Center of Mass

We will need to determine the center of mass of objects for upcoming problems. The center of mass is a point in an object where the mass seems to be concentrated.

Types of Motion - Large Objects

The motion of large objects can be divided into two types, ***translational*** and ***rotational***.

translational motion - the motion of an object from one point to another

rotational motion - the motion of an object about one point (pivot point or fulcrum)

Figure 10.15 The wrench is rotating around the mark on the wrench while the mark is moving in a straight line.

Physics - McGraw-Hill Ryerson, Page 491

Torque occurs when a force is applied to an object and that force causes the object to rotate.

Torque can be defined as:

$$\tau = r F_{\perp} = r F \sin\theta$$

τ* -> torque (Nm)

* this symbol represents the Greek letter *tau*

r -> distance from pivot point to the application of the force (m)

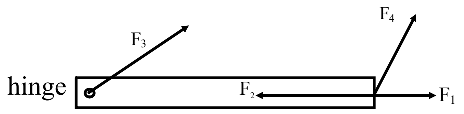
F -> force applied (N)

θ -> angle between r and F when they start at the same point (degrees)

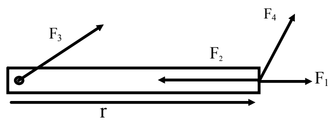
Torque is a ***vector***. The direction of torque is based on the direction in which the force would cause the object to rotate if it were acting alone.

CW: clockwise (-)
CCW: counter-clockwise (+)

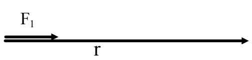
The diagram below shows four forces acting on a door. Which forces will cause the door to rotate?



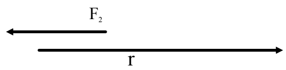
Only the component of F_4 perpendicular to r produces torque.



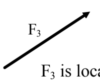
We can verify our previous answers by examining the equation.



F_1: $\theta = 0°$
$\sin 0° = 0$
$\tau = 0$ Nm

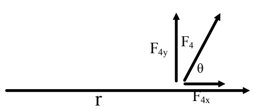


F_2: $\theta = 180°$
$\sin 180° = 0$
$\tau = 0$ Nm



F_3 is located at the pivot point.

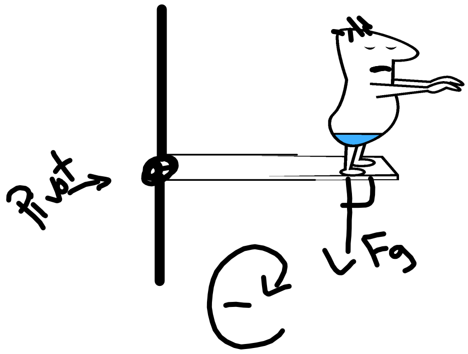
F_3: $r = 0$ m
$\tau = 0$ Nm



F_4: $r \neq 0$ m and $\sin\theta \neq 0$
F_{4x} will cause the door to rotate!

Label the Pivot Point

Example: A 490 N Mr. Martin stands at the end of a diving board at distance of 1.5 m from the point at which it is attached to the tower. What is the torque the man exerts on the board? (735 Nm, CW or -735 Nm)



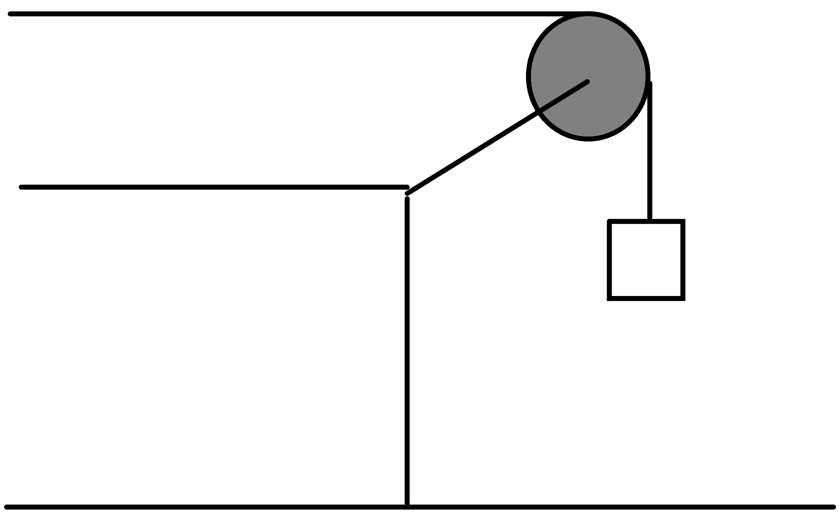
$$\tau = rF\sin\theta$$

$$= (1.5\,m)(490)\sin 90^\circ$$

$$= 735\ Nm\ CW$$

or -735 Nm

Example: A 5.0 kg mass is attached as shown to a pulley of radius 5.0 cm. What torque is produced by the mass? (2.5 Nm, CW or -2.5 Nm)

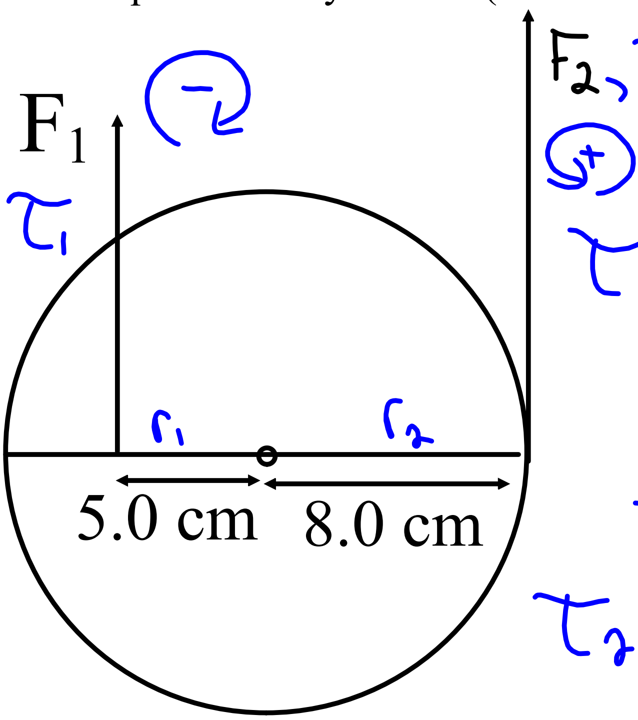


Net Torque

Just as net force sometimes plays a part in a problem, so does net torque. Net torque is the vector sum of all torques.

$$\tau_{net} = \sum \text{Torques}$$

Example: Two forces act on the cylinder as shown in the diagram below. If F_1 = 10 N and F_2 = 15 N, what is the net torque on the cylinder? (0.70 Nm, CCW)



$$\tau_1 = -r_1 F_1$$
$$= -(0.05)(10)$$
$$= -0.5 \text{ Nm}$$

$$\tau_2 = +r_2 F_2$$
$$= (0.08)(15)$$
$$= 1.2 \text{ Nm}$$

$$\tau_{net} = \sum \text{Torques}$$
$$= \tau_1 + \tau_2$$
$$= (-0.5 \text{ Nm}) + 1.2 \text{ Nm}$$
$$\boxed{= +0.70 \text{ Nm}}$$

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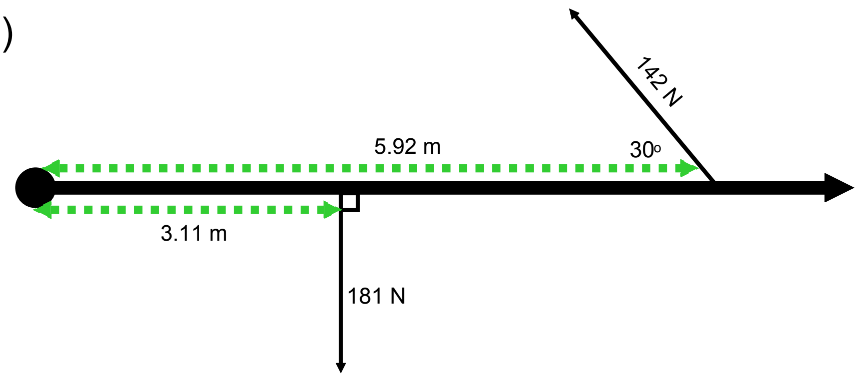
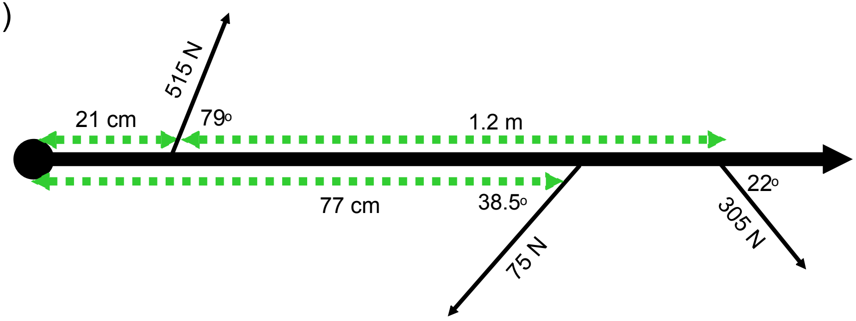
Net Torque Practice

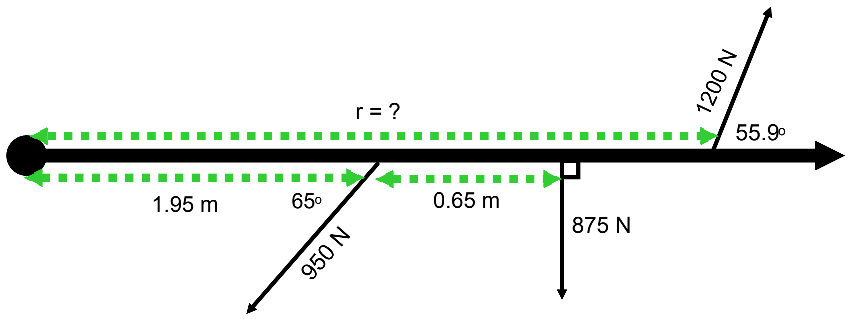
#1)



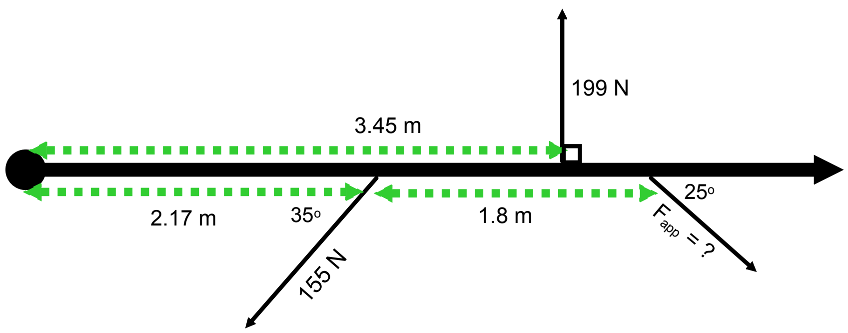
#2)



#3) τ_{net} = 0 Nm



#4) τ_{net} = 0 Nm



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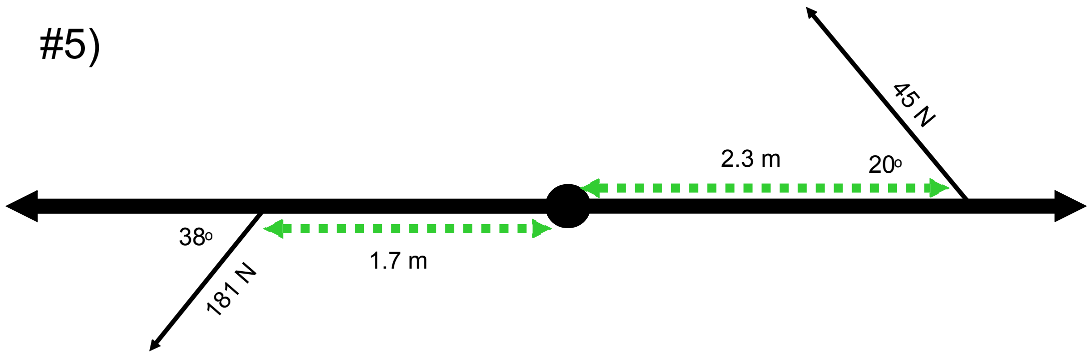
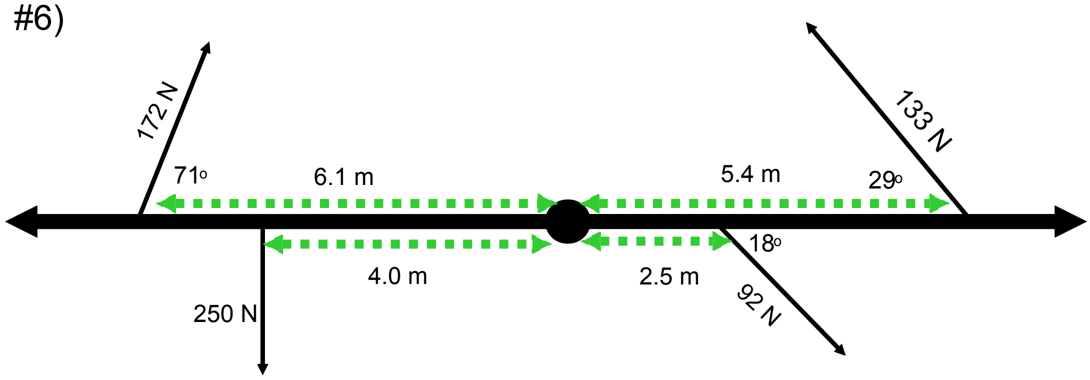
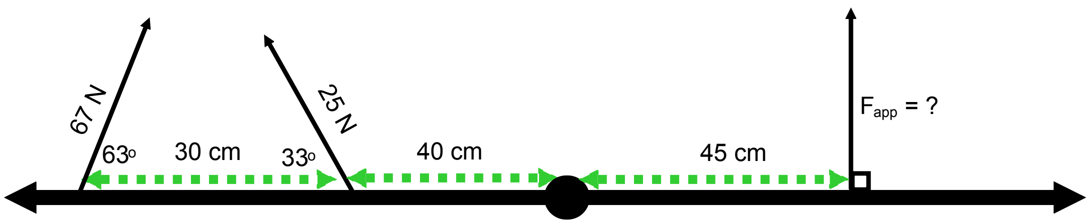
#5)



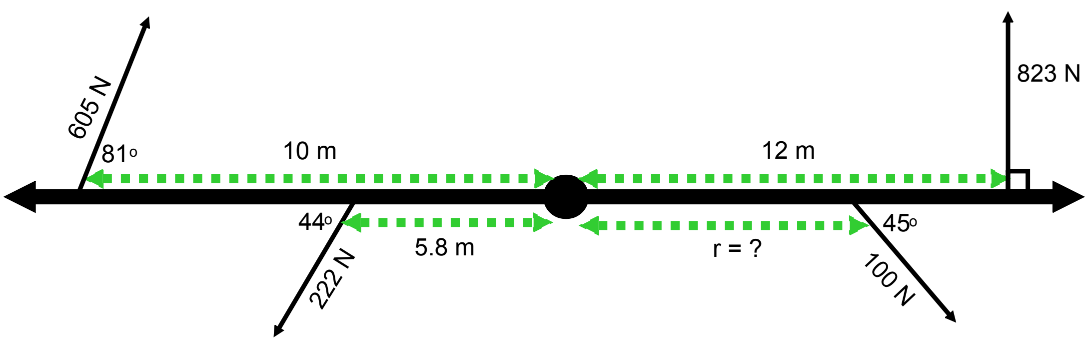
#6)



#7) τ_{net} = 0 Nm

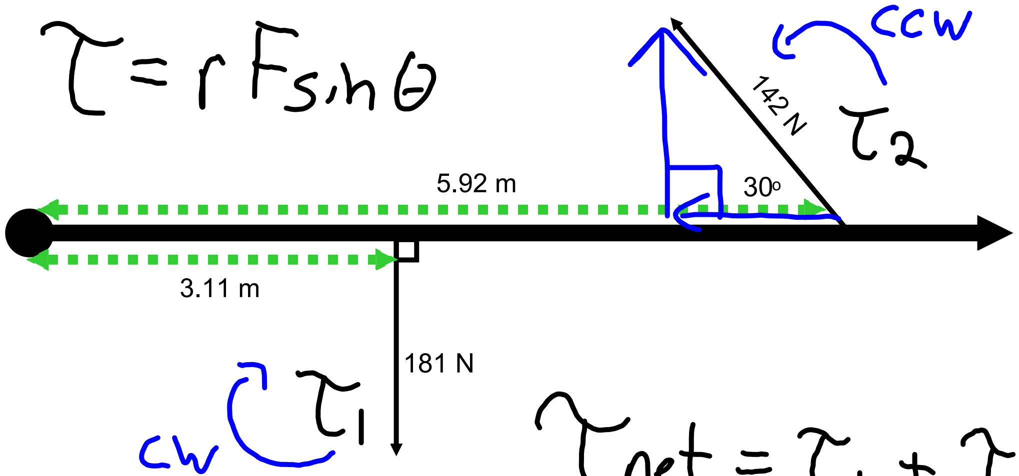


#8) τ_{net} = 0 Nm



Net Torque Practice - Solutions

#1) $\tau = rF\sin\theta$



$\tau_{net} = \tau_1 + \tau_2$

$\tau_1 = -(3.11\,m)(181)\sin 90°$

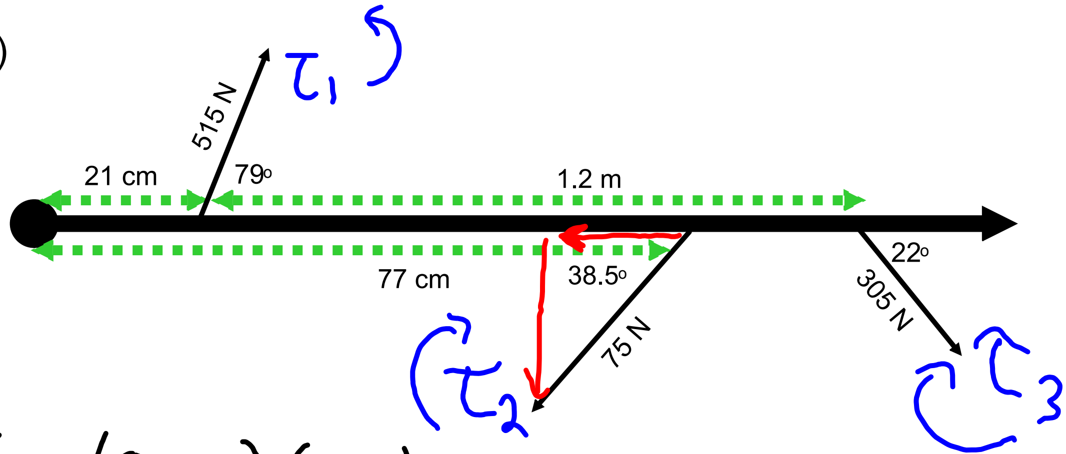
$= -563\ Nm$

$\tau_2 = 420\ Nm \Leftarrow (5.92\,m)(142)\sin 30°$

$\tau_{net} = -563\ Nm + 420\ Nm$

$= -143\ Nm$ or $143\ Nm\ [cw]$

#2)



$$\tau_1 = (0.21)(515)\sin 79$$
$$= 106 \text{ Nm}$$

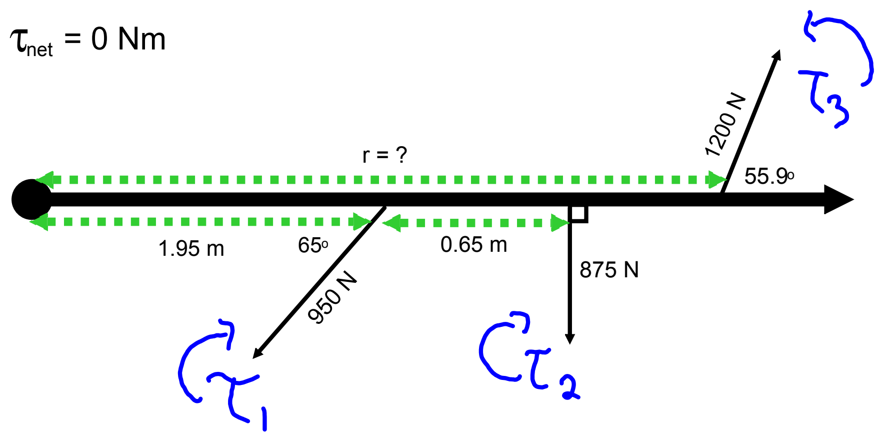
$$\tau_2 = -(0.77)(75)\sin 38.5°$$
$$= -35.9 \text{ Nm}$$

$$\tau_3 = -(1.2 + 0.21)(305)(\sin 22°)$$
$$= -161 \text{ Nm}$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 = 106 \text{ Nm} - 35.9 \text{ Nm} - 161 \text{ Nm}$$

$$\boxed{\tau_{net} = -90.9 \text{ Nm or } 90.9 \text{ Nm [cw]}}$$

#3) $\tau_{net} = 0 \text{ Nm}$



$$\tau_1 = -(1.95)(950)\sin 65°$$
$$= -1679 \text{ Nm}$$

$$\tau_2 = -(1.95 + 0.65)(875)\sin 90°$$
$$= -2275 \text{ Nm}$$

$$\tau_3 = +r(1200)\sin 55.9°$$
$$= 994r$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$

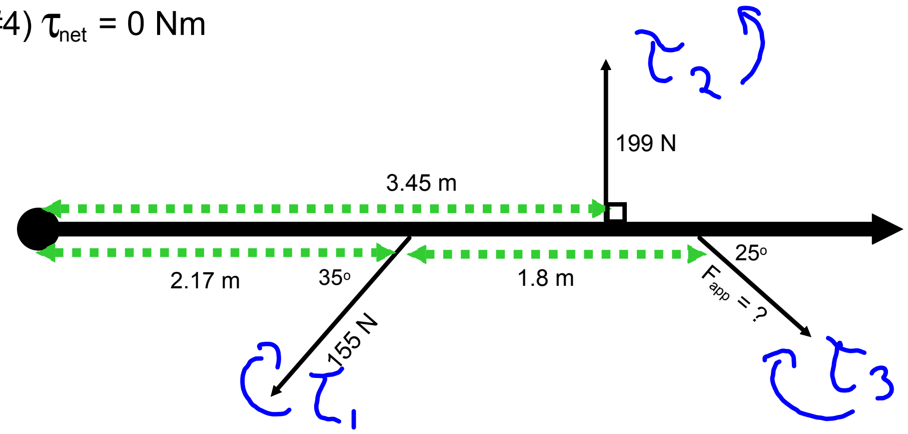
$$0 = -1679 \text{ Nm} - 2275 \text{ Nm} + (994 \text{ N})r$$

$$3954 \text{ Nm} = (994 \text{ N})r$$

$$\frac{3954 \text{ Nm}}{994 \text{ N}} = r$$

$$\boxed{3.98 \text{ m} = r}$$

#4) $\tau_{net} = 0$ Nm



$$\tau_1 = (2.17)(155)\sin 35^\circ$$
$$= -193 \text{ Nm}$$

$$\tau_2 = (3.45)(199)\sin 90^\circ$$
$$= 687 \text{ Nm}$$

$$\tau_3 = -(2.17 + 1.8) F \sin 25^\circ$$
$$= -1.68F$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$

$$0 = -193 \text{ Nm} + 687 \text{ Nm} - 1.68F$$

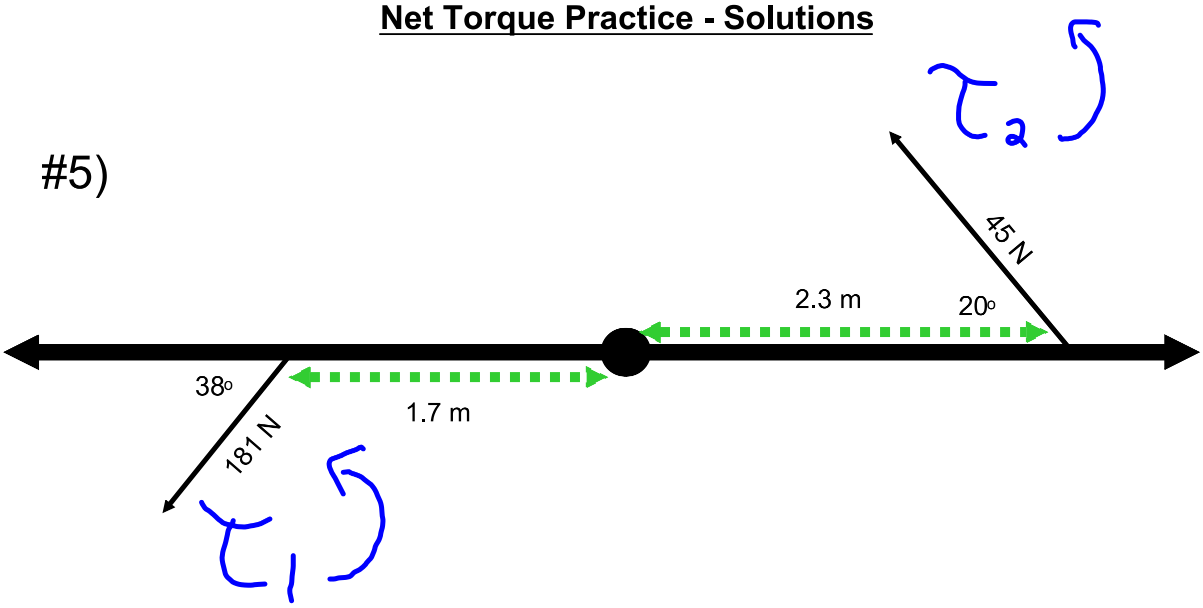
$$-494 = -1.68F$$

$$\frac{-494 \text{ Nm}}{-1.68 \text{ m}} = F$$

$$\boxed{294 \text{ N} = F}$$

Net Torque Practice - Solutions

#5)



$$\tau_1 = (1.7)(181)\sin 38°$$

$$= 189\ Nm$$

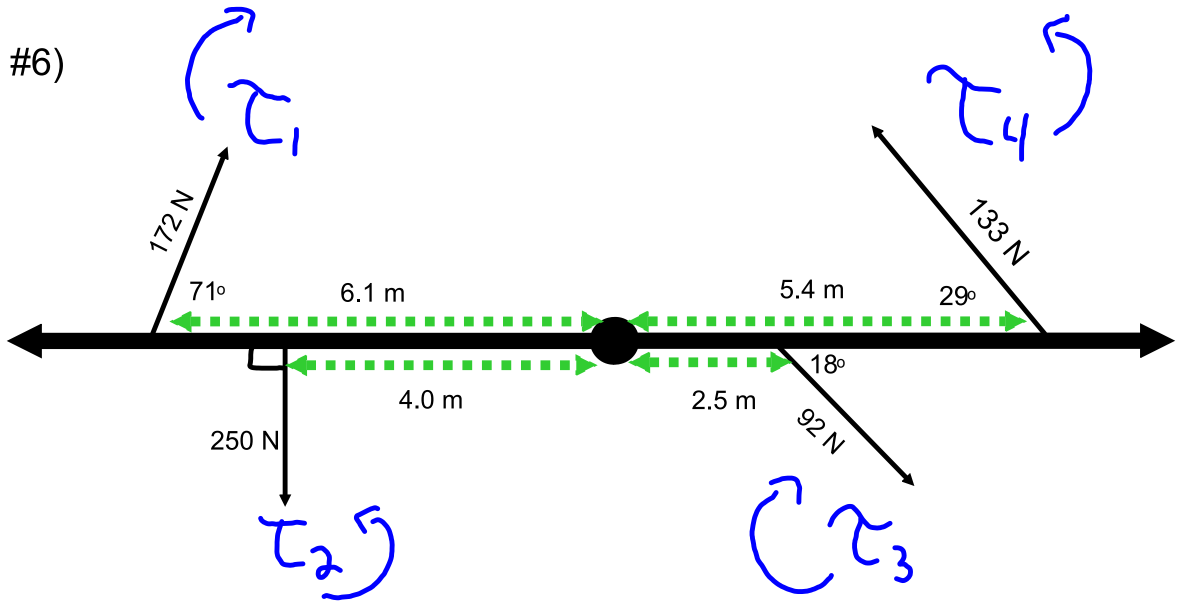
$$\tau_2 = (2.3)(45)\sin 20°$$

$$= 35.4\ Nm$$

$$\tau_{net} = 189\ Nm + 35.4\ Nm$$

$$\boxed{= 224\ Nm\ [ccw]}$$

#6)



$\tau_1 = -(6.1)(172)\sin 71°$
$= -992\ Nm$

$\tau_2 = (4.0)(250)\sin 90°$
$= 1000\ Nm$

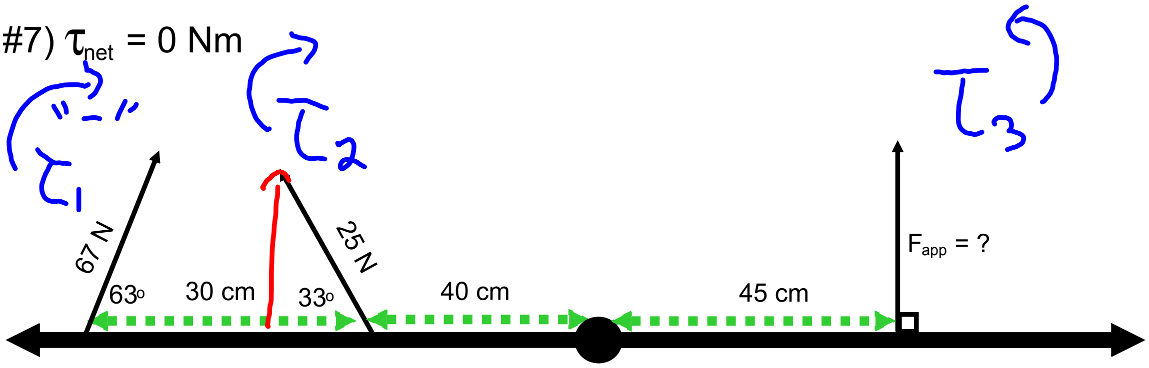
$\tau_3 = -(2.5)(92)\sin 18°$
$= -71.1\ Nm$

$\tau_4 = (5.4)(133)\sin 29°$
$= 348\ Nm$

$\tau_{net} = -992\ Nm + 1000\ Nm - 71\ Nm + 348\ Nm$

$= 285\ Nm$ or $285\ Nm$ [ccw]

#7) $\tau_{net} = 0$ Nm



$$\tau_1 = -(0.30 + 0.40)(67)\sin 63°$$
$$= -41.8 \text{ Nm}$$

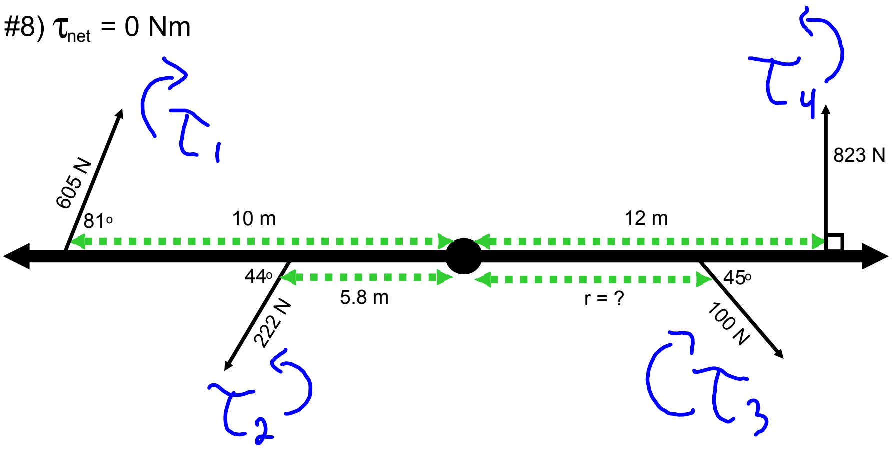
$$\tau_2 = -(0.40)(25)\sin 33°$$
$$= -5.45 \text{ Nm}$$

$$\tau_3 = (0.45\text{m})(F)\sin 90°$$
$$= 0.45F$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$
$$0 = -41.8 \text{ Nm} - 5.45 \text{ Nm} + 0.45F$$
$$47.25 \text{ Nm} = 0.45F$$
$$\frac{47.25 \text{ Nm}}{0.45 \text{ m}} = F$$

$$\boxed{105 \text{ N} = F}$$

#8) $\tau_{net} = 0$ Nm



$$\tau_1 = -(10)(605)\sin 81^\circ$$
$$= -5976 \text{ Nm}$$

$$\tau_2 = (5.8)(222)\sin 44^\circ$$
$$= 894 \text{ Nm}$$

$$\tau_3 = -r(100)\sin 45^\circ$$
$$= -70.7r$$

$$\tau_4 = (12)(823)\sin 90^\circ$$
$$= 9876 \text{ Nm}$$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \tau_4$$

$$0 = -5976 \text{ Nm} + 894 \text{ Nm} - 70.7r + 9876 \text{ Nm}$$

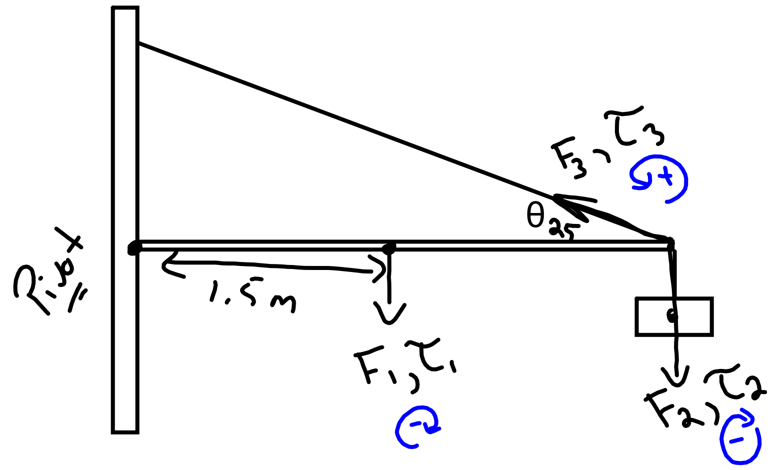
$$0 = 4794 \text{ Nm} - 70.7r$$

$$-4794 \text{ Nm} = -70.7r$$

$$\frac{-4794 \text{ Nm}}{-70.7 \text{ N}} = r$$

$$\boxed{67.8 \text{ m} = r}$$

Example: A uniform beam of mass 50.0 kg and length 3.00 m is attached to a wall with a hinge. The beam supports a sign of mass 300 kg which is suspended from its end. The beam is also supported by a wire that makes an angle of 25° with the beam. Determine the tension in the wire. (7.5×10^3 N)



$\tau_1 = -(1.5)(50)(9.81)$ $\tau_3 = (3)F_3 \sin 25$

$\tau_2 = -(3)(300)(9.81)$

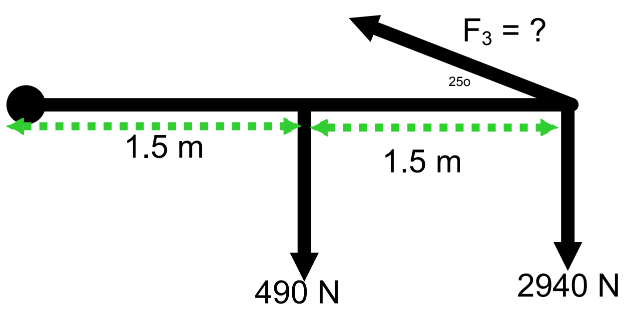
$\tau_{net} = \sum \text{Torques}$

$0 = \tau_1 + \tau_2 + \tau_3$

$0 = -736 - 8829 + 1.26 F_3$

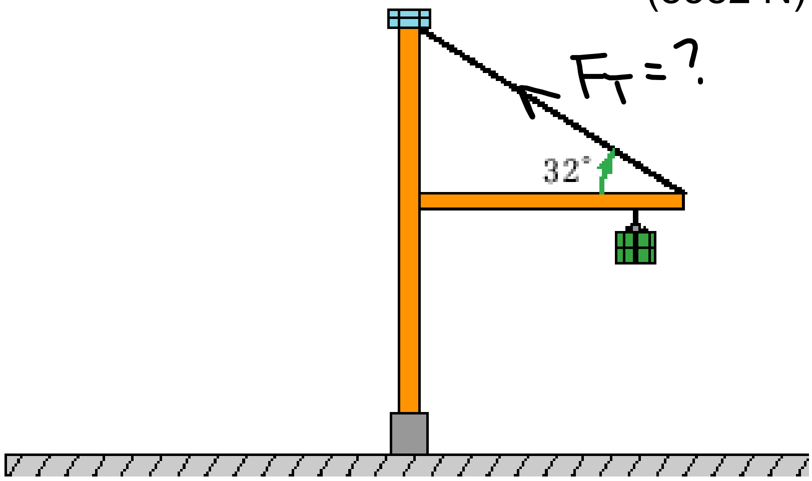
$9565 = 1.26 F_3$

$7591 \text{ N} = F_3$ ← force of Tension



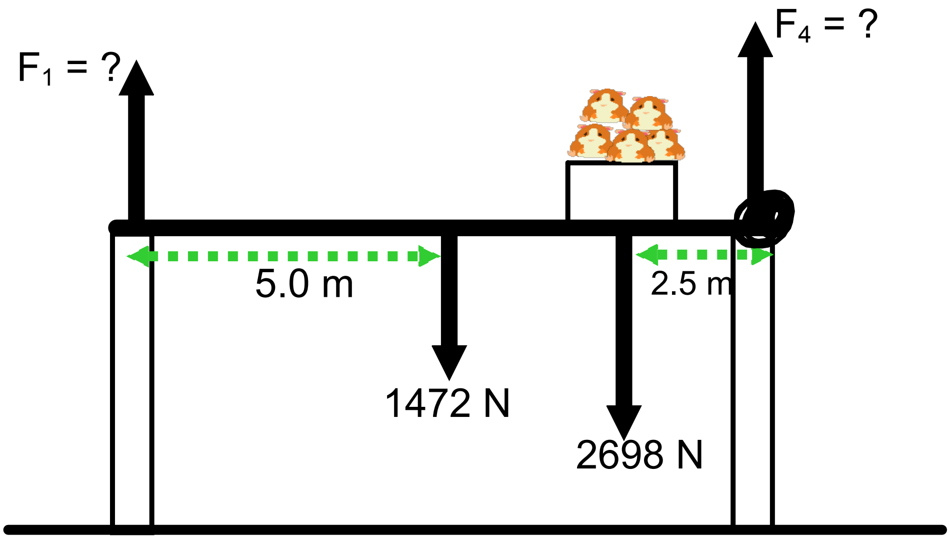
28. A crane with a movable pulley system on a horizontal arm is moving a large container. The 355 kg container is hanging from a cable that is 6.15 m out on the 7.50 m arm. The arm has a mass of 345 kg. A cable that is attached to its end makes an angle 32.0° with the horizontal arm.

(8582 N)



(a) What is the tension in the cable supporting the arm?

<u>Example</u>: A uniform 150 kg beam, 10.0 m long, supports a 275 kg box of hamsters 2.5 m from the right support column. Calculate the magnitude of the forces on the beam exerted by each of the vertical support columns.



27. Mountain climbers have placed a 3.6 kg uniform ladder across an icy crevasse. The edges of the crevasse are 4.1 m apart. The first climber starts to cross the crevasse on the ladder and reaches a point 1.8 m from the edge. The mass of the climber and her gear is 87 kg. With what force is the ice on each side of the crevasse pushing up on the ladder?

497 N 392 N

Static Equilibrium

An object is in static equilibrium if:

1. $\mathbf{v} = 0$ m/s
2. $\mathbf{F_{net}} = 0$ N
3. $\tau_{net} = 0$ Nm

Steps for Solving Static Equilibrium Problems

1. Draw a diagram.
2. Label all forces.
3. Choose a pivot point. It is helpful to place the pivot point where an unknown force exists (in a case of two unknown forces).
4 Label distances from the pivot point to the forces. (r values)
5. Write a τ_{net} equation.
6. Solve the equation(s) for the unknown.

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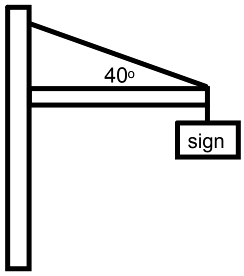
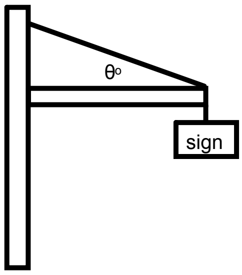
More Torque!

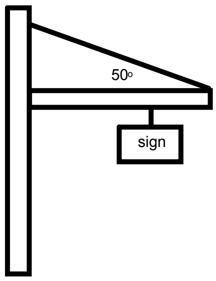
1. Determine the tension in the wire and the components of the force from the hinge. The beam has a mass of 170 kg, the sign has a mass of 75 kg, and the beam is 6.0 m long. (T = 2442 N, F_{hy} = 834 N, F_{hx} = 1870 N)



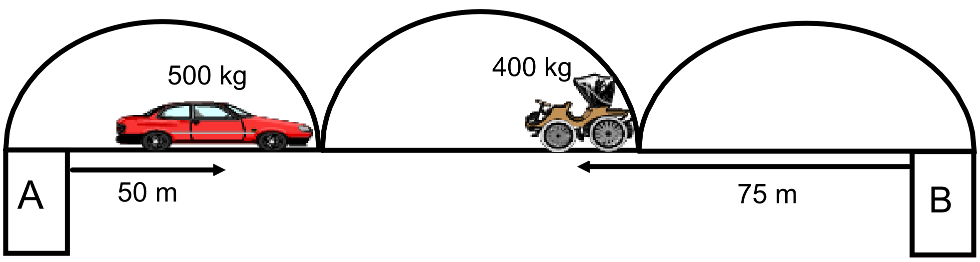
2. The cable in the diagram to the right will break if the tension reaches 1500 N. What is the smallest angle that can be made with the horizontal? The beam is 15 m long and has a weight of 1050 N. The sign has a weight of 500 N. (θ = 43°)



3. If the cable will break under a stress of 2300 N, what is the largest mass that can be hung from the beam? The beam is 150 kg and 8.0 m long. The cable makes an angle of 50° with the beam and the sign is 5.5 m from the left end of the beam. (152 kg)



4. The Morrissey bridge will collapse if column A must support more than 50000 N of weight. The bridge spans 225 m and has a mass of 8500 kg. Will the bridge collapse under the circumstances depicted in the diagram? (No, F_A = 46815 N)



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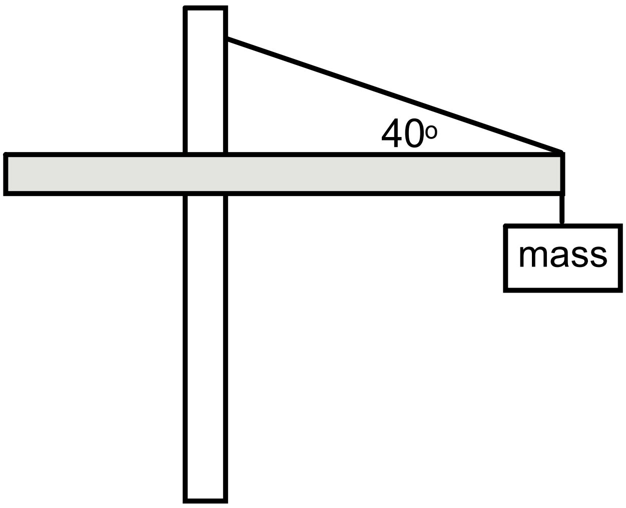
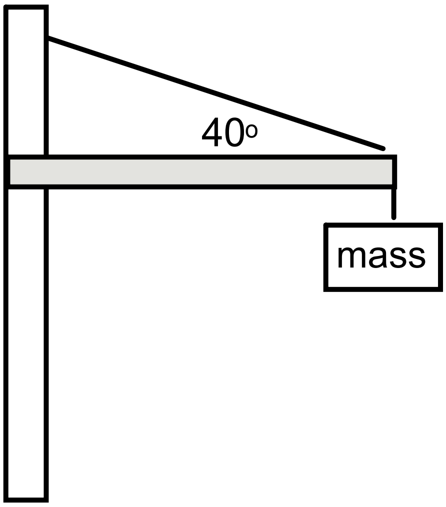
A construction crane is designed such that part of the boom acts as a counterweight. The boom is constructed of uniform material with a linear density of 25 kg/m. The left side of the crane is 10 m long and the right side is 15 m.

a) If the mass at the right end is 300 kg what is the tension in the cable? (T = 6200 N)

b) What is the tension in the cable if there was no left side of the boom? (T = 7400 N)

c) Suppose each cable can support a tension of 12000 N. What is the maximum mass that each crane can support? (Left: 680 kg; Right: 600 kg)





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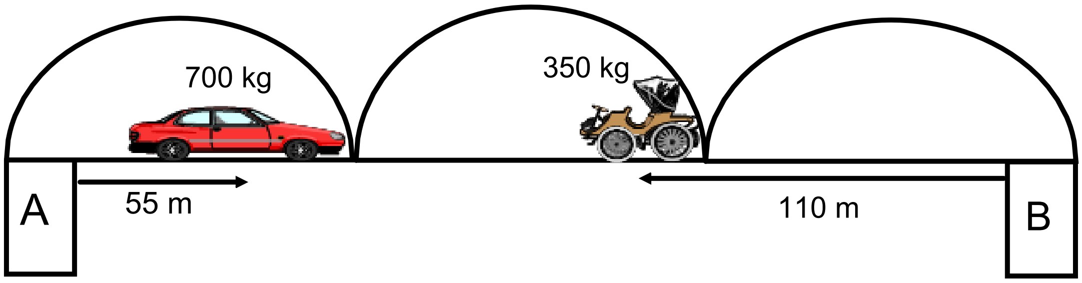
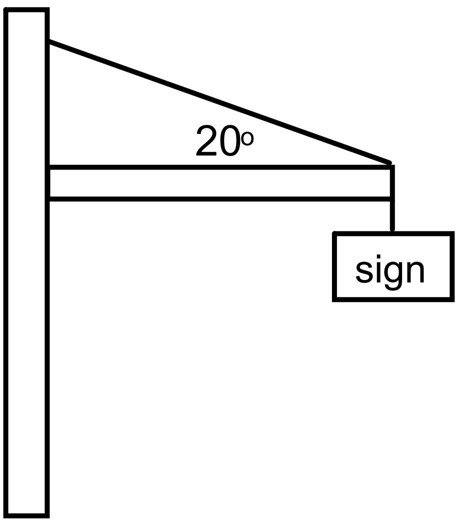
Torque Quiz - Solve on your own paper

1. Find the force each column supports. The bridge spans 300 m and has a mass of 12 500 kg.



2. Determine the tension in the wire. The beam has a mass of 225 kg, the sign has a mass of 1055 kg, and the beam is 15 m long.



3. Find the net torque.