Center of Mass

We will need to determine the center of mass of objects for upcoming problems. The center of mass is a point in an object where the mass seems to be concentrated.

Types of Motion - Large Objects

The motion of large objects can be divided into two types, ***translational*** and ***rotational***.

translational motion - the motion of an object from one point to another

rotational motion - the motion of an object about one point (pivot point or fulcrum)

Figure 10.15 The wrench is rotating around the mark on the wrench while the mark is moving in a straight line.

Physics - McGraw-Hill Ryerson, Page 491

Torque occurs when a force is applied to an object and that force causes the object to rotate.

Torque can be defined as:

$$\tau = r F_{\perp}$$

τ * -> torque (Nm)

* this symbol represents the Greek letter *tau*

r -> distance from pivot point to the application of the force (m)

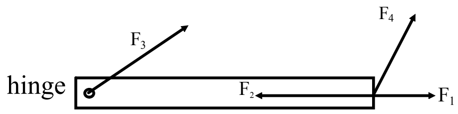
F -> force applied (N)

θ -> angle between r and F when they start at the same point (degrees)

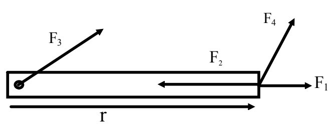
Torque is a ***vector***. The direction of torque is based on the direction in which the force would cause the object to rotate if it were acting alone.

CW: clockwise (-)
CCW: counter-clockwise (+)

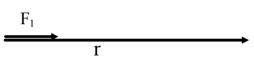
The diagram below shows four forces acting on a door. Which forces will cause the door to rotate?



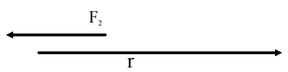
Only the component of F_4 perpendicular to r produces torque.



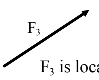
We can verify our previous answers by examining the equation.



F_1: $\theta = 0°$
$\sin 0° = 0$
$\tau = 0$ Nm

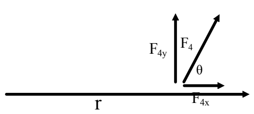


F_2: $\theta = 180°$
$\sin 180° = 0$
$\tau = 0$ Nm



F_3 is located at the pivot point.

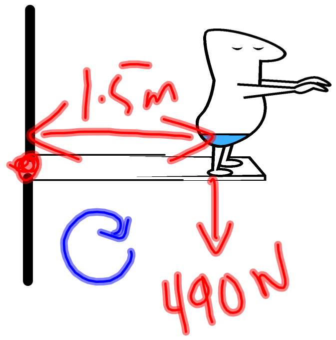
F_3: $r = 0$ m
$\tau = 0$ Nm



F_4: $r \neq 0$ m and $\sin \theta \neq 0$
F_{4x} will cause the door to rotate!

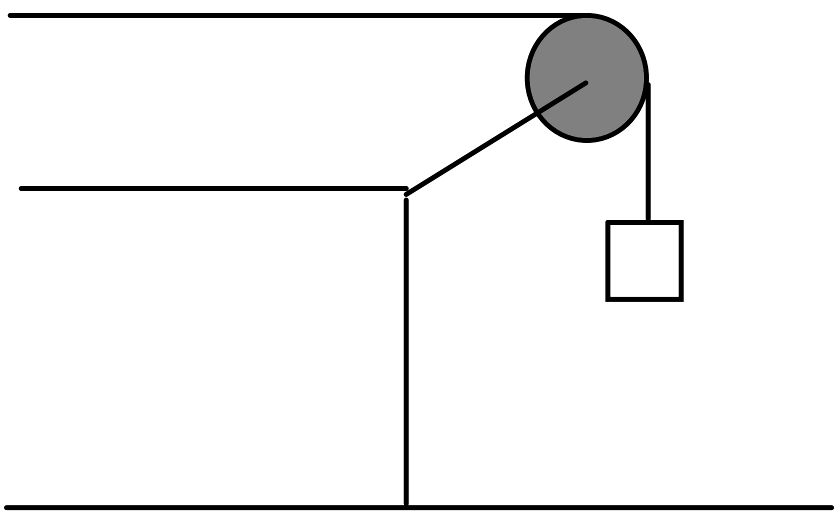
Label the Pivot Point

Example: A 490 N man stands at the end of a diving board at a distance of 1.5 m from the point at which it is attached to the tower. What is the torque the man exerts on the board?
(735 Nm, CW or -735 Nm)

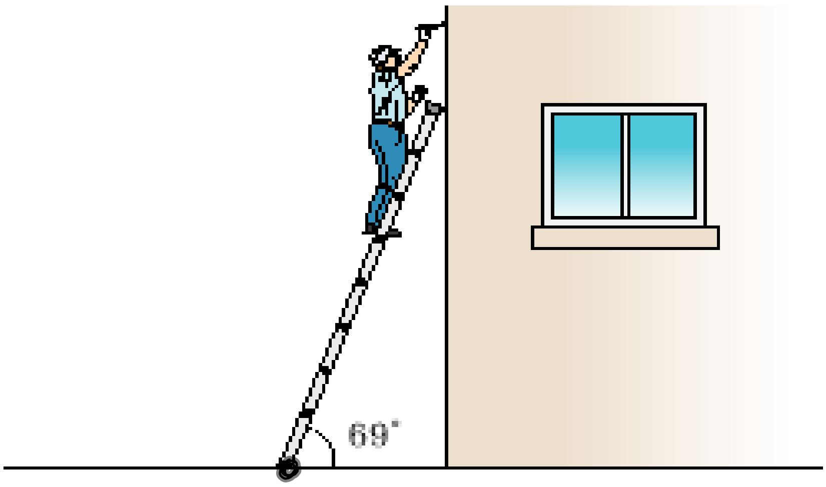


$$\tau = r F_{\perp}$$
$$= (1.5)(490)$$
$$\tau = 735\ Nm\ CW$$
$$\boxed{= -735\ Nm}$$

Example: A 5.0 kg mass is attached as shown to a pulley of radius 5.0 cm. What torque is produced by the mass?
(2.5 Nm, CW or -2.5 Nm)



Example: A 64 kg painter is standing three fourths of the distance up a ladder that is 3.0 m long. If the ladder makes an angle of 69^o with the ground, what torque does the painter's weight exert on the ladder? (5.1×10^2 Nm, CW)



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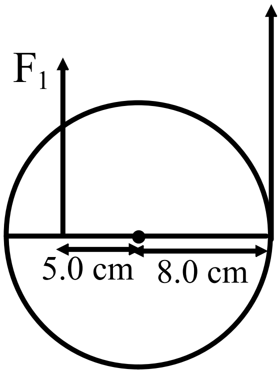
Net Torque

Just as net force sometimes plays a part in a problem, so does net torque. Net torque is the vector sum of all torques.

$$\tau_{net} = \sum \tau_{orques}$$

Example: Two forces act on the cylinder as shown in the diagram below. If F_1 = 10 N and F_2 = 15 N, what is the net torque on the cylinder? (0.70 Nm, CCW)



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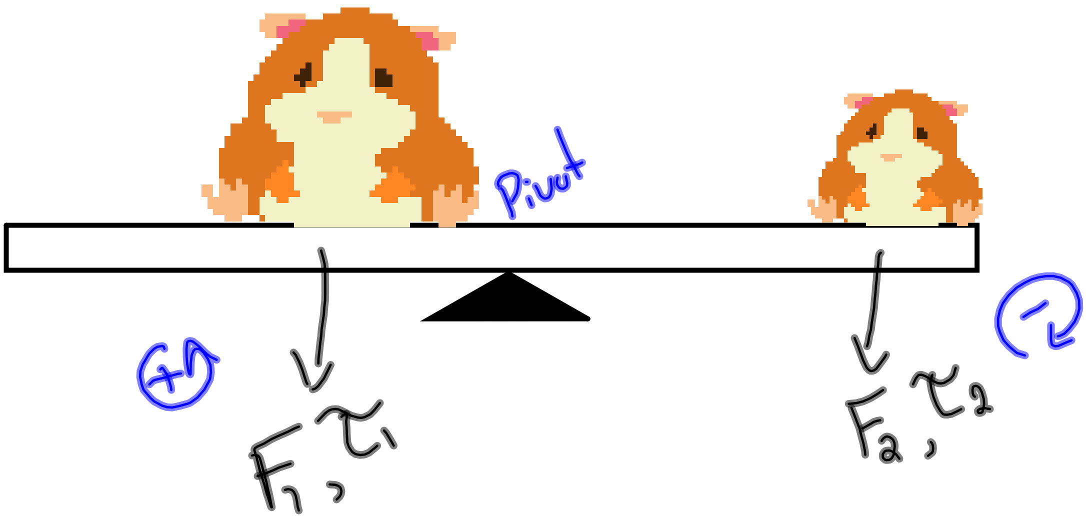
Example: A massless board serves as a seesaw for two giant hamsters as shown below. One hamster has a mass of 30 kg and sits 2.5 m from the pivot point. At what distance from the pivot point must a 25 kg hamster place himself to balance the seesaw? (3.0 m)



$$\tau_{net} = \tau_1 + \tau_2$$

$$= +r_1F_1 + (-r_2F_2)$$

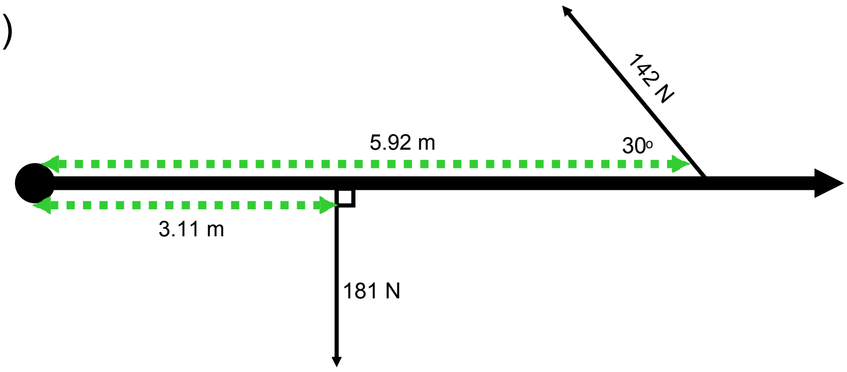
$$0 = (2.5)(30)(9.81) - r_2(25)(9.81)$$

$$0 = 736\ Nm - 245 r_2$$

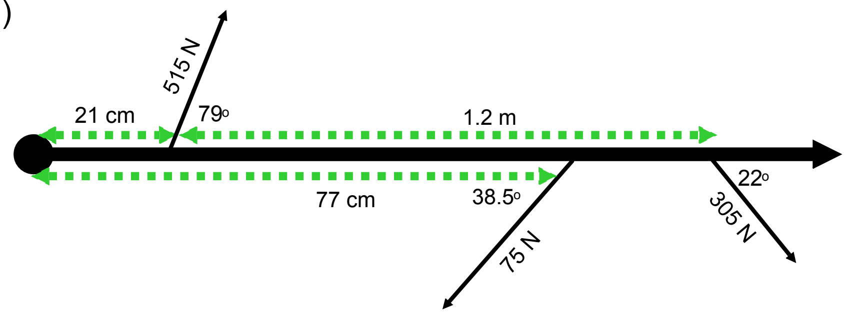
$$\frac{-736}{-245} = r_2 \rightarrow \boxed{3.0\ m = r_2}$$

Net Torque Practice

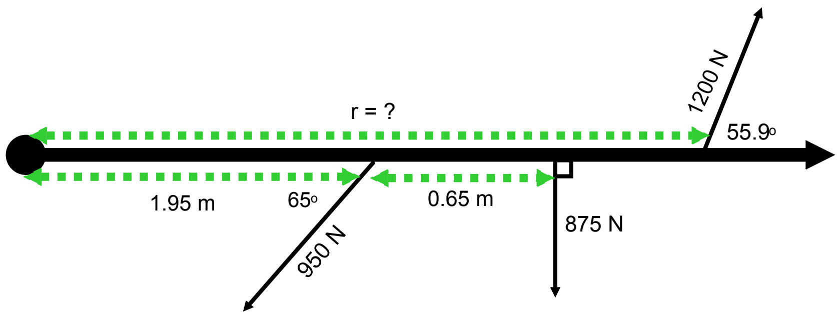
#1)



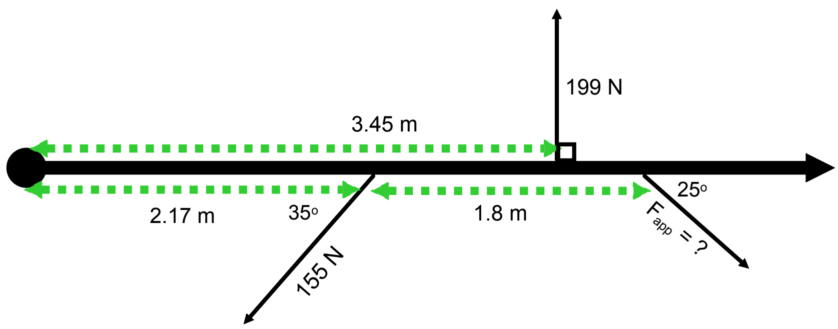
#2)



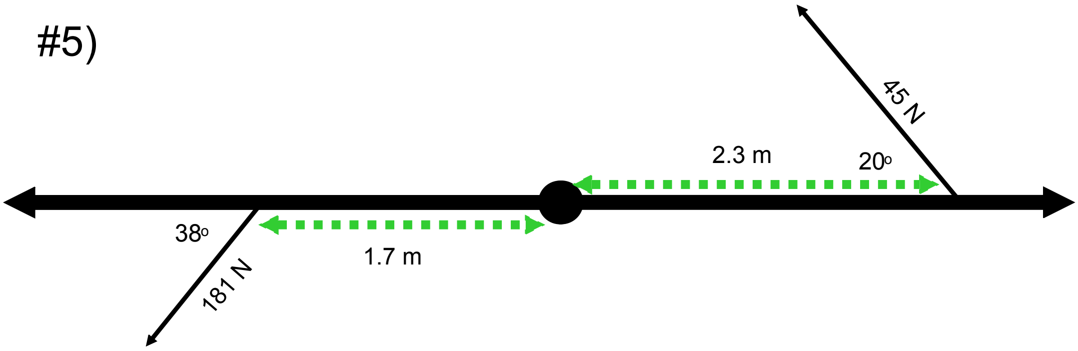
#3) τ_{net} = 0 Nm



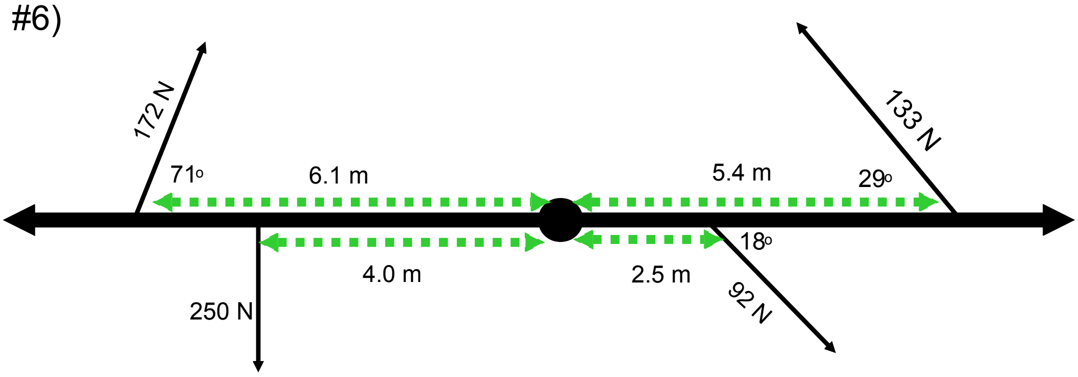
#4) τ_{net} = 0 Nm



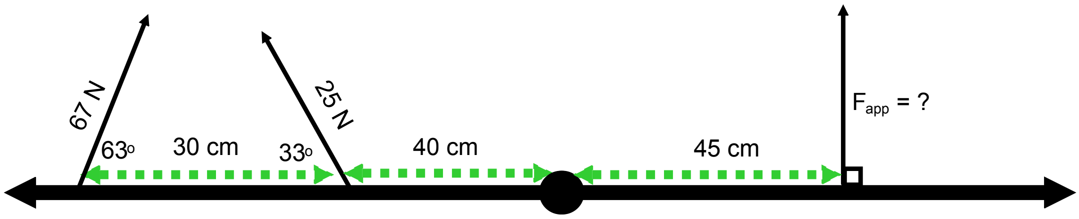
#5)



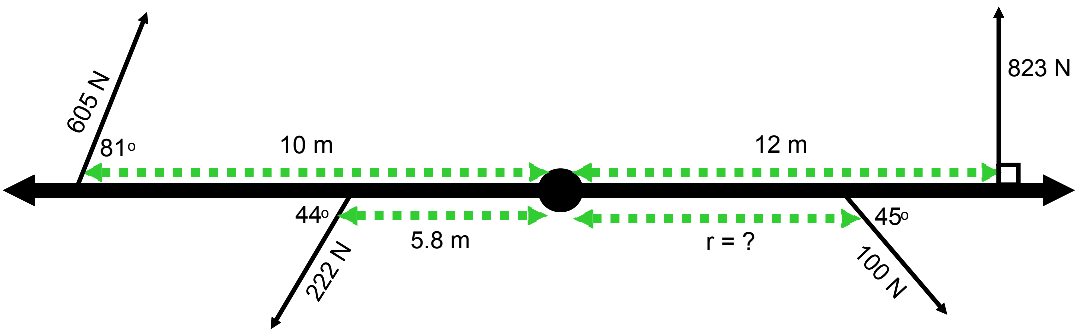
#6)



#7) τ_{net} = 0 Nm

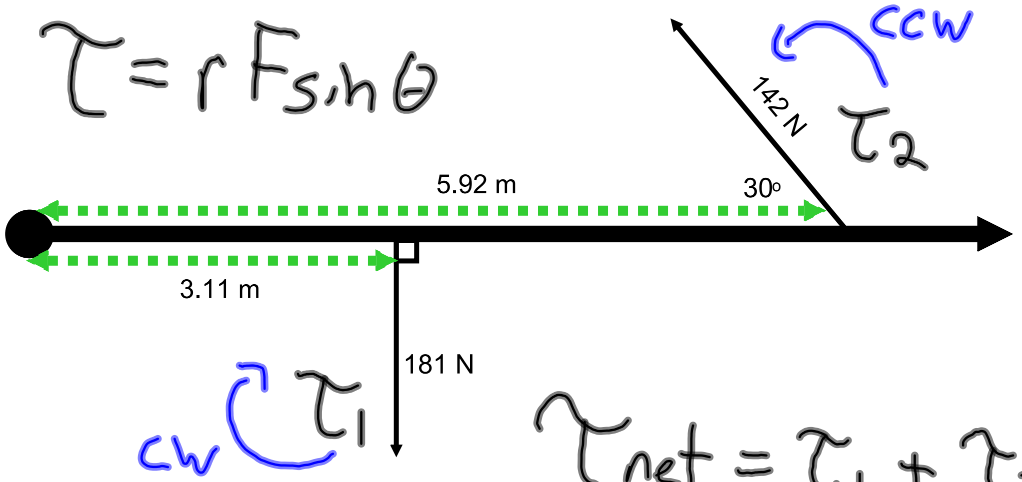


#8) τ_{net} = 0 Nm



Net Torque Practice - Solutions

#1) $\tau = rF\sin\theta$



$\tau_{net} = \tau_1 + \tau_2$

$\tau_1 = -(3.11\,m)(181)\sin 90°$

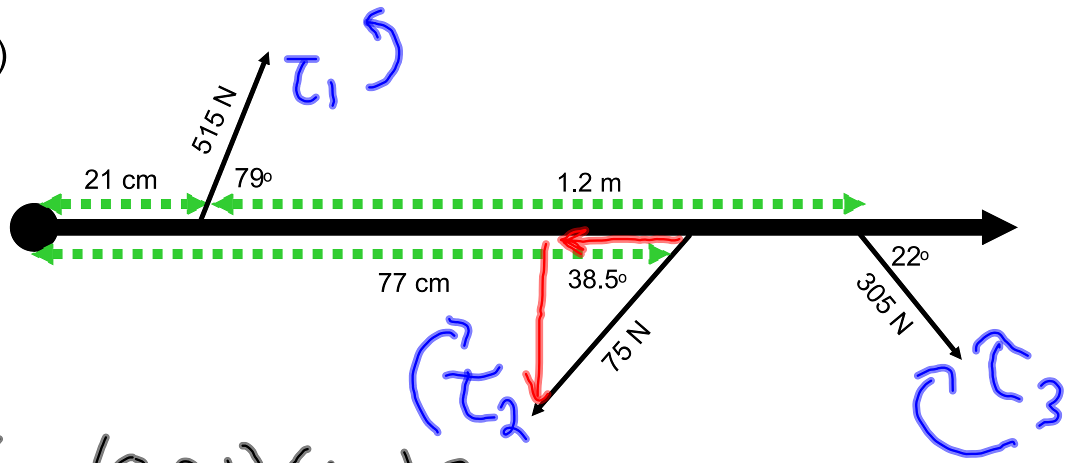
$= -563\ Nm$

$\tau_2 = 420\ Nm \Leftarrow (5.92\,m)(142)\sin 30°$

$\tau_{net} = -563\ Nm + 420\ Nm$

$= -143\ Nm$ or $143\ Nm\ [cw]$

#2)



$\tau_1 = (0.21)(515)\sin 79$

$= 106\ Nm$

$\tau_2 = -(0.77)(75)\sin 38.5°$

$= -35.9\ Nm$

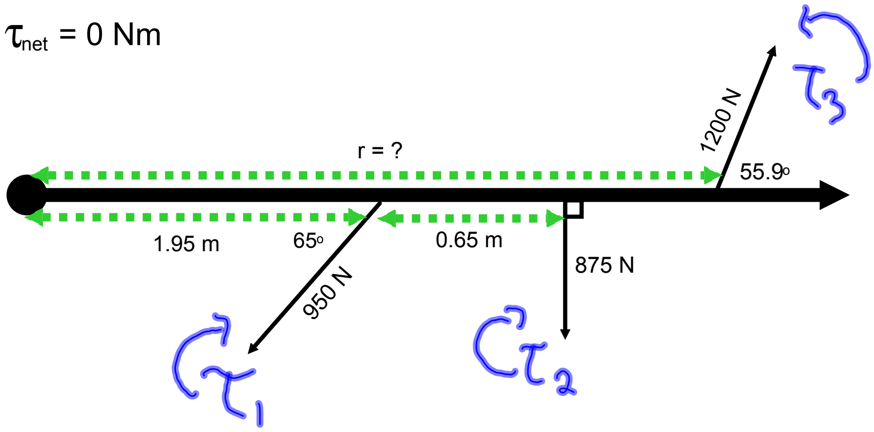
$\tau_3 = -(1.2 + 0.21)(305)(\sin 22°)$

$= -161\ Nm$

$\tau_{net} = \tau_1 + \tau_2 + \tau_3 = 106\ Nm - 35.9\ Nm - 161\ Nm$

$\tau_{net} = -90.9\ Nm$ or $90.9\ Nm$ [cw]

#3) $\tau_{net} = 0$ Nm



$$\tau_1 = -(1.95)(950)\sin 65^\circ$$
$$= -1679 \text{ Nm}$$

$$\tau_2 = -(1.95 + 0.65)(875)\sin 90^\circ$$
$$= -2275 \text{ Nm}$$

$$\tau_3 = +r(1200)\sin 55.9^\circ$$
$$= 994r$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$

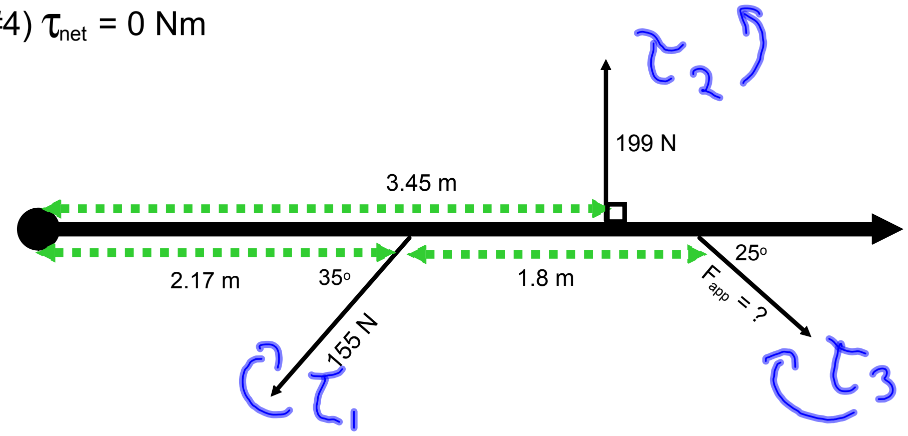
$$0 = -1679 \text{ Nm} - 2275 \text{ Nm} + (994 \text{ N})r$$

$$3954 \text{ Nm} = (994 \text{ N})r$$

$$\frac{3954 \text{ Nm}}{994 \text{ N}} = r$$

$$\boxed{3.98 \text{ m} = r}$$

#4) $\tau_{net} = 0$ Nm



$$\tau_1 = (2.17)(155)\sin 35° = -193 \text{ Nm}$$

$$\tau_2 = (3.45)(199)\sin 90° = 687 \text{ Nm}$$

$$\tau_3 = -(2.17 + 1.8) F \sin 25° = -1.68F$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$

$$0 = -193 \text{ Nm} + 687 \text{ Nm} - 1.68F$$

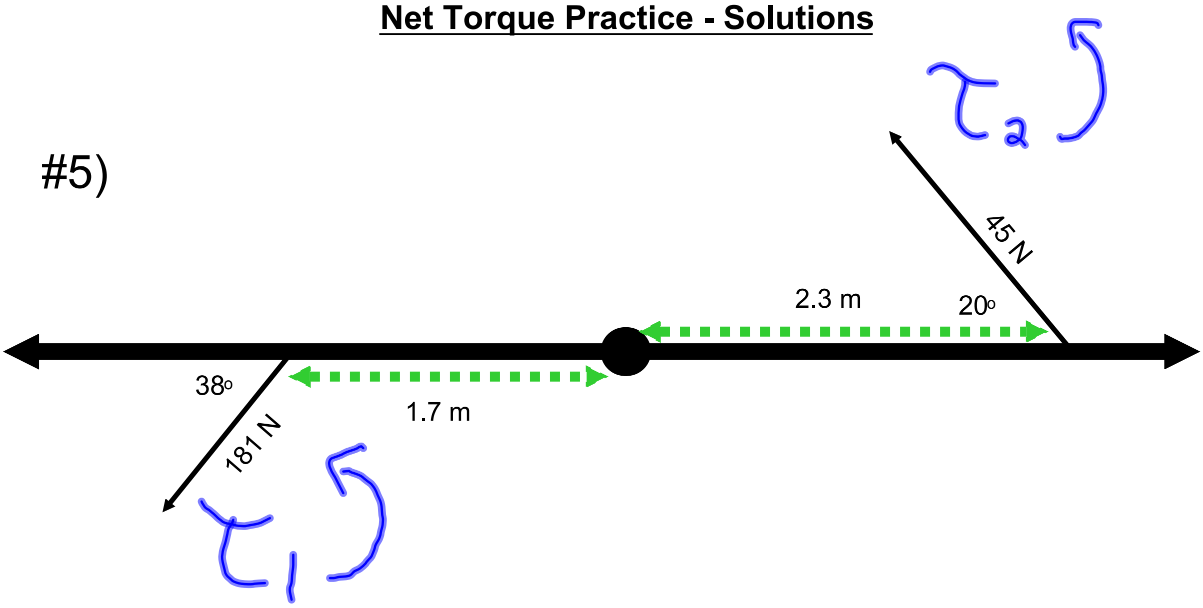
$$-494 = -1.68F$$

$$\frac{-494 \text{ Nm}}{-1.68 \text{ m}} = F$$

$$\boxed{294 \text{ N} = F}$$

Net Torque Practice - Solutions

#5)



$$\tau_1 = (1.7)(181)\sin 38^\circ$$

$$= 189\ \text{Nm}$$

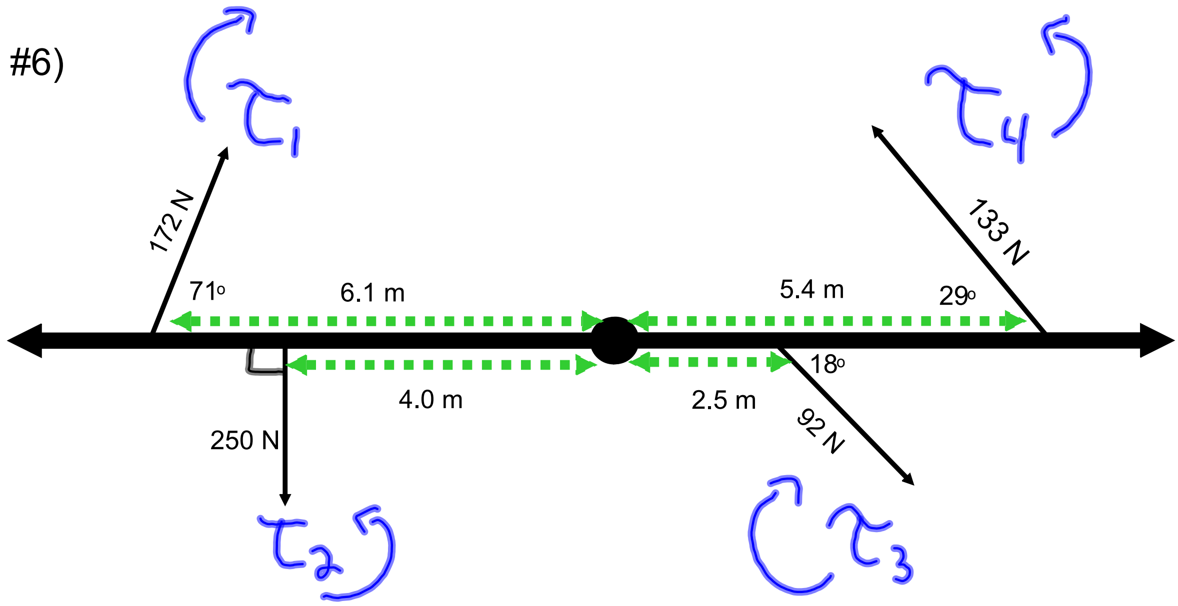
$$\tau_2 = (2.3)(45)\sin 20^\circ$$

$$= 35.4\ \text{Nm}$$

$$\tau_{net} = 189\ \text{Nm} + 35.4\ \text{Nm}$$

$$= 224\ \text{Nm}\ [\text{ccw}]$$

#6)



$$\tau_1 = -(6.1)(172)\sin 71^\circ$$
$$= -992\ Nm$$

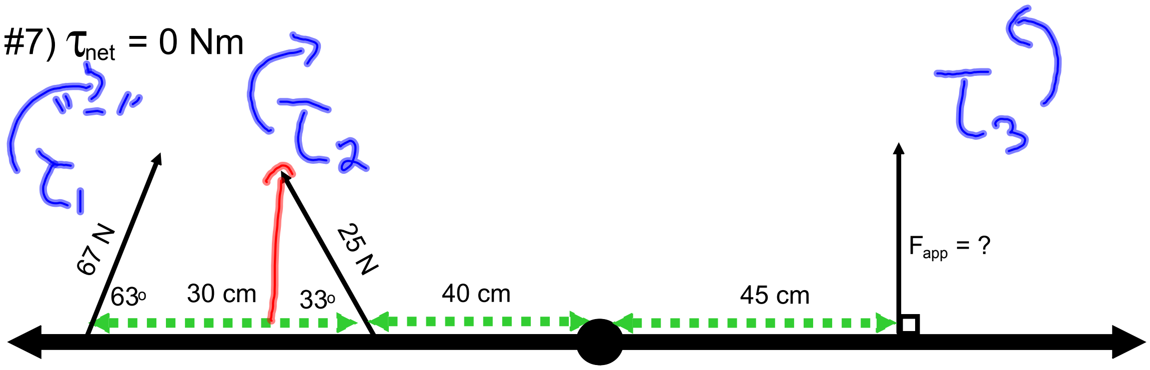
$$\tau_2 = (4.0)(250)\sin 90^\circ$$
$$= 1000\ Nm$$

$$\tau_3 = -(2.5)(92)\sin 18^\circ$$
$$= -71.1\ Nm$$

$$\tau_4 = (5.4)(133)\sin 29^\circ$$
$$= 348\ Nm$$

$$\tau_{net} = -992\ Nm + 1000\ Nm - 71\ Nm + 348\ Nm$$
$$= 285\ Nm \text{ or } 285\ Nm\ [ccw]$$

#7) τ_{net} = 0 Nm



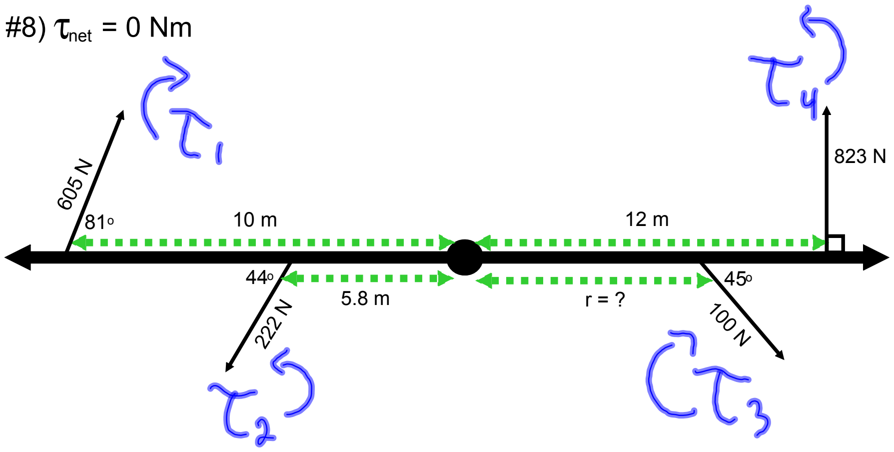
$$\tau_1 = -(0.30+0.40)(67)\sin 63^\circ$$
$$= -41.8\ Nm$$

$$\tau_2 = -(0.40)(25)\sin 33^\circ$$
$$= -5.45\ Nm$$

$$\tau_3 = (0.45m)(F)\sin 90^\circ$$
$$= 0.45F$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$
$$0 = -41.8\ Nm - 5.45\ Nm + 0.45F$$
$$47.25\ Nm = 0.45F$$
$$\frac{47.25\ Nm}{0.45\ m} = F$$
$$\boxed{105\ N = F}$$

#8) $\tau_{net} = 0$ Nm



$$\tau_1 = -(10)(605)\sin 81^\circ$$
$$= -5976 \text{ Nm}$$

$$\tau_2 = (5.8)(222)\sin 44^\circ$$
$$= 894 \text{ Nm}$$

$$\tau_3 = -r(100)\sin 45^\circ$$
$$= -70.7r$$

$$\tau_4 = (12)(823)\sin 90^\circ$$
$$= 9876 \text{ Nm}$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = -5976 \text{ Nm} + 894 \text{ Nm} - 70.7r + 9876 \text{ Nm}$$

$$0 = 4794 \text{ Nm} - 70.7r$$

$$-4794 \text{ Nm} = -70.7r$$

$$\frac{-4794 \text{ Nm}}{-70.7 \text{ N}} = r$$

$$\boxed{67.8 \text{ m} = r}$$