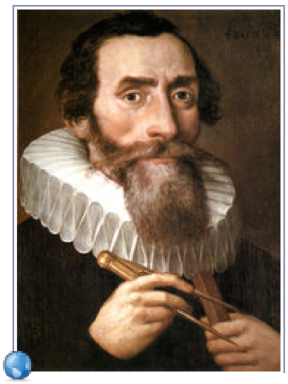
Universal Gravitation (Page 574 MHR)

Chapter 12

Theories of Planetary Motion

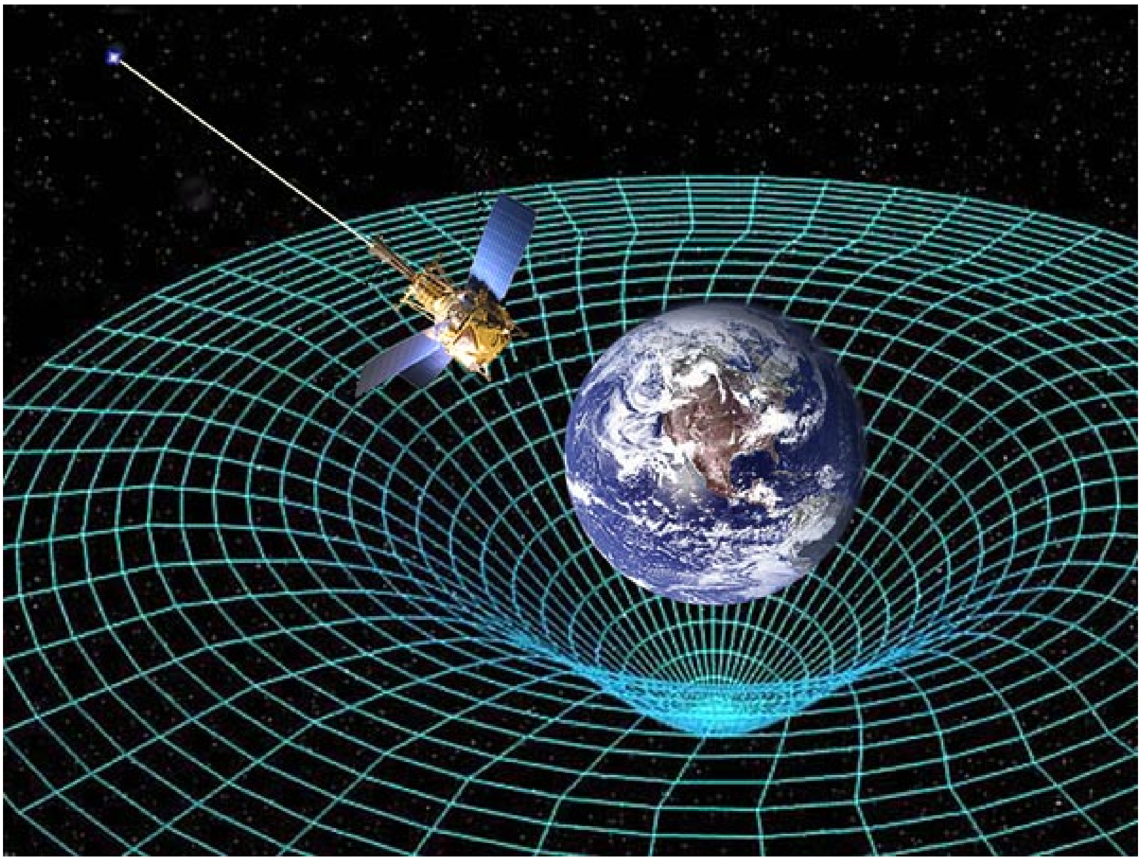
1. Tycho Brahe (1546-1601)

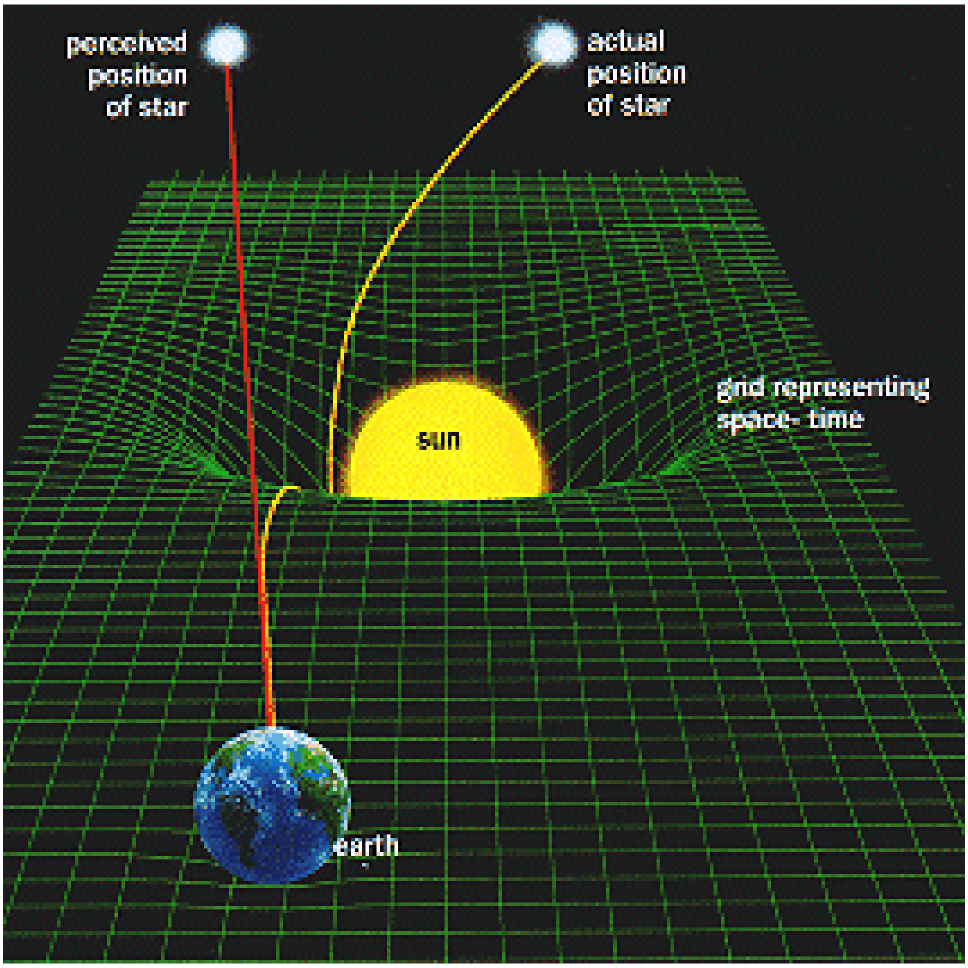
   Brahe believed in the geocentric theory of planetary motion - Earth as the center of the universe.



2. Johannes Kepler (1571-1630)

   Kepler believed in the heliocentric theory of planetary motion - Sun as the center of the universe.

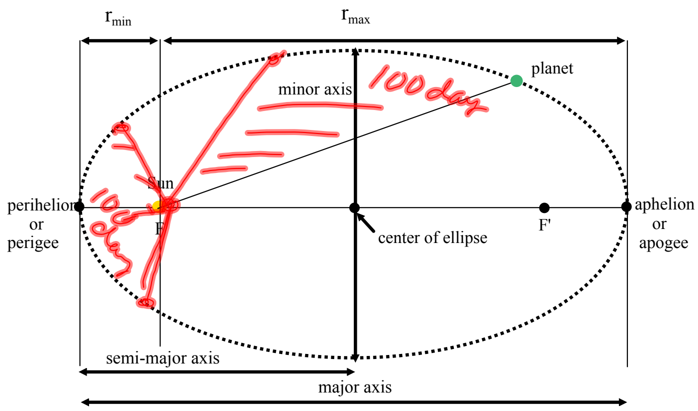




perceived
position
of star
actual
position
of star
grid representing
space- time
sun
earth

Kepler's Three Laws of Planetary Motion

1. Law of Orbits - The paths of the planets are ellipses with the center of the Sun at one focus.



eccentricity - a quantity indicating how non-circular an ellipse is
- based on a scale from 0 to 1 where a value nearer 1 implies a higher degree of non-circularity

2. Law of Areas - An imaginary line from the sun to a planet sweeps out equal areas in equal time intervals. Therefore, planets move faster when closest to the sun and slowest when farthest away.

Simulation

3. Law of Periods - The ratio of the square of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun.

$$\boxed{\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}}$$

T_A -> period of object A (s, min, h, etc)
T_B -> period of object B (s, min, h, etc)
r_A -> mean orbital radius (m, km, etc)
r_B -> mean orbital radius (m, km, etc)

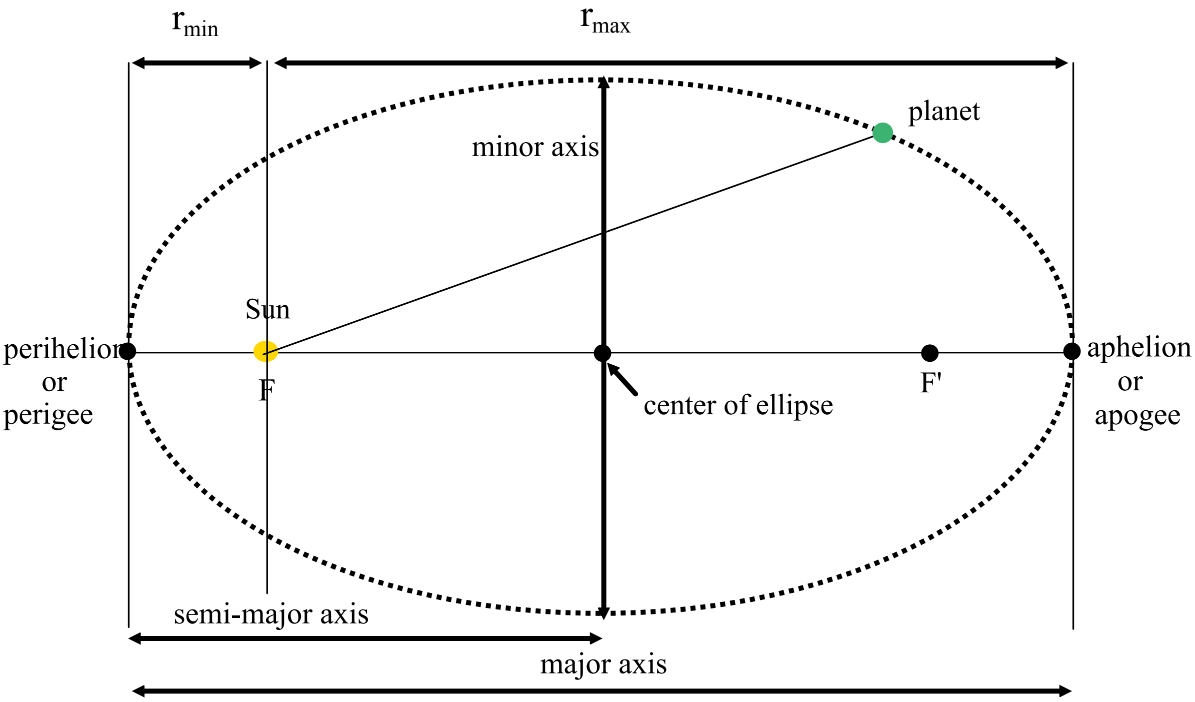
NOTE: The first two laws apply to each planet, satellite or moon. The third relates to the motion of two objects which are orbiting the same central body. The central body does not have to be the sun.

NOTE: One astronomical unit (AU) is the average distance from Earth to the Sun.

$$\boxed{1 \text{ AU} = 1.496 \times 10^{11} \text{ m}}$$

NOTE: Physical data can be found on Page 573 and 955.

Elliptical Orbit



Example:
Galileo discovered four moons of Jupiter. Io, which he measured to be 4.2 units from the center of Jupiter, has a period of 1.8 days. He measured the radius of Ganymede's orbit as 10.7 units. Use Kepler's third law to find the period of Ganymede. (7.3 days)

Example:
The fourth moon of Jupiter, Callisto, has a period of 16.7 days. Find its average distance from Jupiter using the same units Galileo used.
(19 units)

Example:
On average, Mars is 1.52 AU. Predict the time required for Mars to circle the sun in Earth days. (684 days)

Example:
Uranus requires 84 years to circle the sun. Find Uranus' orbital radius as a multiple of Earth's orbital radius. (19$\cancel{r_E}$) AU

Universal Law of Gravitation

Newton -> explained the motion of the planets
-> a force of attraction between the sun and planets keep the planets traveling along their elliptical paths

1 $F \propto m_1 m_2$

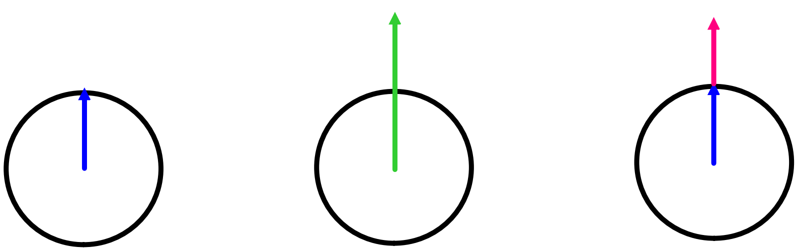
* Force of attraction is directly proportional to the product of the masses of the objects.

2 $F \propto 1/r^2$ $F = \frac{m_1 m_2}{r^2}$

* Gravitation force is inversely proportional to the distance squared.

$\Rightarrow F = Gm_1m_2/r^2$

F = gravitational force (N)
m_1 = mass of object 1 (kg)
m_2 = mass of object 2 (kg)
r = distance between masses (m)
G = universal gravitational constant (6.672×10^{-11} Nm^2/kg^2)



radius | orbital radius | radius + distance above surface

Proportionality Questions

$F = G\frac{m_1 m_2}{r^2}$

By what factor will the force of gravity on a person change if the mass of this person doubles?

$F \propto m_1 m_2$ Force doubled

$F \propto 1/r^2$

Universal Gravitation Handout

Text Pages 580 # 1 - 8

Read Pg 580, 583

By what factor will the force of gravity on a person change if the person orbits a distance three times the Earth's radius?

$\frac{1}{r^2}$ 9 times less

$F_{new} = \frac{2}{9} \times 900\,N$

$= 200\,N$

$$F = \frac{G M_1 M_2}{r^2}$$

Sun's Mass
1.989×10^{30} Kg
Radius $= 6.96 \times 10^{8}$ m

$M_E = 5.974 \times 10^{24}$ Kg

$r = 1.5 \times 10^{11}$ m $\Longrightarrow$ (1 AU)

$$F = \frac{6.672 \times 10^{-11} (5.974 \times 10^{24})(1.989 \times 10^{30})}{(1.5 \times 10^{11})^2}$$

$$F = \frac{7.928 \times 10^{44}}{(1.5 \times 10^{11})^2}$$

$$\boxed{= 3.52 \times 10^{22} \text{ N}}$$

Moon: 7.349×10^{22} Kg
Diameter: 3476 Km $= 3.476 \times 10^{6}$ m

Earth-Moon Dist $=$ 384 400 Km
$= 3.844 \times 10^{8}$ m

Sun Mass $= 1.989 \times 10^{30}$ Kg

Sun Radius $= 6.96 \times 10^{8}$ m

Proton Mass $\doteq 1.673 \times 10^{-27}$ Kg

Electron Mass $= 9.109 \times 10^{-31}$ Kg

Force of Gravity between Earth & Sun: 3.52×10^{22} N

" " " " Earth & Moon: 1.97×10^{20} N

Tidal Force: Change in gravitation force with distance (Force Gradient)

-4.70×10^{11} N/m Sun

$$F = G M_E M_P (r^{-2})$$

-1.03×10^{12} N/m

Moon

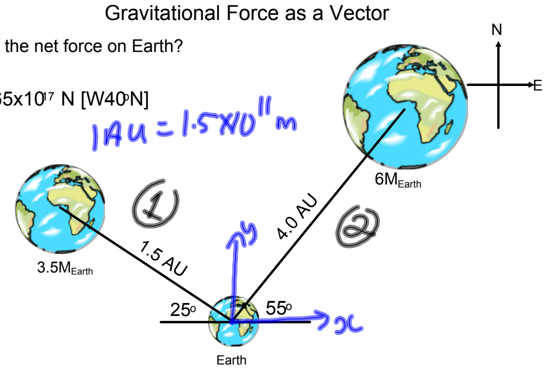
$$\frac{dF}{dr} = \frac{-2 M_E M_P}{r^3}$$

Tides are caused by the Moon because the change in force is greater per unit distance than that of the Sun.

Gravitational Force as a Vector

What is the net force on Earth?

F = 1.65x10[17] N [W40ºN]



$$F_1 = \frac{G(3.5M_E)(M_E)}{(1.5\ AU)^2} \qquad F_2 = \frac{G(6M_E)(M_E)}{(4.0AU)^2}$$

$$= \frac{G\,3.5M_E^2}{2.25} \qquad = 0.375GM^2$$

$$= 1.56GM^2$$

$$F_{1x} = -1.56GM^2\cos 25^\circ \qquad F_{2x} = 0.375GM^2\cos 55^\circ$$

$$= -1.41GM^2 \qquad = 0.215GM^2$$

$$F_{1y} = +1.56GM^2\sin 25^\circ \qquad F_{2y} = 0.375GM^2\sin 55^\circ$$

$$= +0.695GM^2 \qquad = 0.307GM^2$$

$$F_x = F_{1x} + F_{2x} \qquad F_y = F_{1y} + F_{2y}$$

$$= 0.695 + 0.307$$

$$F_x = -1.41GM^2 + 0.215GM^2 \qquad = 1.002GM^2$$

$$= -1.195GM^2$$

$$F_{net} = \sqrt{F_x^2 + F_y^2} \qquad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$F_{net} = \sqrt{(-1.195GM^2)^2 + (1.002GM^2)^2}$$

$$= \sqrt{1.428G^2M^4 + 1.004G^2M^4}$$

$$= \sqrt{2.432G^2M^4}$$

$$= 1.56GM^2 \qquad \theta = \tan^{-1}\left(\frac{1.002\cancel{GM^2}}{1.195\cancel{GM^2}}\right)$$

$$\theta = 40^\circ$$

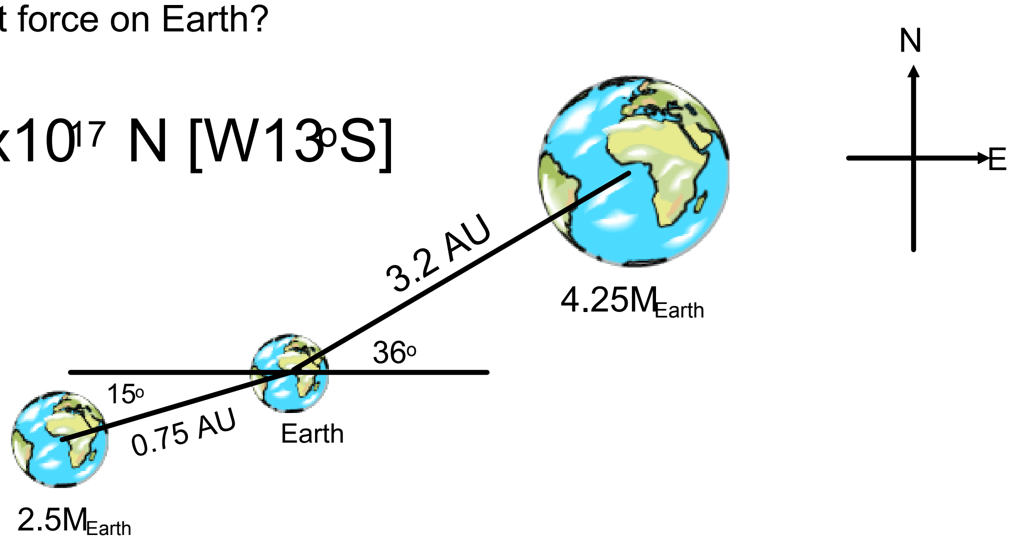
$$F_{(Newtons)} = \frac{1.56(\overset{G}{6.672\times10^{-11}})(\overset{M_E}{5.974\times10^{24}})^2}{(1.5\times10^{11}m)^2}$$

$$= \frac{3.71\times10^{39}}{(1.5\times10^{11})^2}$$

$$\boxed{\vec{F} = 1.65\times10^{17}N\ [W\ 40^\circ N]}$$

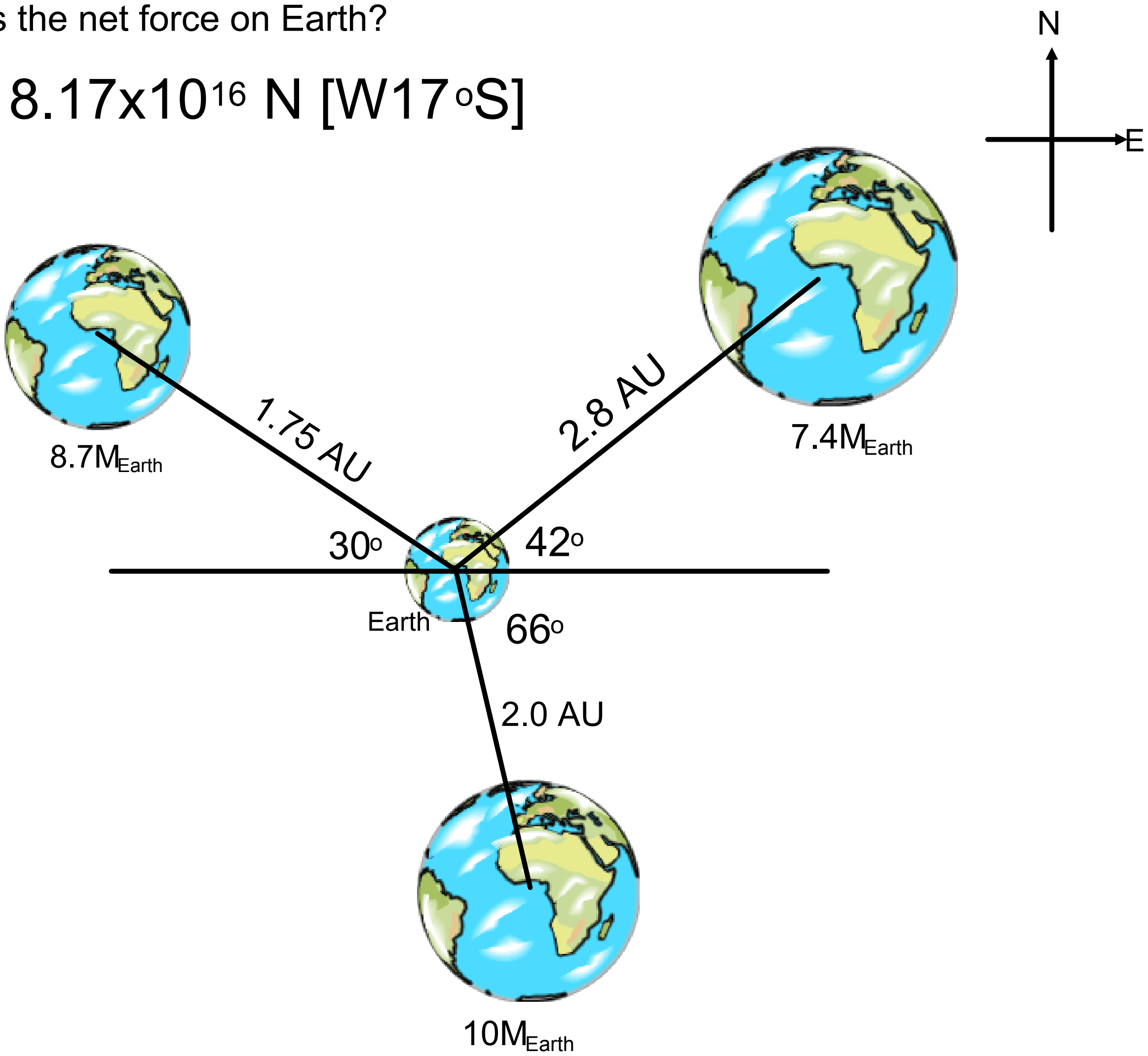
What is the net force on Earth?

$\boldsymbol{F} = 4.3 \times 10^{17}$ N [W13^{o}S]



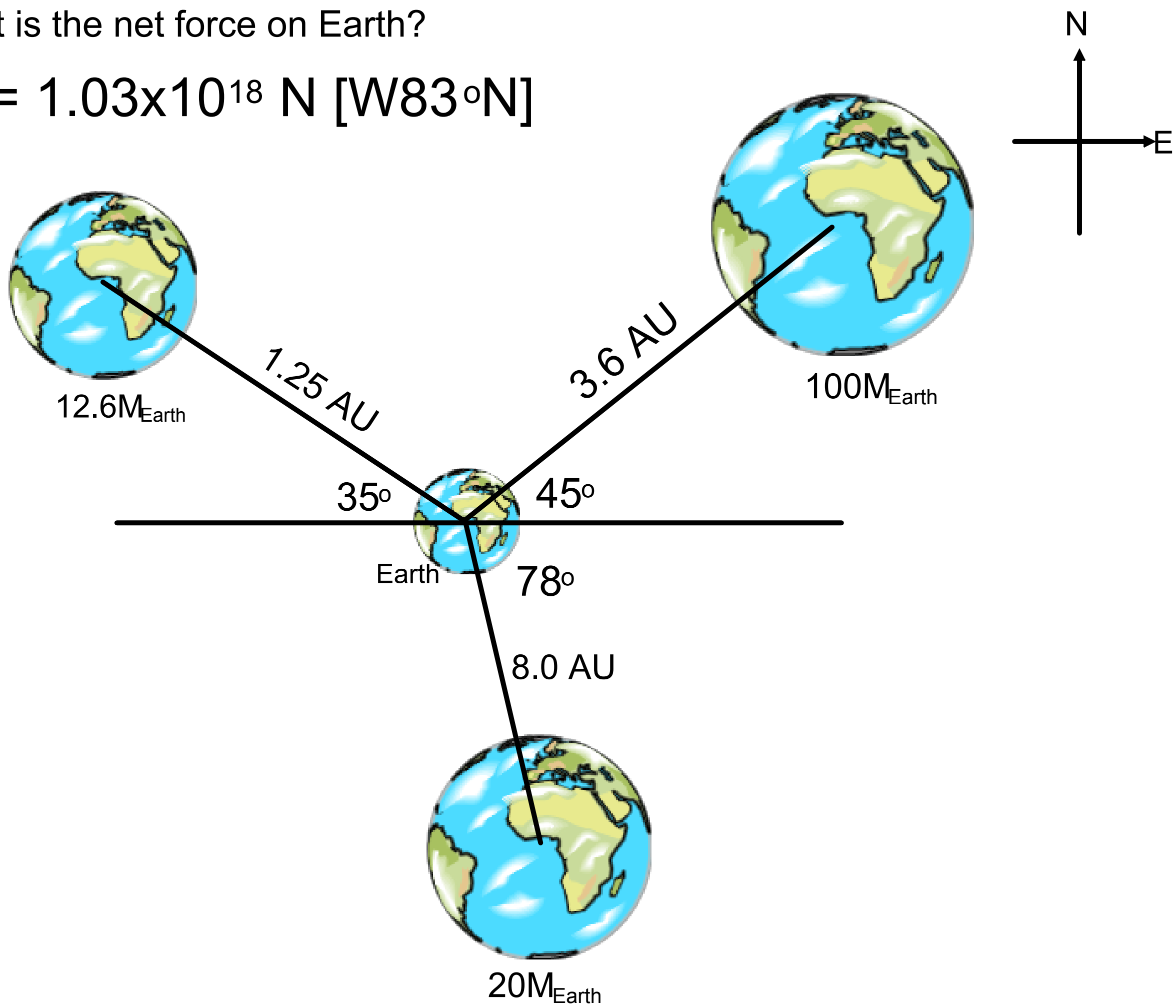
What is the net force on Earth?

$\boldsymbol{F} = 8.17\text{x}10^{16}$ N [W17^{o}S]



What is the net force on Earth?

$\boldsymbol{F} = 1.03 \times 10^{18}$ N [W83^{o}N]



Finding the Formula for "g"

$$W = m_{object}\, g \qquad\qquad F = \frac{G m_{object}\, m_{planet}}{r^2}$$

$$W = F$$

$$\cancel{m_{object}}\, g = \frac{G \cancel{m_{object}}\, m_{planet}}{r^2}$$

$$\boxed{g = \frac{G M_p}{r^2}}$$

What is the gravitational acceleration 1000km above the Earth's surface?

$g = 7.33 m/s^2$

What is the acceleration of gravity on the surface of the Moon? $g = 1.62 m/s^2$

Suppose a new planet is discovered that has a radius 3 times that of the Earth and a mass 5 times that of the Earth. What is the acceleration of gravity on that planet's surface?

$g = 5.45 m/s^2$

How far from the Earth's surface do you have to go to experience 0.5*g*? $r = 2.64 \times 10^6 m$

What is the acceleration of the Moon towards the Earth? Earth towards the Moon?

$g_{m \to E} = 2.69 \times 10^{-3} m/s^2$ | $g_{E \to m} = 3.33 \times 10^{-5} m/s^2$

A cannon is fired from the surface of Mars. The speed is 25 m/s at an angle of 30° to the surface. What is the maximum height of the cannon ball? How far from the cannon does the ball land? (Mass of Mars = 6.421×10^{23} kg, Radius = 3.39×10^6 m) $y_{max} = 21 m$ $x = 148 m$

Text Pg. 595 (read the reflection).
#'s 22 - 28, 30 - 33.

Suppose a planet is discovered that is 2.5 times as massive as the Earth. What would its radius have to be for its gravitational acceleration to be the same as Earth's?

$$g = \frac{G M_p}{r^2}$$

$$9.81 = \frac{(6.672 \times 10^{-11})(2.5)(5.97 \times 10^{24})}{r^2}$$

$$r = \sqrt{\frac{(6.672 \times 10^{-11})(2.5)(5.97 \times 10^{24})}{9.81}} \qquad r = 1.0 \times 10^{7}\ \text{m}$$

A planet is found to be 3.75 times the radius of Earth. For it to have g = 9.81 m/s^2, what must its mass be relative to the Earth's?

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Deriving Kepler's Third Law

Finding Masses of Solar System Objects: What force is need to keep an object moving in a circle?

Force of Gravity = Centripetal Force

$$v = \sqrt{\frac{GM_p}{r}}$$

$$\frac{GM_1M_p}{r^2} = \frac{M_1v^2}{r}$$

$$\frac{G\cancel{M_1}M_p}{r^{\cancel{2}}} = \frac{\cancel{M_1}v^2}{\cancel{r}}$$

$$\frac{GM_p}{r} = v^2 *$$

← Orbital velocity of an object a distance ***r*** from the centre of a planet with mass $\boldsymbol{M_p}$.

Recall that: $v = \dfrac{2\pi r}{T}$

$$\frac{GM_p}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM_p}{r} = \frac{4\pi^2 r^2}{T^2}$$

Rearrange to get:

$$\frac{GM_p}{4\pi^2} = \frac{r^3}{T^2} \quad *$$

The le-hand-side is a constant for a given planet.

Example: A star at the edge of the Andromeda galaxy appears to be orbiting the centre of that galaxy at a speed of about 2.0×10^{2} km/s. The star is 5×10^{9} AU from the center of the galaxy. Calculate a rough estimate of the mass of the Andromeda galaxy. (4×10^{41} kg)

$$v = 2.0 \times 10^{2}\ \text{km/s}$$

$$v = 2.0 \times 10^{5}\ \text{m/s}$$

$$r = 5 \times 10^{9}\ \text{AU} \times 1.5 \times 10^{11}\ \frac{\text{m}}{\text{AU}}$$

$$r = 7.5 \times 10^{20}\ \text{m}$$

$$\frac{G M_A}{r} = v^2 \qquad M_A = \frac{r v^2}{G}$$

$$M_A = \frac{(7.5 \times 10^{20})(2 \times 10^{5})^2}{6.672 \times 10^{-11}}$$

$$\boxed{= 4.5 \times 10^{41}\ \text{kg}}$$