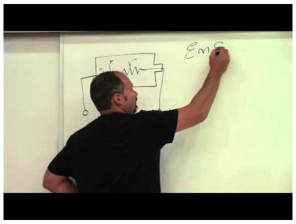
We can plug this value into the equation given above to find the internal resistance.

$$\begin{aligned}V &= Emf - Ir\\ r &= \frac{Emf - V}{I}\\ r &= \frac{5\,\text{V} - 4.5\,\text{V}}{1.5\,\text{A}}\\ r &= .33\,\Omega\end{aligned}$$

Now we can find find the power dissipated by the resistor

$$\begin{aligned}P &= IV\\ P &= I(Ir)\\ P &= I^2 r\\ P &= (1.5\,\text{A})^2 * .33\,\Omega\\ P &= .75\,\text{W}\end{aligned}$$

Watch this Explanation

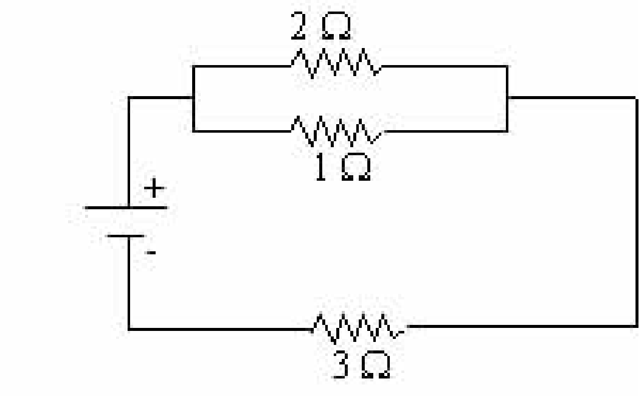


MEDIA

Click image to the left for more content.

Time for Practice

1. In the circuit shown here, the battery produces an *emf* of 1.5 V and has an internal resistance of 0.5 Ω.



   a. Find the total resistance of the external circuit.
   b. Find the current drawn from the battery.
   c. Determine the terminal voltage of the battery

   d. Show the proper connection of an ammeter and a voltmeter that could measure voltage across and current through the 2 Ω resistor. What measurements would these instruments read?

2. Students are now measuring the terminal voltage of a battery hooked up to an external circuit. They change the external circuit four times and develop the **Table** (10.2); using this data, complete the following:
   a. Graph this data, with the voltage on the vertical axis.
   b. Use the graph to determine the emf of the battery.
   c. Use the graph to determine the internal resistance of the battery.
   d. What voltage would the battery read if it were not hooked up to an external circuit?

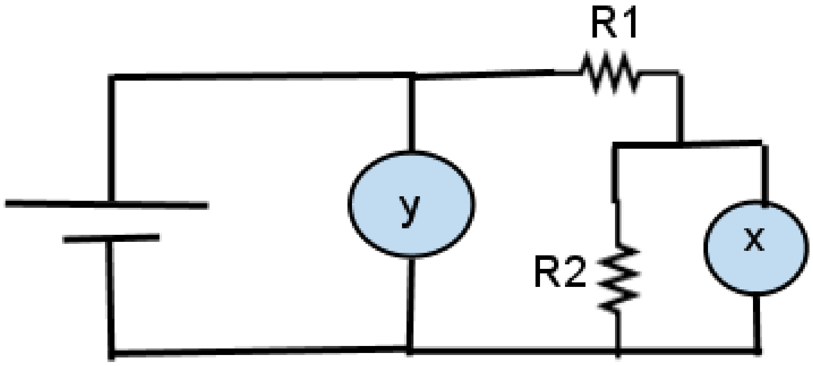
TABLE 10.2:

Terminal Voltage (v)	Current (a)
14.63	.15
14.13	.35
13.62	.55
12.88	.85

3. You have a battery with an emf of 12 V and an internal resistance of 1.00 Ω. Some 2.00 Aare drawn from the external circuit.
   a. What is the terminal voltage
   b. The external circuit consists of device X, 0.5 A and 6 V; device Y, 0.5 A and 10 V, and two different resistors. Show how this circuit is connected.
   c. Determine the values of the two resistors.

Answers to Selected Problems

1. a. 3.66 Ω b. 0.36A c. 1.32 V
2. b. 15V c. 2.5Ω d. 15V
3. a.10V b.



c. $R_1 = 2.6\Omega$ and $R_2 = 6\Omega$

10.8 Resistors in Series

Students will learn how to analyze and solve problems involving circuits with resistors in series.

Key Equations

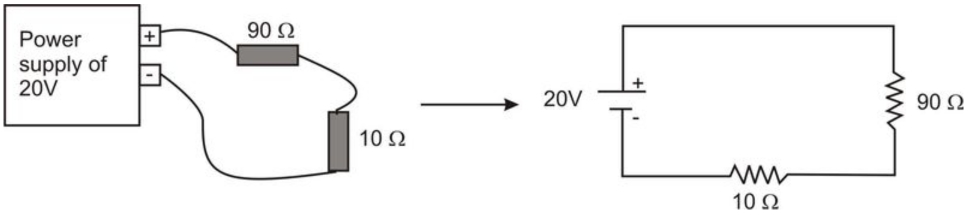
$R_{eqs} = R_1 + R_2 + R_3 + \dots$ Equivalent resistance for resistors in series

Guidance

Resistors in Series: All resistors are connected end to end. There is only one river, so they all receive the same current. But since there is a voltage drop across each resistor, they may all have different voltages across them. The more resistors in series the more rocks in the river, so the less current that flows.

Example 1

A circuit is wired up with two resistors in series.



Both resistors are in the same #8216;river#8217;, so both have the same current flowing through them. Neither resistor has a direct connection to the power supply so neither has 20V across it. But the combined voltages across the individual resistors add up to 20V.

Question: What is the total resistance of the circuit?

Answer: The total resistance is $R_{eqs} = R_1 + R_2 = 90\,\Omega + 10\,\Omega = 100\,\Omega$

Question: What is the total current coming out of the power supply?

Answer: Use Ohm#8217;s Law $(V = IR)$ but solve for current $(I = V/R)$.

$$I_{total} = \frac{V_{total}}{R_{total}} = \frac{20\,V}{100\,\Omega} = 0.20\,A$$

Question: How much power does the power supply dissipate?

Answer: $P = IV$, so the total power equals the total voltage multiplied by the total current. Thus, $P_{total} = I_{total}V_{total} = (0.20\,A)(20V) = 4.0\,W$. So the Power Supply is outputting 4W (i.e. 4 Joules of energy per second).

Question: How much power does each resistor dissipate?

Answer: Each resistor has different voltage across it, but the same current. So, using Ohm#8217;s law, convert the power formula into a form that does not depend on voltage.

$$
\begin{aligned}
P &= IV = I(IR) = I^2R.\\
P_{90\,\Omega} &= I_{90\,\Omega}^2 R_{90\,\Omega} = (0.2\,A)^2(90\,\Omega) = 3.6\,W\\
P_{10\,\Omega} &= I_{10\,\Omega}^2 R_{10\,\Omega} = (0.2\,A)^2(10\,\Omega) = 0.4\,W
\end{aligned}
$$

*Note: If you add up the power dissipated by each resistor, it equals the total power outputted, as it should#8211;Energy is always conserved.

Question: How much voltage is there across each resistor?

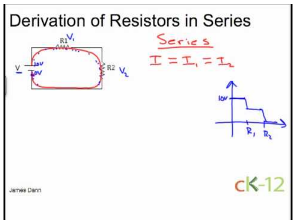
Answer: In order to calculate voltage across a resistor, use Ohm#8217;s law.

$$
\begin{aligned}
V_{90\,\Omega} &= I_{90\,\Omega} R_{90\,\Omega} = (0.2\,A)(90\,\Omega) = 18\,V\\
V_{10\,\Omega} &= I_{10\,\Omega} R_{10\,\Omega} = (0.2\,A)(10\,\Omega) = 2\,V
\end{aligned}
$$

*Note: If you add up the voltages across the individual resistors you will obtain the total voltage of the circuit, as you should. Further note that with the voltages we can use the original form of the Power equation $(P = IV)$, and we should get the same results as above.

$$
\begin{aligned}
P_{90\,\Omega} &= I_{90\,\Omega} V_{90\,\Omega} = (18\,V)(0.2\,A) = 3.6\,W\\
P_{10\,\Omega} &= I_{10\,\Omega} V_{10\,\Omega} = (2.0\,V)(0.2\,A) = 0.4\,W
\end{aligned}
$$

Watch this Explanation

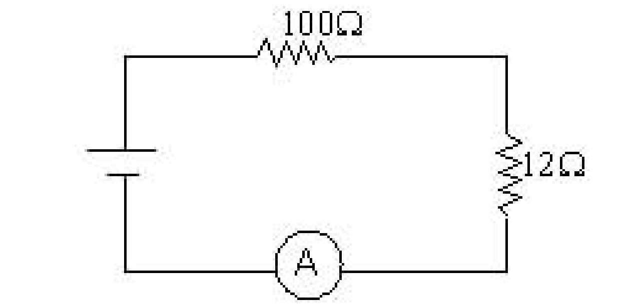


MEDIA

Click image to the left for more content.

Time for Practice

1. Regarding the circuit below.
   a. If the ammeter reads 2 A, what is the voltage?
   b. How many watts is the power supply supplying?
   c. How many watts are dissipated in each resistor?



2. Five resistors are wired in series. Their values are 10Ω, 56Ω, 82Ω, 120Ω and 180Ω.
    a. If these resistors are connected to a 6 V battery, what is the current flowing out of the battery?
    b. If these resistors are connected to a 120 V power suppluy, what is the current flowing out of the battery?
    c. In order to increase current in your circuit, which two resistors would you remove?
3. Given the resistors above and a 12 V battery, how could you make a circuit that draws 0.0594 A?

Answers to Selected Problems

1. a. 224 V b. 448 W c. 400 W by 100 Ω and 48 W by 12 Ω
2. a. 0.013 A b. 0.27 A c. 120Ω and 180Ω
3. need about 202Ω of total resistance. So if you wire up the 120Ω and the 82Ω in series, you'll have it.

10.9 Resistors in Parallel

Students will learn how to analyze and solve problems involving circuits with resistors in parallel.

Key Equations

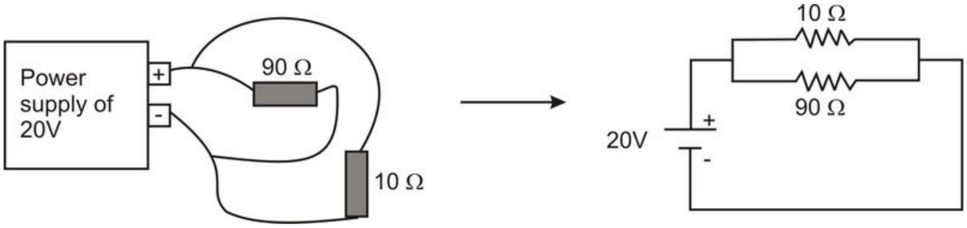
$\frac{1}{R_{eqp}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$ Equivalent resistance for resistors connected in parallel

Guidance

Resistors in Parallel: All resistors are connected together at both ends. There are many rivers (i.e. The main river branches off into many other rivers), so all resistors receive different amounts of current. But since they are all connected to the same point at both ends they all receive the same voltage.

Example 1

A circuit is wired up with 2 resistors in parallel.



Both resistors are directly connected to the power supply, so both have the same 20V across them. But they are on different #8216;rivers#8217; so they have different current flowing through them. Lets go through the same questions and answers as with the circuit in series.

Question: What is the total resistance of the circuit?

Answer: The <u>total</u> resistance is $\frac{1}{R_{eqp}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{90\Omega} + \frac{1}{10\Omega} = \frac{1}{90\Omega} + \frac{9}{90\Omega} = \frac{10}{90\Omega}$ thus, $R_{eqp} = \frac{90\Omega}{10} = 9\Omega$

*Note: Total resistance for a circuit in parallel will always be smaller than smallest resistor in the circuit.

Question: What is the total current coming out of the power supply?

Answer: Use Ohm#8217;s Law $(V = IR)$ but solve for current $(I = V/R)$.

$$I_{total} = \frac{V_{total}}{R_{total}} = \frac{20V}{9\Omega} = 2.2A$$

Question: How much power does the power supply dissipate?

Answer: $P = IV$, so the total power equals the total voltage multiplied by the total current. Thus, $P_{total} = I_{total}V_{total} = (2.2A)(20V) = 44.4W$. So the Power Supply outputs 44W (i.e. 44 Joules of energy per second).

Question: How much power is each resistor dissipating?

Answer: Each resistor has different current across it, but the same voltage. So, using Ohm#8217;s law, convert the power formula into a form that does not depend on current. $P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}$ Substituted $I = V/R$ into the power formula. $P_{90\Omega} = \frac{V_{90\Omega}^2}{R_{90\Omega}} = \frac{(20V)^2}{90\Omega} = 4.4W; P_{10\Omega} = \frac{V_{10\Omega}^2}{R_{10}\Omega} = \frac{(20V)^2}{10\Omega} = 40W$

*Note: If you add up the power dissipated by each resistor, it equals the total power outputted, as it should#8211;Energy is always conserved.

Question: How much current is flowing through each resistor?

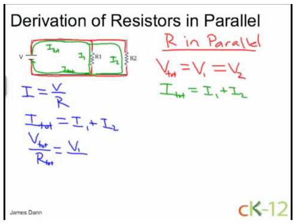
Answer: Use Ohm#8217;s law to calculate the current for each resistor.

$$I_{90\Omega} = \frac{V_{90\Omega}}{R_{90\Omega}} = \frac{20V}{90\Omega} = 0.22A \qquad I_{10\Omega} = \frac{V_{10\Omega}}{R_{10\Omega}} = \frac{20V}{10\Omega} = 2.0A$$

Notice that the 10Ω resistor has the most current going through it. It has the least resistance to electricity so this makes sense.

*Note: If you add up the currents of the individual #8216;rivers#8217; you get the total current of the of the circuit, as you should.

Watch this Explanation

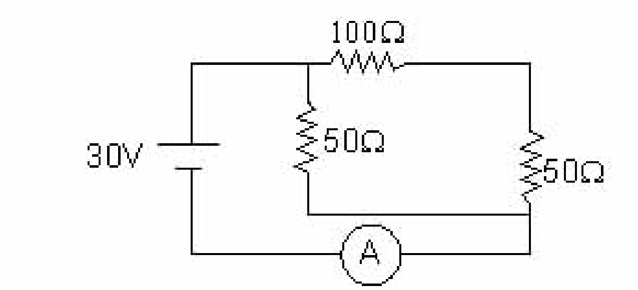


MEDIA

Click image to the left for more content.

Time for Practice

1. Three 82 Ω resistors and one 12 Ω resistor are wired in parallel with a 9 Vbattery.
   a. Draw the schematic diagram.
   b. What is the total resistance of the circuit?
2. What does the ammeter read and which resistor is dissipating the most power?



3. Given three resistors, 200 Ω, 300 Ω and 600 Ω and a 120 Vpower source connect them in a way to heat a container of water as rapidly as possible.
   a. Show the circuit diagram
   b. How many joules of heat are developed after 5 minutes?

Answers to Selected Problems

1. b. 8.3 W
2. 0.8A and the 50 Ω on the left
3. part 2 43200J.

10.10 Resistor Circuits

Students will use what they have learned from the previous lessons (ohm's law, resistors in series and resistors in parallel) and apply that knowledge to understand and solve more complicated resistor circuits.

Key Equations

$V = IR$

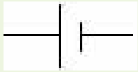
$P = IV.$

$$R_{eqs} = R_1 + R_2 + R_3 + \dots$$

$$\frac{1}{R_{eqp}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

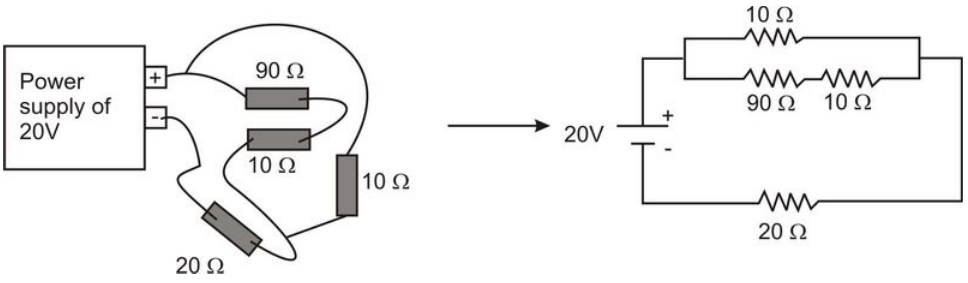
Guidance

TABLE 10.3: **Table of electrical symbols and units**

Name	Electrical Symbol	Units	Analogy
Voltage (*V*		Volts (V)	A water dam with pipes coming out at different heights. The lower the pipe along the dam wall, the larger the water pressure, thus the higher the voltage. **Examples**: Battery, the plugs in your house, etc.
Current (I)		Amps (A) $A = \text{C}/\text{s}$	A river of water. Objects connected in series are all on the same river, thus receive the same current. Objects connected in parallel make the main river branch into smaller rivers. These guys all have different currents. **Examples**: Whatever you plug into your wall sockets draws current
Resistance (R)		Ohm (Ω)	If current is analogous to a river, then resistance is the amount of rocks in the river. The bigger the resistance the less current that flows **Examples**: Light bulb, Toaster, etc.

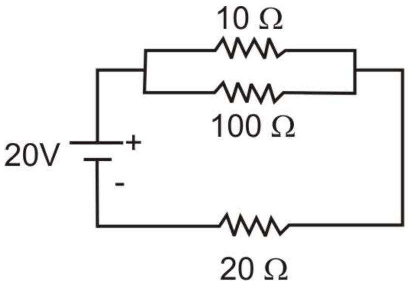
Example 1

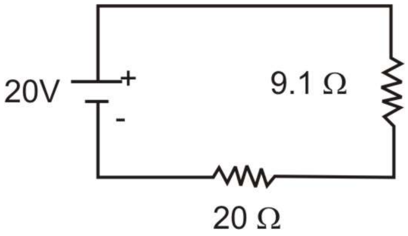
A more complicated circuit is analyzed.

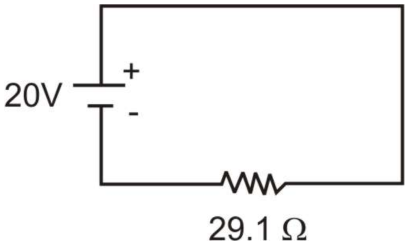


Question: What is the total resistance of the circuit?

Answer: In order to find the total resistance we do it in steps (see pictures. First add the 90Ω and 10Ω in series to make one equivalent resistance of 100Ω (see diagram at below). Then add the 100Ω to the 10Ω in parallel to get one resistor of 9.1Ω. Now we have two resistors in series, simply add them to get the total resistance of 29.1Ω.







Question: What is the total current coming out of the power supply?

Answer: Use Ohm#8217;s Law ($V = IR$) but solve for current ($I = V/R$). $I_{total} = \frac{V_{total}}{R_{total}} = 20V/2.91\Omega = 0.69\ Amps$

Question: What is the power dissipated by the power supply?

Answer: $P = IV$, so the total power equals the total voltage multiplied by the total current. Thus, $P_{total} = I_{total}V_{total} = (0.69A)(20V) = 13.8W$.

Question: How much power is the 20Ω resistor dissipating?

Answer: The 20Ω has the full 0.69Amps running through it because it is part of the #8216;main river#8217; (this is not the case for the other resistors because the current splits). $P_{20\Omega} = I^2_{20\Omega}R_{20\Omega} = (0.69A)^2(20\Omega) = 9.5W$

Question: If these resistors are light bulbs, order them from brightest to least bright.

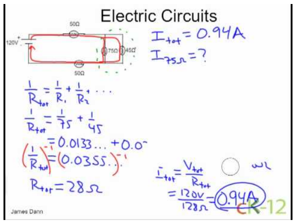
Answer: The brightness of a light bulb is directly given by the power dissipated. So we could go through each resistor as we did the 20Ω guy and calculate the power then simply order them. But, we can also think it out. For the guys in parallel the current splits with most of the current going through the 10Ω path (less resistance) and less going through the $90\Omega + 10\Omega$ path. Well the second path is ten times the resistance of the first, so it will have one tenth of the total current. Thus, there is approximately and 0.069 Amps going through the 90Ω and 10Ω path and 0.621Amps going through the 10Ω path.

$$P_{10\Omega} = I^2_{10\Omega}R_{10\Omega} = (0.621A)^2(10\Omega) = 3.8W$$
$$P_{90+10\Omega} = I^2_{90+10\Omega}R_{90+10\Omega} = (0.069A)^2(100\Omega) = 0.5W$$

We now know that the 20Ω is the brightest, 10Ω is second and then the 90Ω and last the 10Ω (-these last two have same current flowing through them, so 90Ω is brighter due to its higher resistance).

*Note: Adding up these two plus the 9.5W from the 20Ω resistor gives us 13.8W, which is the total power previously calculated, so we have confidence everything is good.

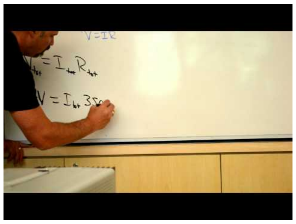
Example 2



MEDIA

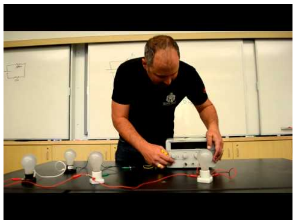
Click image to the left for more content.

Watch this Explanation



MEDIA

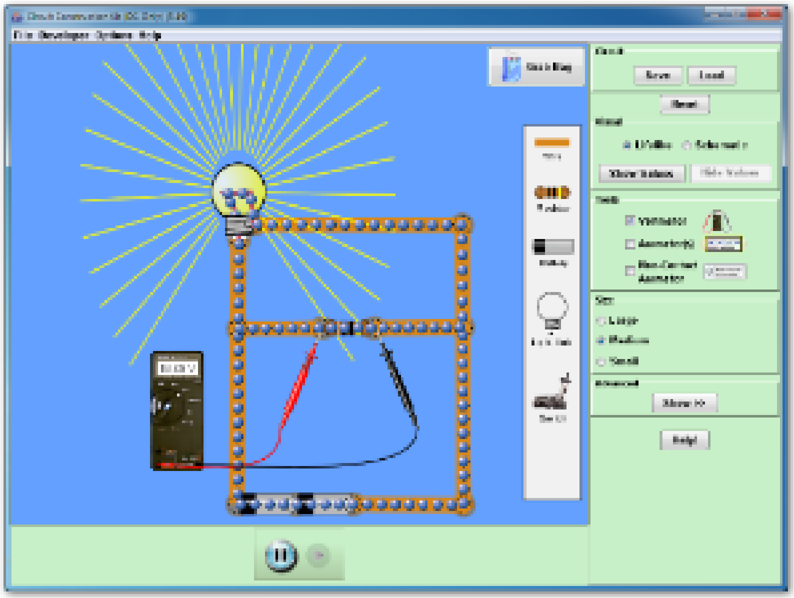
Click image to the left for more content.



MEDIA

Click image to the left for more content.

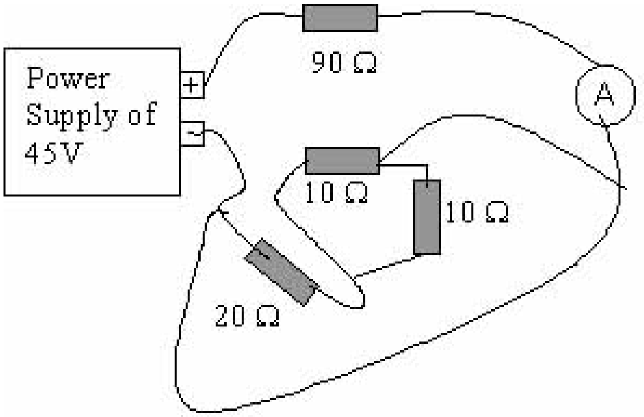
Simulation



Circuit ConstructionKit(DCOnly) (PhETSimulation)

Time for Practice

1. What will the ammeter read for the circuit shown to the right?



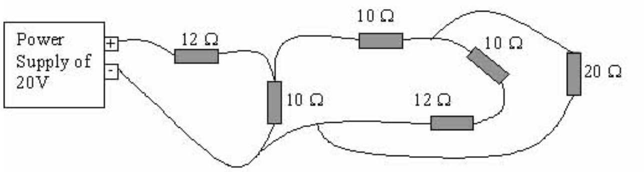
2. You can use the simulation below to check your answer. Click on the blue arrow and select the part of the circuit you want to track. Then scroll down to the Data tab and you can see the current and voltage in different parts of the circuit.
3.

Multimedia

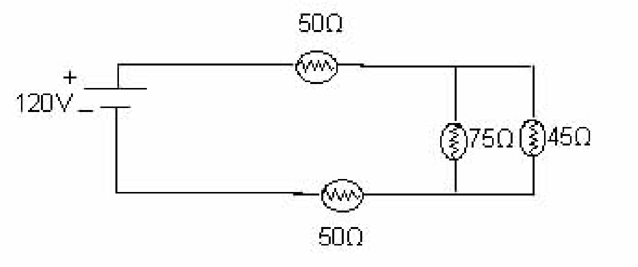
MEDIA

Click image to the left for more content.

4. Draw the schematic of the following circuit.

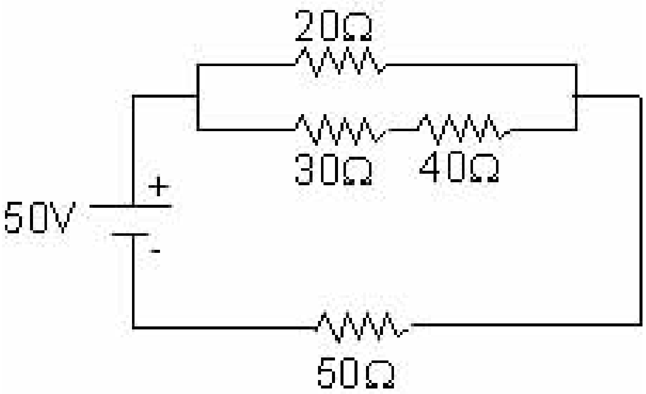


5. Analyze the circuit below.



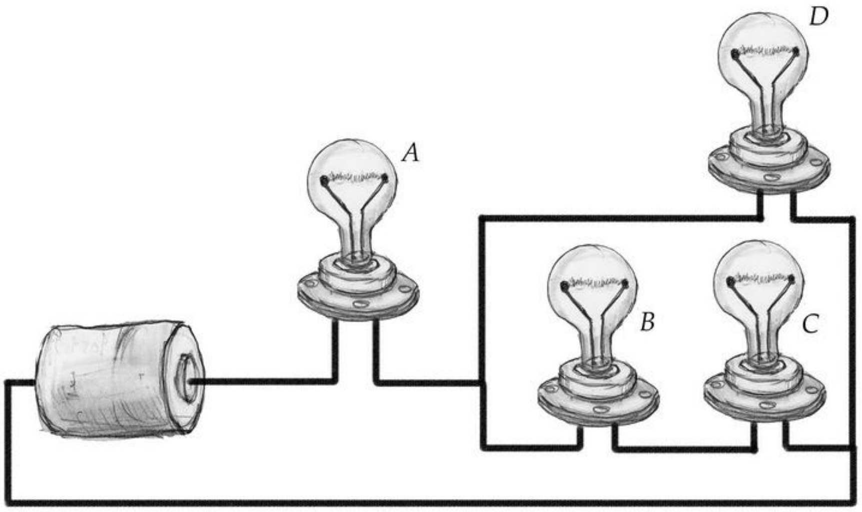
   a. Find the current going out of the power supply
   b. How many Joules per second of energy is the power supply giving out?
   c. Find the current going through the 75 Ω light bulb.
   d. Find the current going through the 50 Ω light bulbs (hint: it#8217;s the same, why?).
   e. Order the light bulbs in terms of brightness
   f. If they were all wired in parallel, order them in terms of brightness.

6. 4. Find the total current output by the power supply and the power dissipated by the 20 Ω resistor.

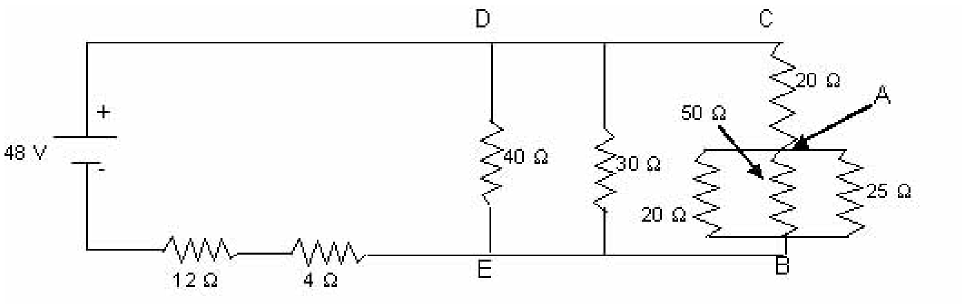


7. 5. You have a 600 V power source, two 10 Ω toasters that both run on 100 V and a 25 Ωresistor.

   a. Show me how you would wire them up so the toasters run properly.
   b. What is the power dissipated by the toasters?
   c. Where would you put the fuses to make sure the toasters don#8217;t draw more than 15 Amps?
   d. Where would you put a 25 Amp fuse to prevent a fire (if too much current flows through the wires they will heat up and possibly cause a fire)?



8. 6. Look at the following scheme of four identical light bulbs connected as shown. Answer the questions below giving a justification for your answer:
   a. Which of the four light bulbs is the brightest?
   b. Which light bulbs are the dimmest?
   c. Tell in the following cases which other light bulbs go out if:
      a. bulb *A* goes out
      b. bulb *B* goes out
      c. bulb *D* goes out
   d. Tell in the following cases which other light bulbs get dimmer, and which get brighter if:
      a. bulb *B* goes out
      b. bulb *D* goes out
9. 7. Refer to the circuit diagram below and answer the following questions.
   a. What is the resistance between *A* and *B*?
   b. What is the resistance between *C* and *B*?
   c. What is the resistance between *D* and *E*?
   d. What is the the total equivalent resistance of the circuit?
   e. What is the current leaving the battery?
   f. What is the voltage drop across the 12 Ω resistor?
   g. What is the voltage drop between *D* and *E*?
   h. What is the voltage drop between *A* and *B*?
   i. What is the current through the 25 Ω resistor?
   j. What is the total energy dissipated in the 25 Ω if it is in use for 11 hours?



10. 8. You are given the following three devices and a power supply of exactly 120 v. * Device X is rated at 60 V and 0.5 A* Device Y is rated at 15 w and 0.5 A* Device Z is rated at 120 V and 1800 w Design a circuit that obeys the following rules: you may only use the power supply given, one sample of each device, and an extra, single resistor of any value (you choose). Also, each device must be run at their rated values.

Answers to Selected Problems

1. 0.5A
2. .
3. a. 0.94 A b. 112 W c. 0.35 A d. 0.94 A e. $50, 45, 75\ \Omega$ f. both 50 Ω resistors are brightest, then 45 Ω, then 75 Ω
4. a. 0.76 A b. 7.0 W
5. b. 1000 W
6. .
7. a. 9.1 Ω b 29.1 Ω c. 10.8 Ω d.26.8 Ω e. 1.8A f. 21.5V g. 19.4V h. 6.1V i. 0.24A j. 16 kW
8. .

10.11 Capacitors

Students will learn how a capacitor works and how to solve basic problems involving the classic two plate capacitor.

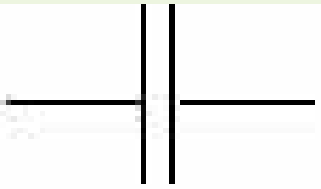
Key Equations

$$\begin{cases} \varepsilon_0 = 8.85 \times 10^{-12} \text{F/m} & \text{Constant called the permittivity of free space} \\ Q = CV & \text{Charge stored in a capacitor} \\ C = \frac{\kappa \varepsilon_0 A}{d} & \text{For two parallel metal plates, area A, separation } d, \text{dielectric } \kappa \end{cases}$$

Guidance

A capacitor is a device that stores charge. It is typically two very large flat plates wrapped in a cylinder with a dielectric substance in between the two. The dielectric allows the plates to be at a high voltage (and thus store more charge) without arcing between them. The capacitance, literally tells you how much charge it can hold. The capacitance of a capacitor only depends on its geometry. Here some important things to keep in mind in regards to capacitors:

- Q refers to the amount of positive charge stored on the high voltage side of the capacitor; an equal and opposite amount, Q, of negative charge is stored on the low voltage side of the capacitor.
- Current can flow *into* a capacitor from either side, but current doesn#8217;t flow across the capacitor from one plate to another. The plates do not touch, and the substance in between is insulating, not conducting.
- One side of the capacitor fills up with negative charge, and the other fills up with positive charge. The reason for the thin, close plates is so that you can use the negative charge on one plate to attract and hold the positive charge on the other plate. The reason for the plates with large areas is so that you can spread out the charge on one plate so that its self-repulsion doesn#8217;t stop you from filling it with more charge.
- Typical dielectric constants #954; are roughly 5.6 for glass and 80 for water. What these #8220;dielectric#8221; substances do is align their electric polarity with the electric field in a capacitor (much like atoms in a magnetic material) and, in doing so, reduces the electric field for a given amount of charge. Thereby allowing for more charge to be stored for a given Voltage.
- The electrical circuit symbol for a capacitor is two flat plates, mimicking the geometry of a capacitor, which typically consists of two flat plates separated by a small distance. The plates are normally wrapped around several times to form a cylindrical shape.



Example 1

You create a simple capacitor by placing two .25m square metal plates .01m apart and then connecting each plate to one end of a battery. (a) If the battery can create a voltage drop of 12V, how much charge can be stored in the capacitor. (b) If you immersed the whole system in water, how much more charge could you store on the capacitor?

Solution

(a): To solve this problem, we can use the equations give above. First we'll find the capacitance of the two plates based on their dimensions. Since there is not dielectric between the plates to start, the dielectric constant is 1.

$$\begin{aligned} C &= \frac{\kappa \varepsilon_o A}{d} \\ C &= \frac{1 * 8.85 * 10^{-12}\,\text{F/m} * (.25\,\text{m})^2}{.01\,\text{m}} \\ C &= 5.5 * 10^{-11}\,\text{F} \end{aligned}$$

Now we can find out how much charge can be stored on the capacitor.

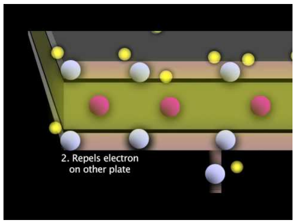
$$\begin{aligned} Q &= CV \\ Q &= 5.5 * 10^{-11}\,\text{F} * 12\,\text{V} \\ Q &= 6.63 * 10^{-10}\,\text{C} \end{aligned}$$

(b): The new capacitance will just be 80x the original capacitance because that is the dielectric constant of water. We can use this to calculate how much charge can be stored on the submerged capacitor.

$$\begin{aligned} Q &= 80CV \\ Q &= 80 * 5.5 * 10^{-11}\,\text{F} * 12\,\text{V} \\ Q &= 5.31 * 10^{-8}\,\text{F} \end{aligned}$$

As you can see, the capacitance of these plates is pretty small. Most capacitors have very thin, but very long metal sheets that are rolled up into a cylinder and separated by only a few millimeters; this provides a much greater capacitance in a much smaller volume.

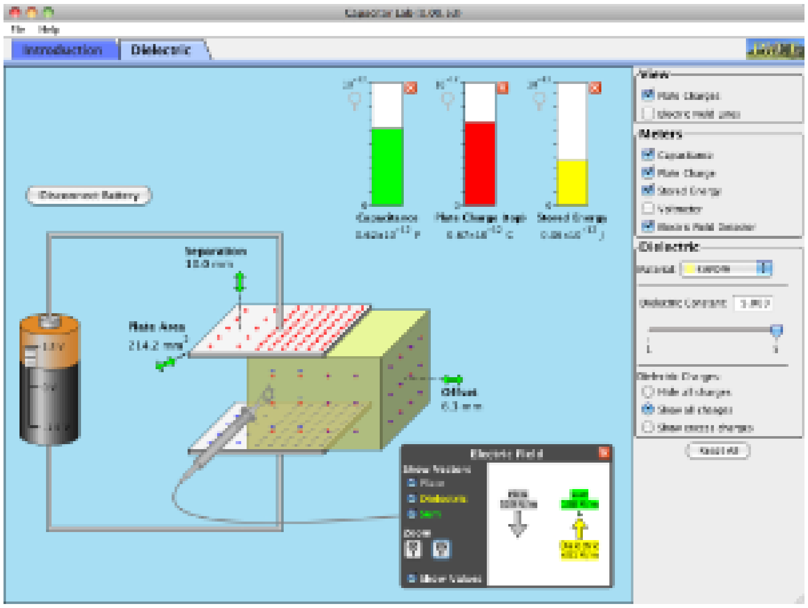
Watch this Explanation



MEDIA

Click image to the left for more content.

Simulation



Capacitor Lab (PhET Simulation)

Time for Practice

1. Design a parallel plate capacitor with a capacitance of 100 mF. You can select any area, plate separation, and dielectric substance that you wish.
2. Show, by means of a sketch illustrating the charge distribution, that two identical parallel-plate capacitors wired in parallel act exactly the same as a single capacitor with twice the area.
3. A certain capacitor can store 5 C of charge if you apply a voltage of 10 V.
    a. How many volts would you have to apply to store 50 C of charge in the same capacitor?
    b. Why is it harder to store more charge?
4. A capacitor is charged and then unhooked from the battery. A dielectric is then inserted using an insulating glove.
    a. Does the electric field between plates increase or decrease? Why?
    b. did it take negative work, no work or positive work when the dielectric was inserted? (.e. did it get #8216;sucked in#8217; or did you have to #8216;push it in#8217; or neither) Explain.

    The capacitor is now reattached to the battery

    a. Does the voltage of the capacitor increase, decrease or stay the same? Explain
    b. The dielectric is now removed from the middle with battery attached. What happens?

Answers to Selected Problems

1. .
2. .
3. a. 100 V b. Because as charges build up they repel each other from the plate and a greater voltage is needed to create a stronger electric field forcing charge to flow
4. a. decrease b. negative work ('sucked in') c. Goes back up to what it was before inserted dielectric d. V is constant, since capacitance goes down, charge must go down. There's probably a discharge across the capacitor.

10.12 Capacitor Energy

Students learn how energy is stored in a capacitor and how to calculate said energy.

Key Equations

The electric potential energy, E_C, stored in the capacitor is given by

$$E_C = \frac{1}{2}CV^2$$

Guidance

Suppose we have two parallel metal plates set a distance d from one another. We place a positive charge on one of the plates and a negative charge on the other. In this configuration, there will be a uniform electric field between the plates pointing from, and normal to, the plate carrying the positive charge. The magnitude of this field is given by

$$E = \frac{V}{d}$$

where V is the potential difference (voltage) between the two plates.

The amount of charge, Q, held by each plate is given by

$$Q = CV$$

where again V is the voltage difference between the plates and C is the capacitance of the plate configuration. Capacitance can be thought of as the capacity a device has for storing charge . In the parallel plate case the capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}$$

where A is the area of the plates, d is the distance between the plates, and ε_0 is the permittivity of free space whose value is $8.85 \times 10^{-12} C/V \cdot m$.

The electric field between the capacitor plates stores energy.

Where does this energy come from? Recall, that in our preliminary discussion of electric forces we assert that "like charges repel one another". To build our initial configuration we had to place an excess of positive and negative charges, respectively, on each of the metal plates. Forcing these charges together on the plate had to overcome the mutual repulsion that the charges experience; this takes work. The energy used in moving the charges onto the plates gets stored in the field between the plates. It is in this way that the capacitor can be thought of as an energy storage device. This property will become more important when we study capacitors in the context of electric circuits in the next several Concepts.

Note: Many home-electronic circuits include capacitors; for this reason, it can be dangerous to mess around with old electronic components, as the capacitors may be charged even if the unit is unplugged. For example, old computer monitors (not flat screens) and TVs have capacitors that hold dangerous amounts of charge hours after the power is turned off.

Example 1

You have a capacitor with capacitance 100nF. If you connected it to a 12V battery, how much energy is stored in the capacitor when it is fully charged? If you were to submerge this capacitor in water, how much energy could be stored in it?

Solution

We'll just use the equation given above to calculate the energy stored on the capacitor.

$$E_C = \frac{1}{2}CV^2$$
$$E_C = \frac{1}{2}100 * 10^{-9}\,\mathrm{F} * 12\,\mathrm{V}$$
$$E_C = 6 * 10^{-7}\,\mathrm{J}$$

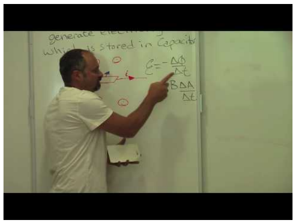
By adding a dielectric, we increase the capacitance of the capacitor by a factor of the dielectric constant. The dielectric constant of water is 80, so the new capacitance will be 80 times the original capacitance.

$$E_C = \frac{1}{2}80CV^2$$
$$E_C = \frac{1}{2}80 * 100 * 10^{-9}\,\mathrm{F} * 12\,\mathrm{V}$$
$$E_C = 4.8 * 10^{-5}\,\mathrm{J}$$

Watch this Explanation

MEDIA

Click image to the left for more content.



MEDIA

Click image to the left for more content.

Time for Practice

1. You have a 5μFcapacitor.
    a. How much voltage would you have to apply to charge the capacitor with 200 C of charge?
    b. Once you have finished, how much potential energy are you storing here?
    c. If all this energy could be harnessed to lift 100 lbs. into the air, how high would you be lifted?
2. A certain capacitor can store 500 J of energy (by storing charge) if you apply a voltage of 15 V. How many volts would you have to apply to store 1000 J of energy in the same capacitor? (Important: why isn#8217;t the answer to this just 30 V?)
3. Marciel, a bicycling physicist, wishes to harvest some of the energy he puts into turning the pedals of his bike and store this energy in a capacitor. Then, when he stops at a stop light, the charge from this capacitor can flow out and run his bicycle headlight. He is able to generate 18 V of electric potential, on average, by pedaling (and using magnetic induction). If Mars wants to provide 0.5 A of current for 60 seconds at a stop light, what should the capacitance of his capacitor be?

Answers to Selected Problems

1. a. 4×10^7 V b. 4×10^9 J c.About 9000 km
2. 21 V, V is squared.
3. 3.3 F

10.13 Capacitors Circuits

Students will learn to break down and solve capacitor circuits.

Key Equations

$$C = \frac{Q}{V}$$ Definition of Capacitance

$$C_{eqp} = C_1 + C_2 + C_3 + \dots$$ Capacitors in parallel add like resistors in series

$$\frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$ Capacitors in series add like resistors in parallel

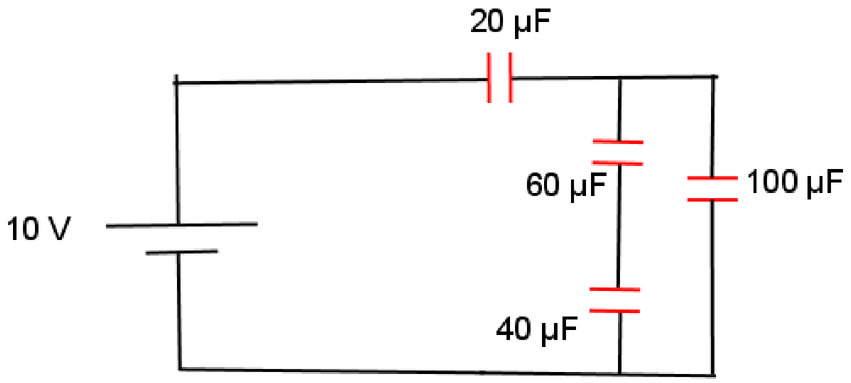
Guidance

When a capacitor is placed in a circuit, current does not actually travel across it. Rather, equal and opposite charge begins to build up on opposite sides of the capacitor — mimicking a current — until the electric field in the capacitor creates a potential difference across it that balances the voltage drop across any parallel resistors or the voltage source itself (if there are no resistors in parallel with the capacitor). The ratio of charge on a capacitor to potential difference across it is called capacitance.

It is important to break down a complicated circuit into the equivalent capacitance using the rules for capacitors in series and capacitors in parallel. Also remember that capacitors in parallel have the same voltage while capacitors in series have the same charge.

Example 1

In the circuit shown below, determine (a) the total capacitance and (b) the total charge stored.



Solution

(a): In solving this problem, we'll call the 20 μF capacitor C_1, the 60 μF capacitor C_2, the 40 μF capacitor C_3, and the 100 μF capacitor C_4.

Our first step will be to find the equivalent capacitance of C_2 and C_3.

$$\frac{1}{C_{2,3}} = \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{2,3}} = \frac{1}{60\ \mu\text{F}} + \frac{1}{40\ \mu\text{F}}$$

$$\frac{1}{C_{2,3}} = \frac{5}{120\ \mu\text{F}}$$

$$C_{2,3} = 24\ \text{F}$$

Next, we'll combine the capacitance of $C_{2,3}$ and C_4.

$$C_{2,3,4} = C_{2,3} + C_4$$

$$C_{2,3,4} = 24\ \mu\text{F} + 100\ \mu\text{F}$$

$$C_{2,3,4} = 124\ \mu\text{F}$$

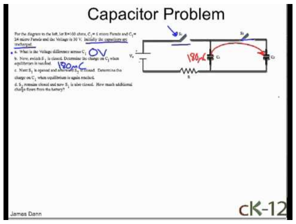
Finally, we can combine $C_{2,3,4}$ with C_1 to find the total capacitance.

$$\begin{aligned}\frac{1}{C_{eq}} &= \frac{1}{C_1}+\frac{1}{C_{2,3,4}}\\ \frac{1}{C_{eq}} &= \frac{1}{20\ \mu\text{F}}+\frac{1}{124\ \mu\text{F}}\\ \frac{1}{C_{eq}} &= .058\ \mu\text{F}^{-1}\\ C_{eq} &= 17.22\ \mu\text{F}\end{aligned}$$

(b): Now we can use this value to find the total charge stored on all the capacitors by also using the voltage provided on the diagram.

$$\begin{aligned}Q &= CV\\ Q &= 17.22\ \mu\text{F} * 10\ \text{V}\\ Q &= 172.2\mu\text{C}\end{aligned}$$

Watch this Explanation


Capacitor Problem

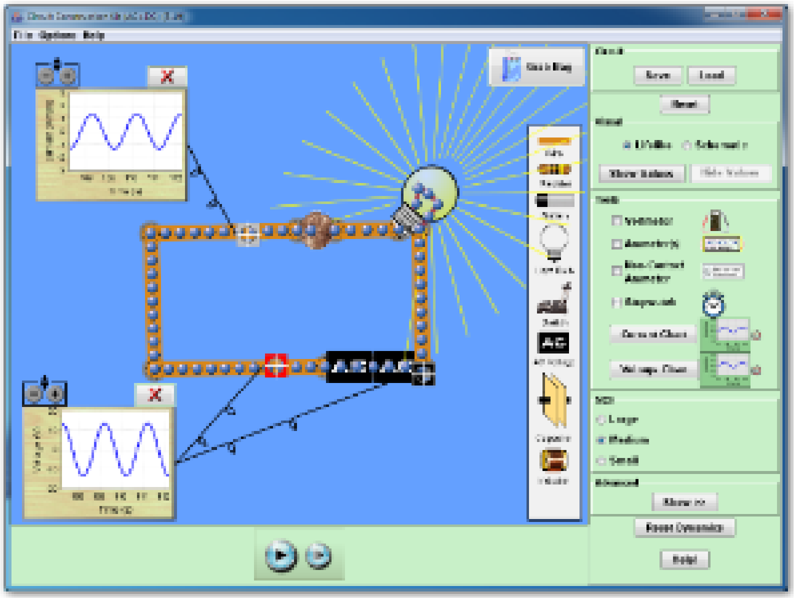
James Dann

cK-12

MEDIA

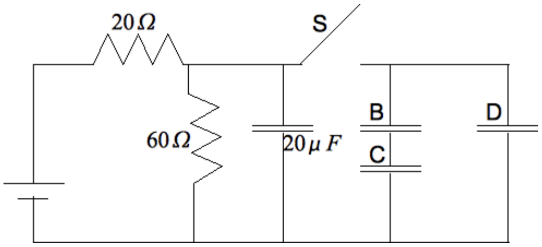
Click image to the left for more content.

Simulation



Circuit ConstructoinKit(AC+DC)(PhET Simlation)

Time for Practice

1. 

2. Consider the figure above with switch, *S*, initially open and the power supply set to 24 V:
    a. What is the voltage drop across the 20 resistor?
    b. What current flows thru the 60Ω resistor?
    c. What is the voltage drop across the 20 microfarad capacitor?
    d. What is the charge on the capacitor?
    e. How much energy is stored in that capacitor?
    f. Find the capacitance of capacitors *B*, *C*, and *D* if compared to the 20μFcapacitor where...
        a. *B* has twice the plate area and half the plate separation
        b. *C* has twice the plate area and the same plate separation
        c. *D* has three times the plate area and half the plate separation
3. Now the switch in the previous problem is closed.
    a. What is the total capacitance of branch with B and C?
    b. What is the total capacitance of the circuit?
    c. What is the voltage drop across capacitor *B*?

Answers to Selected Problems

1. a. 6V b. 0.3A c. 18V d. 3.6×10^{-4}C e. 3.2×10^{-3}J f. i) 80μF ii) 40μF iii) 120μF
2. a. 26.7μF b. 166.7μF

10.14 Capacitors in Series and Parallel

Students will understand and apply the equations governing capacitors hooked up in series and parallel.

Key Equations

$$C_{eqp} = C_1 + C_2 + C_3 + \ldots$$ [5] Capacitors in parallel add like resistors in series

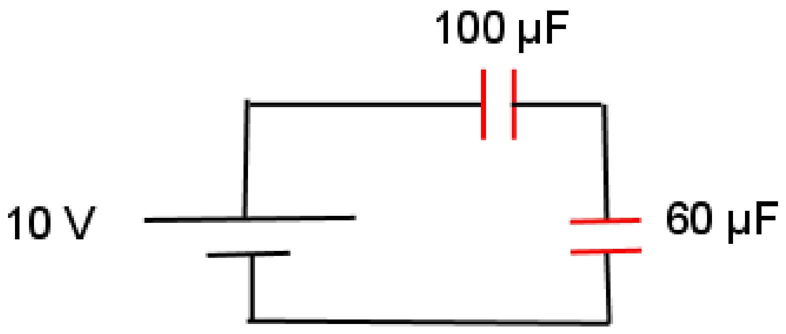
$$\frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$ [6] Capacitors in series add like resistors in parallel

Guidance

Capacitors in parallel have the same voltage, but different charge stored. Capacitors in series have the same charge stored, but different voltages. Remember that if a capacitor are hooked up to the battery they will have the same voltage as the battery. If the capacitor is unhooked from a battery and other capacitors are attached to it, then the voltage can change but the total amount of charge must remain constant. Charge conservation holds that charge can not be created or destroyed. When solving problems involving capacitor circuits, we use the equation for the charge on a capacitor much like we use Ohm's Law.

Example 1

Two capacitors, one of 10;μF (C_1) and one of 60 μF (C_2), are connected to a 10V battery in series. A diagram of the circuit is shown below. Determine (a) the total capacitance, (b) the charge stored on the 100 μF capacitor, and (c) the voltage drop across the 60 μF.



Solution

(a): To find the total capacitance, we'll use the equation give above for determining the equivalent capacitance of capacitors in series.

$$\begin{aligned}\frac{1}{C_{eqs}} &= \frac{1}{C_1}+\frac{1}{C_2}\\ \frac{1}{C_{eqs}} &= \frac{1}{100\,\mu\text{F}}+\frac{1}{60\,\mu\text{F}}\\ C_{eqs} &= 37.5\,\mu\text{F}\end{aligned}$$

(b): Since charge is the same across capacitors in series, we can use the charge found using the total capacitance and the total voltage drop to find the charge in the C_1 capacitor.

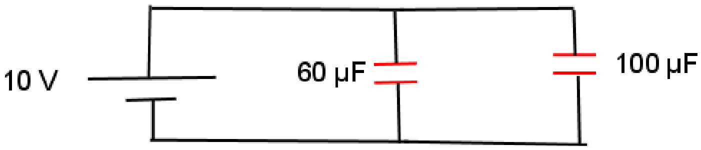
$$\begin{aligned}Q &= C_{eq}V\\ Q &= 37.5\,\mu\text{F} * 10\text{ V}\\ Q &= 375\,\mu\text{C}\end{aligned}$$

(c): Since we know the charge and the capacitance of C_2, we can find the voltage drop.

$$\begin{aligned}Q &= C_2V_2\\ V_2 &= \frac{Q}{C_2}\\ V_2 &= \frac{375\,\mu\text{C}}{60\,\mu\text{F}}\\ V_2 &= 6.2\text{ V}\end{aligned}$$

Example 2

The two capacitors used in the previous example problem are now connected to the battery in parallel. What is (a) the total capacitance and (b) the charge on C_1. A diagram of the circuit is shown below.



Solution

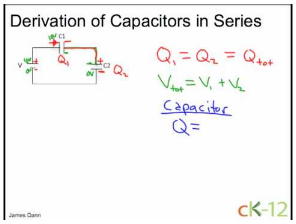
(a): To find the total capacitance, we'll us the equation given above for capacitors in parallel.

$$
\begin{aligned}
C_{eqp} &= C_1 + C_2\\
C_{eqp} &= 100\,\mu\text{F} + 60\,\mu\text{F}\\
C_{eqp} &= 160\,\mu\text{F}
\end{aligned}
$$

(b): Now, since the voltage across capacitors in parallel is equal, we can find the charge on C_2.

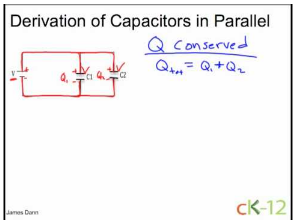
$$
\begin{aligned}
Q_2 &= C_2 V\\
Q_2 &= 60\,\mu\text{F} * 10\,\text{V}\\
Q_2 &= 600\,\mu\text{C}
\end{aligned}
$$

Watch this Explanation



MEDIA

Click image to the left for more content.

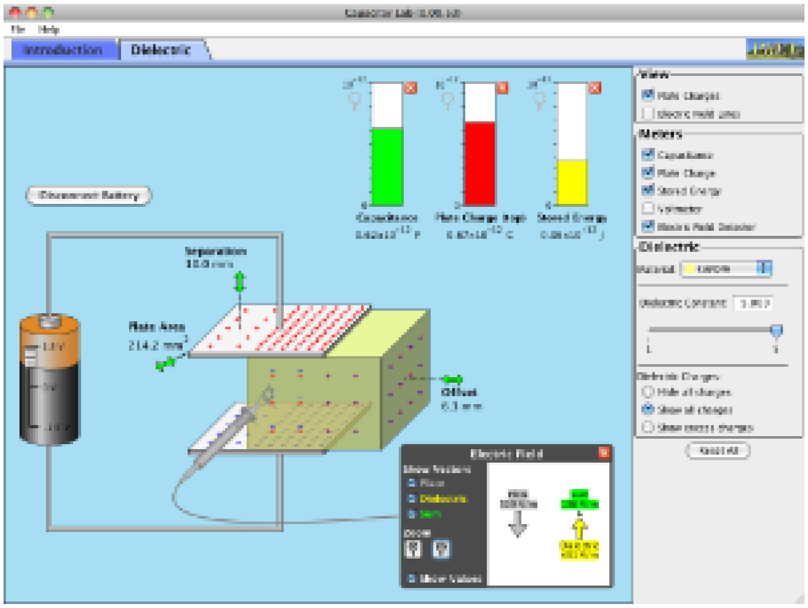


MEDIA

Click image to the left for more content.

Simulation

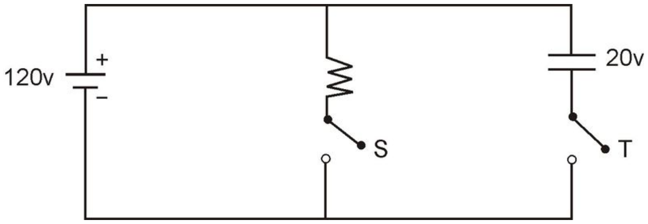
Note: go to the third tab to see circuits with multiple capacitors.



Capacitor Lab (PhET Simulation)

Time for Practice

1. You have two 42μF and one 39μF all wired in parallel. Draw the schematic and calculate the total capacitance of the system .
2. You have two 42μF and one 39μF all wired in series. Draw the schematic and calculate the total capacitance of the system .
3. Given a capacitor with 1 cm between the plates a field of 20,000 N/Cis established between the plates.
   a. What is the voltage across the capacitor?



   b. If the charge on the plates is 1μC, what is the capacitance of the capacitor?
   c. If two identical capacitors of this capacitance are connected in series what it the total capacitance?
   d. Consider the capacitor connected in the following circuit at point B with two switches S and T, a 20Ω resistor and a 120 Vpower source:
      a. Calculate the current through and the voltage across the resistor if S is open and T is closed
      b. Repeat if S is closed and T is open

Answers

1. 123μF
2. 0.073μF
3. a. 6V b. 0.3A c. 18V d. 3.6×10^{-4}C e. 3.2×10^{-3}J f. i) 80μF ii) 40μF iii) 120μF

10.15 RC Time Constant

Students will learn about the RC time constant and how to solve various problems involving resistor-capacitor circuits.

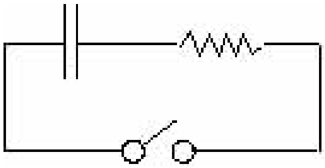
Key Equations

$Q(t) = Q_0 e^{\frac{-t}{\tau}}$ Discharge rate, where $\tau = RC$

$I(t) = I_0 e^{\frac{-t}{\tau}}$ Electric current flow varies with time in a like manner

Guidance

When a capacitor is initially uncharged, it is very easy to stuff charge in. As charge builds, it repels new charge with more and more force. Due to this effect, the charging of a capacitor follows a logarithmic curve. When you pass current through a resistor into a capacitor, the capacitor eventually #8220;fills up#8221; and no more current flows. A typical RC circuit is shown below; when the switch is closed, the capacitor discharges with an exponentially decreasing current.



Example 1

In the circuit diagram shown above, the resistor has a value of 100 Ω and the capacitor has a capacitance of 500 μF. After the switched is closed, (a) how long will it be until the charge on the capacitor is only 10% of what it was when the switch was originally closed? If the capacitor was originally charged by a 12 V battery, how much charge will be left on it at this time?

Solution

(a): To solve the first part of the problem, we'll use the equation that gives charge as a function of time.

$$\begin{aligned}
Q(t) &= Q_o e^{\frac{-t}{\tau}} && \text{start with the equation give above}\\
.1Q_o &= Q_o e^{\frac{-t}{\tau}} && \text{substitute } .1Q_o \text{ for } Q(t) \text{ because that's the charge at the time we want to find}\\
.1 &= e^{\frac{-t}{\tau}} && \text{simplify the equation}\\
t &= -\tau \ln(.1) && \text{solve for time}\\
t &= -RC \ln(.1) && \text{substitute in the value for } \tau\\
t &= -100\ \Omega * 500\ \mu\text{F} * \ln(.1) && \text{substitute in all the known values}\\
t &= .12\ \text{s}
\end{aligned}$$

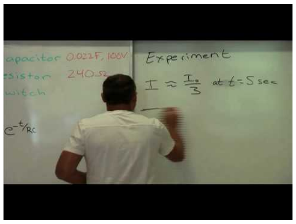
(b): Solving the second part of this problem will be a two step process. We will use the capacitance and the voltage drop to determine how much charge was originally on the capacitor (Q_o).

$$\begin{aligned}
Q_o &= CV\\
Q_o &= 500\ \mu\text{F} * 12\ \text{V}\\
Q_o &= .006\ \text{C}
\end{aligned}$$

Now we can plug in the time we found in part A to the equation for charge as a function of time.

$$\begin{aligned}
Q(t) &= Q_o e^{\frac{-t}{\tau}}\\
Q(.12\ \text{s}) &= .006\ \text{C} e^{\frac{-.12\ \text{s}}{100\ \Omega * 500\ \mu\text{F}}}\\
Q(.12\ \text{s}) &= 5.44 * 10^{-4}\ \text{C}
\end{aligned}$$

Watch this Explanation

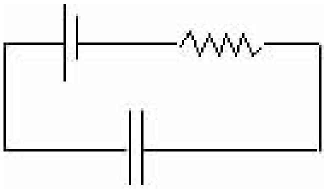


MEDIA

Click image to the left for more content.

Time for Practice

1. Design a circuit that would allow you to determine the capacitance of an unknown capacitor.



2. The power supply (i.e. the voltage source) in the circuit below provides 10 V. The resistor is 200Ω and the capacitor has a value of 50μF.
    a. What is the voltage across the capacitor *immediately* after the power supply is turned on?
    b. What is the voltage across the capacitor after the circuit has been hooked up for a long time?
3. Marciel, a bicycling physicist, wishes to harvest some of the energy he puts into turning the pedals of his bike and store this energy in a capacitor. Then, when he stops at a stop light, the charge from this capacitor can flow out and run his bicycle headlight. He is able to generate 18 Vof electric potential, on average, by pedaling (and using magnetic induction).
    a. If Mars wants to provide 0.5 A of current for 60 seconds at a stop light, how big a 18 V capacitor should he buy (i.e. how many farads)?
    b. How big a resistor should he pass the current through so the RC time is three minutes?
4. A simple circuit consisting of a 39μF and a 10kΩresistor. A switch is flipped connecting the circuit to a 12 V battery.
    a. How long until the capacitor has 2/3 of the total charge across it?
    b. How long until the capacitor has 99% of the total charge across it?
    c. What is the total charge possible on the capacitor?
    d. Will it ever reach the full charge in part c.?
    e. Derive the formula for V(t) across the capacitor.
    f. Draw the graph of V vs. t for the capacitor.
    g. Draw the graph of V vs. t for the resistor.
5. If you have a 39μF capacitor and want a time constant of 5 seconds, what resistor value is needed?

Answers to Selected Problems

1. .
2. a. 0 V b. 10 V
3. 3.3 Fb. 54 Ω
4. a. 0.43 seconds b. 1.8 seconds c. 4.7×10^{-4}C d. No, it will asymptotically approach it. e. The graph is same shape as the Q(t) graph. It will rise rapidly and then tail off asymptotically towards 12 V. f. The voltage across the resistor is 12 V minus the voltage across the capacitor. Thus, it exponentially decreases approaching the asymptote of 0 V.
5. about 128kΩ

10.16 Energy Efficiency

Students will learn how to properly think about efficiency and how to calculate the efficiency of electrical devices.

Key Equations

$P = V \cdot I$; Power in electricity is the voltage multiplied by the current

$E = P \cdot \Delta t$; the electrical energy used is equal to the power dissipated multiplied by the time the circuit is running

$Eff = \frac{P_{out}}{P_{in}}$; Efficiency is the Power out divided by the Power input

Assuming same time periods: $Eff = \frac{E_{out}}{E_{in}} = \frac{Work}{E_{in}}$

Guidance

Conservation of Energy & Electrical Efficiency:
Electrical energy is useful to us mostly because it is easy to transport and can be easily converted to or from other forms of energy. Of course, conversion involves waste, typically as heat.
Electrical energy consumed can be determined by multiplying power by time $(E = P\Delta t)$. Recall the equations for mechanical and thermal energy/work $(PE = mgh, KE = 1/2mv^2, Q = mc\Delta T)$. An important idea is the *efficiency* of an electrical device: the fraction of electrical energy consumed that goes into doing useful work (E_{out}/E_{in}), expressed as a percentage.

Example 1

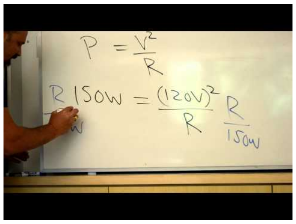
You use a 100 W electric motor to lift a 10 kg mass 5 m, and it takes 20 s.

The electrical energy consumed is $E_{elec} = P \cdot t = (100\ W)(20\ s) = 2000\ J$

The work done is against gravity, so we use $PE = mgh = (10\ kg)(10\ m/s^2)(5\ m) = 500\ J$

The efficiency is $E_{out}/E_{in} = (500\ J)/(2000\ J) = 0.25 = 25\%$ efficient

Watch this Explanation



MEDIA

Click image to the left for more content.

Time for Practice

1. The useful work a light bulb does is emitting light (duh). The rest is #8220;wasted#8221; as heat.

   a. If a standard incandescent 60 W bulb is on for 1 minute and generates 76 J of light energy, what is its efficiency? How much heat energy is produced?
   b. If the efficiency of a CFL (c ompact fl uorescent) bulb is 20%, how many joules of light energy will a 60 W bulb produce?
2. Most microwave ovens are 1000 W devices. You heat 1 cup (8 fl. oz., or 30 ml) of room temperature water $(20°C)$for 30 seconds in a microwave.
   a. What current does the oven draw?
   b. If the water heats up to $90°C$, what is the heating efficiency of the oven?
3. The 2010 Toyota Prius $(m = 1250\ kg)$battery is rated at 201.6 V with a capacity of 6.5 Ah.
   a. What is the total energy stored in this battery on a single charge?
   b. If you used the battery alone to accelerate to 65 mph one time (assuming no friction), what percentage of the battery capacity would you use?

Answers to Selected Problems

1. a. 2.1%, 3524 J b. 720 J
2. a. 8.3A b. 29%
3. a. $4.7 \times 10^6\ J$ b. 11.2%