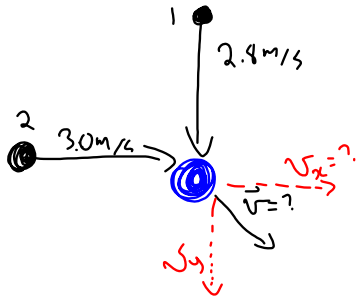
Two Dimensional (2D) Collisions

In order to solve two dimensional collision problems, write a conservation of momentum equation for the horizontal components of the momenta and a conservation of momentum equation for the vertical components of the momenta.

A 4.0 kg object is travelling south at a speed of 2.8 m/s when it collides with a 6.0 kg object travelling east at a velocity of 3.0 m/s. If these objects stick together upon collision, at what velocity do the combined masses move?

2.1 m/s [E 32° S]



Calculate Conservation of Momentum in x-direction (East).

N ↑ → E

East

Before Collision

$v_{1E} = 0$ m/s $\quad v_{1N} = -2.8$ m/s

$v_{2E} = 3.0$ m/s $\quad v_{2N} = 0$ m/s

$$m_1 v_{1E} + m_2 v_{2E} = (m_1 + m_2) v_E$$

(stick together)

$$(4.0)(0) + (6)(3) = (4+6) v_E$$

$$18 = 10 v_E$$

$$1.8 \text{ m/s} = v_E$$

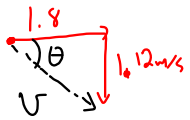
North

$$m_1 v_{1N} + m_2 v_{2N} = (m_1 + m_2) v_N$$

$$(4)(-2.8) + (6)(0) = (10) v_N$$

$$-11.2 = 10 v_N$$

$$-1.12 \text{ m/s} = v_N$$



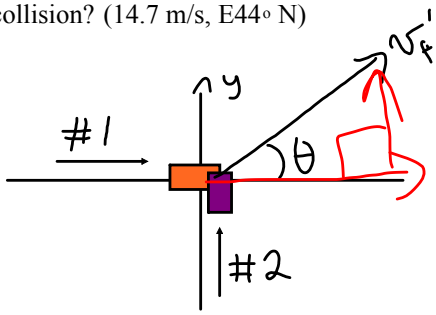
$$v^2 = v_E^2 + v_N^2$$

$$v = \sqrt{(1.8)^2 + (-1.12)^2} = 2.1 \text{ m/s}$$

$$\theta = \tan^{-1}\left|\frac{v_N}{v_E}\right| = \tan^{-1}\left|\frac{1.12}{1.8}\right| = 32^\circ$$

$$\vec{v} = 2.1 \text{ m/s } [E\ 32^\circ S]$$

Example: A 1325 kg car moving north at 27.0 m/s collides with a 2165 kg car moving east at 17.0 m/s. They stick together. In what direction and with what speed do they move after the collision? (14.7 m/s, E44° N)



$v_{1E} = 17$ m/s

$v_{2E} = 0$ m/s

$v_{1N} = 0$ m/s

$v_{2N} = 27$ m/s

Conservation of mom. East-West

$$m_1 v_{1E} + m_2 v_{2E} = m_1 v_{1E}' + m_2 v_{2E}'$$

(same)

$$(2165)(17) + 0 = (2165 + 1325) v_{fE}$$

$$36805 = 3490\, v_{fE}$$

$$10.5 \text{ m/s} = v_{fE}$$

North - South

$$m_1 v_{1N} + m_2 v_{2N} = m_1 v_{1N}' + m_2 v_{2N}'$$

$$0 + (1325)(27) = (2165 + 1325) v_{fN}$$

$$35775 = 3490 v_{fN}$$

$$10.3 \text{ m/s} = v_{fN}$$

$$|v_f| = \sqrt{v_{fE}^2 + v_{fN}^2}$$

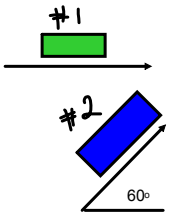
$$= \sqrt{(10.5)^2 + (10.3)^2}$$

$$= 14.7 \text{ m/s}$$

$$\theta = \tan^{-1}\left|\frac{v_{fN}}{v_{fE}}\right| = \tan^{-1}\left|\frac{10.3}{10.5}\right| = 44^\circ$$

$$\boxed{v_f = 14.7 \text{ m/s} \; [E\ 44^\circ\ N]}$$

Example: A 1200 kg car is moving east at 30.0 m/s and collides with a 3600 kg car moving at 20.0 m/s in a direction 60.0 ◦ N of E. The vehicles interlock and move off together. Find their common velocity. (19.8 m/s, 40.9 ◦ N of E)



$v_{1E} = 30.0\ m/s$

$v_{1N} = 0\ m/s$

$v_{2E} = 20\cos 60$

$v_{2N} = 20\sin 60$

$v_{1E}' = v_{2E}' = v_{fE}$

$v_{1N}' = v_{2N}' = v_{fN}$

East - West (x-dir)

$m_1 v_{1E} + m_2 v_{2E} = m_1 v_{1E}' + m_2 v_{2E}'$

$(1200)(30) + (3600)(20\cos 60) = 4800 v_{fE}$ ($m_1 + m_2$)

$36000 + 36000 = 4800 v_{fE}$

$72000 = 4800 v_{fE}$

$15\ m/s = v_{fE}$

North - South

$m_1 v_{1N} + m_2 v_{2N} = (m_1 + m_2) v_{fN}$

$0 + (3600)(20\sin 60) = 4800 v_{fN}$

$62354 = 4800 v_{fN}$

$13\ m/s = v_{fN}$

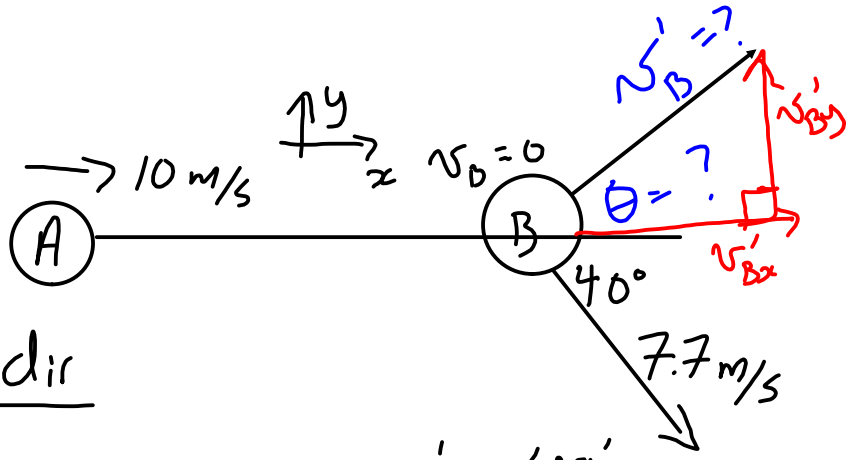
$|v_f| = \sqrt{v_{fE}^2 + v_{fN}^2}$

$= \sqrt{(15)^2 + (13)^2} = 19.8\ m/s$

$\theta = \tan^{-1}\left|\frac{v_{fN}}{v_{fE}}\right| = \tan^{-1}\left|\frac{13}{15}\right| = 41°$

$v_f = 19.8\ m/s\ [E\ 41°\ N]$

#35)



x-dir

$$m_A v_{Ax} + m_B v_{Bx} = m_A v_{Ax}' + m_B v_{Bx}'$$

$$10 + 0 = 7.7\cos 40 + v_{Bx}'$$

$$10 = 5.9 + v_{Bx}'$$

$$4.1 \text{ m/s} = v_{Bx}'$$

y-dir

$$m_A v_{Ay} + m_B v_{By} = m_A v_{Ay}' + m_B v_{By}'$$

$$0 = -7.7\sin 40 + v_{By}'$$

$$4.9 \text{ m/s} = v_{By}'$$

$$v_B' = \sqrt{(v_{Bx}')^2 + (v_{By}')^2}$$

$$= \sqrt{40.8} = 6.4 \text{ m/s}$$

$$\theta = \tan^{-1}\left|\frac{v_{By}'}{v_{Bx}'}\right| = 50°$$

$v_B' = 6.4$ m/s 50° clockwise
counter