Physics 122/121

Unit 1

Dynamics Extension

definition of equlibrium: *the state of an object when the vector sum of all the forces acting on it is zero.*

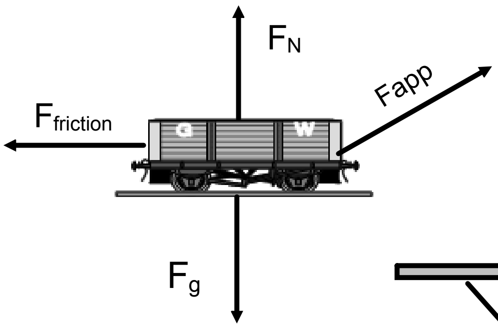
If an object is at ***rest*** and is in ***equilibrium***, then we say that it is in a state of "***static equilibrium***."

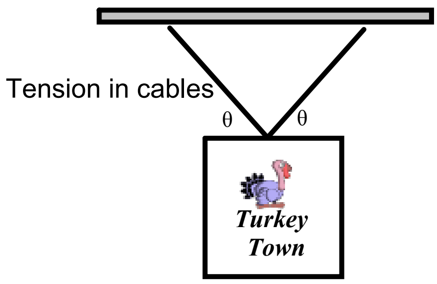
<u>Equilibrant:</u> is the one vector, when added to 2 or more other vectors produces a state of equilibrium. It is equal to the resultant but opposite in direction.

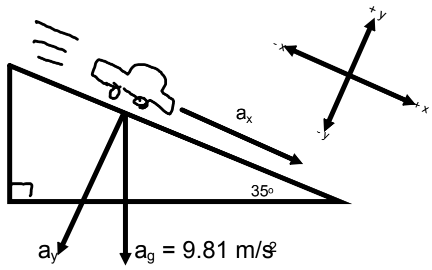
<u>Try</u> - Three forces act simultaneously on point P. The first force is 10 N east. The second force is 15 N south. The third force is 28 N, E46°S . Find the resultant force. (46 N, E50°S). Find the equilibrant.(46N, W50°N)

Three Types of Force Problems

1 - Pushing or pulling an object along a horizontal surface.
2 - Tension and hanging signs.
3 - Objects on an incline.



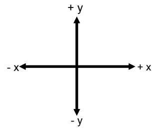




Force Problems - Type I

A 55 kg snow blower is pushed along the ground at an angle of 35 to the horizontal with an applied force of 175 N.

a) Find the F_{ax} and F_{ay}.
b) Calculate F_N.
c) Find the force of friction if $\mu = 0.19$.
d) Find the F_{netx}.
e) Find a_x.



a) $F_{ax} = F_a \cos\theta$
$= 175 \cos 35 = 143\text{ N}$

$F_{ay} = F_a \sin\theta = 175 \sin 35$
$= 100\text{N}$

b) $F_N = ?$

$\sum F$ in y-direction

$F_{nety} = F_{ay} + F_N + F_g$ ← mg

$0 = -100\text{ N} + F_N - (55)(9.81)$

$100 + 540 = F_N$

$640\text{ N} = F_N$

c) $F_f = ?$

$F_f = \mu F_N$

$F_f = 0.19(640)$
$= 122\text{N}$

d) $F_{netx} = \sum$ Forces in x-dir

$F_{netx} = F_{ax} + F_f$
$= 143 + (-122)$
$= 21\text{ N}$

e) $a_x = ?$

$F_{netx} = ma$

$21 = 55a$

$0.38\text{ m/s}^2 = a$

A 35 kg wagon is pulled along the ground at an angle of 25° to the horizontal with an applied force of 97 N.

a) Find the F_{ax} and F_{ay}.

b) Calculate F_N.

c) Find the force of friction if $\mu = 0.22$.

d) Find the F_{netx}.

e) Find a_x.

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Force Problems - Type I

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17. A student pushes a 25 kg lawn mower with a force of 150 N. The handle makes an angle of 35° to the horizontal.

(a) Find the vertical and horizontal components of the applied force.
(b) Calculate the normal force supporting the lawn mower while it is being pushed.
(c) Calculate the net force propelling the mower if a frictional force of 85 N exists.
(d) Calculate the horizontal acceleration of the lawn mower. (Remember: Only part of the $F_{applied}$ is parallel to the direction of horizontal acceleration.)

a) 86 N, down
1.2 x 10^2 N, right
b) 3.3 x 10^2 N, up
c) 38 N, right
d) 1.5 m/s^2, right

24. A toboggan with a mass of 15 kg is being pulled with an applied force of 45 N at an angle of 40° to the horizontal. What is the acceleration if the force of friction opposing the motion is 28 N?

0.43 m/s^2, right

25. A grocery cart is being pushed with a force of 450 N at an angle of 30.0° to the horizontal. If the mass of the cart and the groceries is 42 kg,
(a) Calculate the force of friction if the coefficient of friction is 0.60.
(b) Determine the acceleration of the cart.

a) 3.8 x 10^2 N, left
b) 0.23 m/s^2, right

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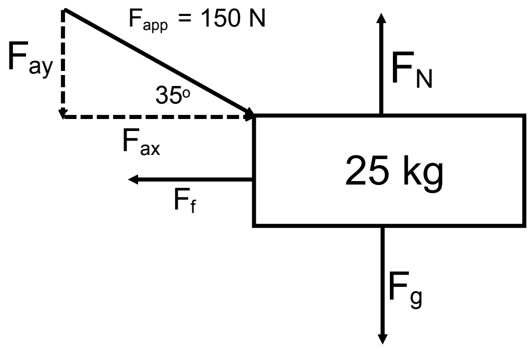
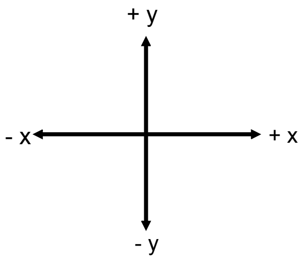
36. A 45.0 kg box is pulled with a force of 205 N by a rope held at an angle of 46.5° to the horizontal. The velocity of the box increases from 1.00 m/s to 1.50 m/s in 2.50 s. Calculate
(a) the net force acting horizontally on the box.
(b) the frictional force acting on the box.
(c) the horizontal component of the applied force.
(d) the coefficient of kinetic friction between the box and the floor.

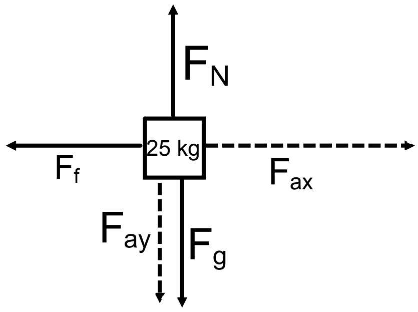
a) 9.0 N, right
b) 132 N, left
c) 141 N, right
d) 0.451

#17





Free Body Diagram



(a) $F_{ax} = +150\cos(35)$
$= 123\text{ N}$

$F_{ay} = -150\sin(35)$
$= -86\text{ N}$

(b) $F_{nety} = F_{ay} + F_g + F_N$
$0 = -86\text{ N} - mg + F_N$
$0 = -86 - 25(9.81) + F_N$
$0 = -86 - 245.25 + F_N$
$0 = -331.25 + F_N$
$+330\text{ N} = F_N$

(c) F_{netx} = Sum of horizontal forces
$= F_{ax} + F_f$
$= 123 + -85$
$= +38\text{ N}$

(d) $F_{net} = ma$

$$a = \frac{F_{net}}{m}$$

$$a = \frac{+38\ N}{25\ kg}$$

$a = +1.5\text{ m/s}^2$

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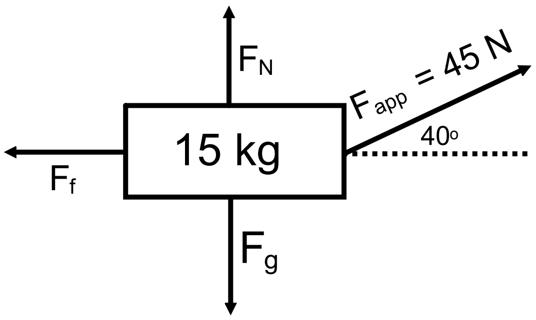
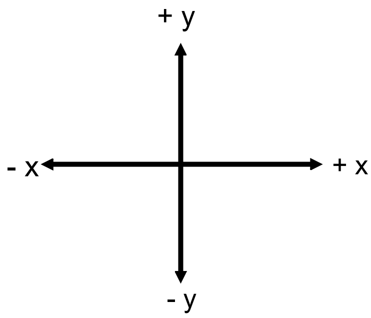
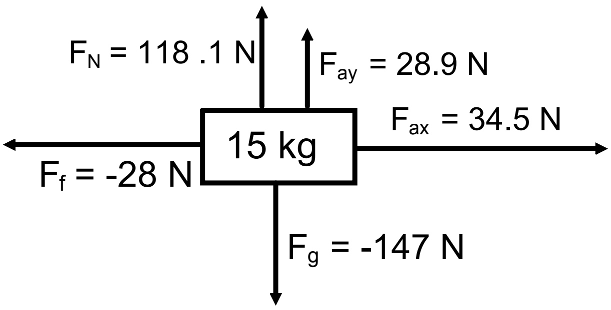






$$F_{netx} = F_{ax} + F_f$$
$$= 34.5 + (-28)$$
$$= +\ 6.5\ \text{N}$$

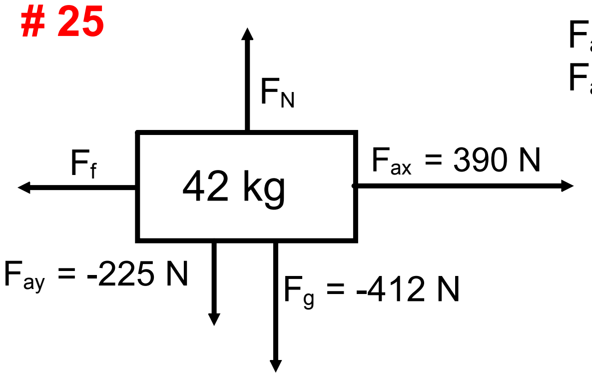
$$F_{net} = ma$$

$$a = \frac{F_{net}}{m}$$

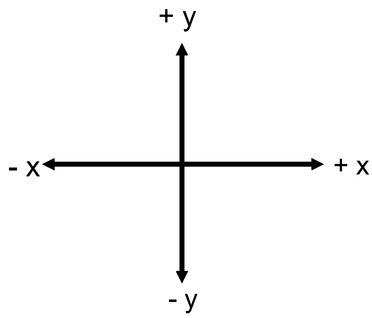
a = +6.5 N/15 kg

a = +0.43 m/s^2

 # 25



$F_{ax} = 450\cos(30)$ N
$F_{ay} = 450\sin(30)$ N



(a) $F_f = \mu F_N$, $\mu = -0.60$

$F_{nety} = F_{ay} + F_g + F_N$
$0 = -412\text{ N} + -225\text{ N} + F_N$
$F_N = +637$ N

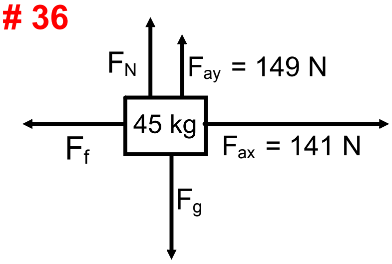
$F_f = 0.60(637\text{ N})$
$F_f = 380$ N [left]

(b) $a = \dfrac{F_{net}}{m} = F_{ax} + F_f$

$a = (389.7\text{ N} + -382\text{ N}) \div 42\text{ kg}$

$a = +0.19\text{ m/s}^2$

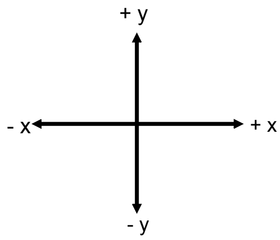
36



$F_{app} = 205$ N @ 46.5°

$F_{ax} = 205\cos(46.5)$

$F_{ay} = 205\sin(46.5)$



(a) $F_{net_x} = ma_x$ ← find acceleration

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{1.50m/s - 1.00m/s}{2.50s}$$

$a = 0.20$ m/s²

$F_{net} = (45kg)(0.20m/s^2)$
$= 9.0$ N

(c) $F_{ax} = 141$ N

(d) $F_{kf} = \mu F_N$

$F_{net_y} = F_{ay} + F_g + F_N$

$0 = -441\text{ N} + 149\text{ N} + F_N$

$F_N = +292$ N

$$\mu = \frac{F_f}{F_N}$$

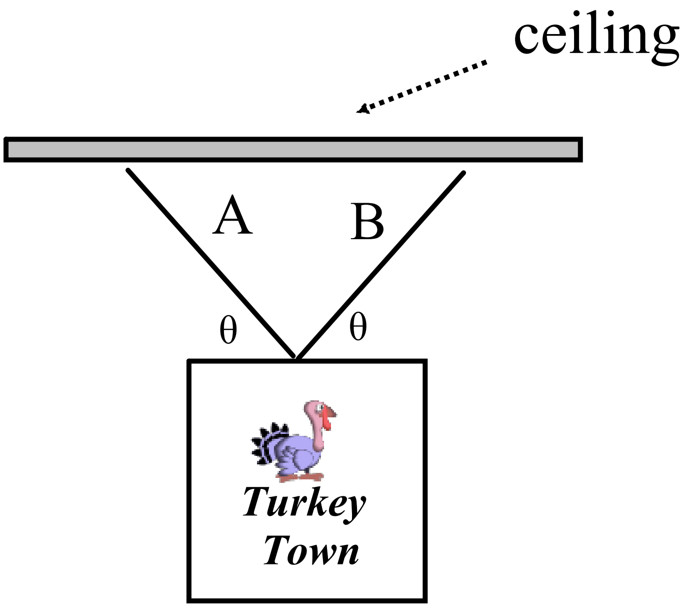
can use "+132" as the formula implies the magnitude of F_f.

$$\mu = \frac{+132N}{292N}$$

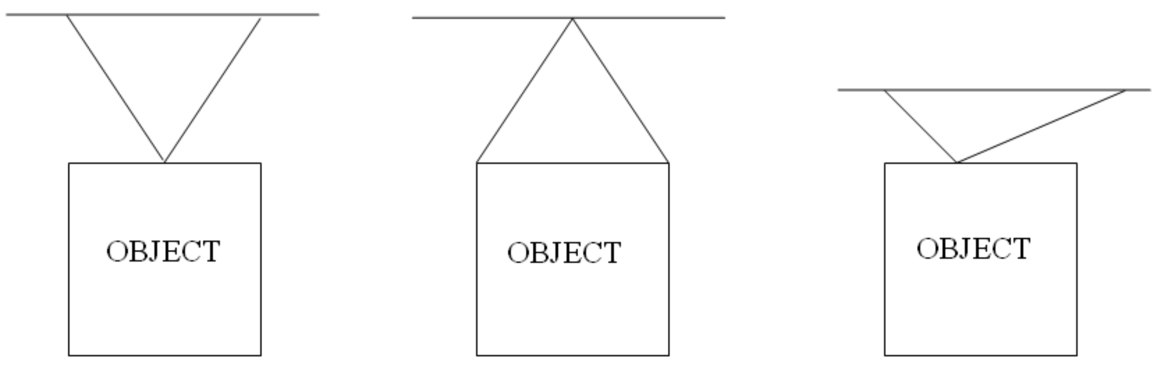
$\mu = +0.451$

Type II - Signs/Pictures/Hanging Objects

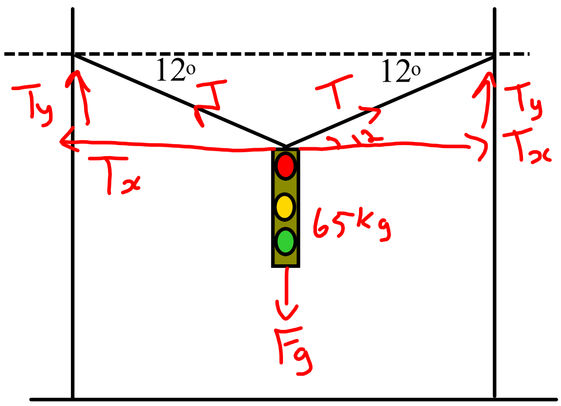
If an object is hung by a rope (wire, chain, etc.), we can resolve the force of tension along the rope.



An object can be hung in a variety of ways.



Example



A traffic light hangs in the center of the road from cables as shown in the figure. (a) If the mass of the traffic light is 65 kg, what is the magnitude of the force that each cable exerts on the light to prevent it from falling? (b) What is the tension in each cable?

(a) The y-component of the tension in each cable must add together to support the light's weight; the light is in static equilibrium. Since the angles are the same the tension in each cable and their components are the same.

$$\sum F_{net\,y} = T_y + T_y + F_g$$

$$0 = 2T_y + F_g$$

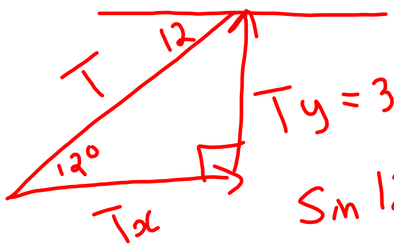
$$0 = 2T_y - (65)(9.81)$$

$$0 = 2T_y - 638$$

$$638 = 2T_y$$

$$\boxed{319\text{ N} = T_y}$$

(b) Use trig to solve for the tension in each cable.

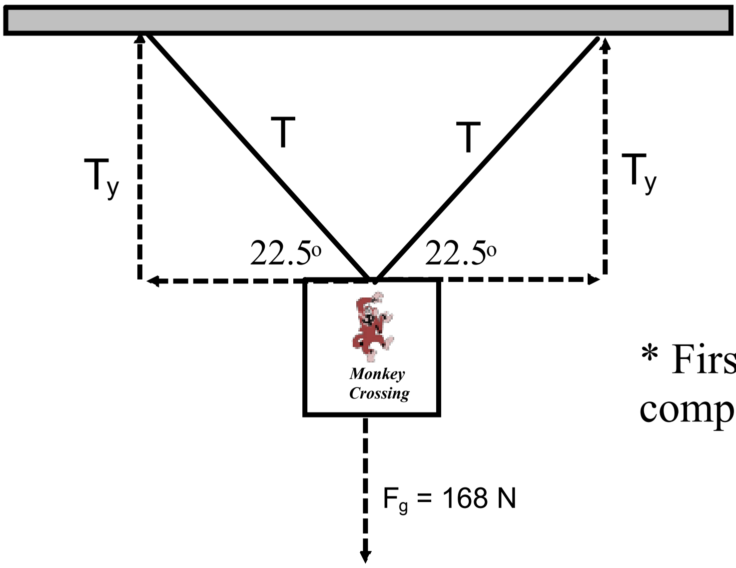


$$T_y = 319$$

$$\sin 12 = \frac{319}{T}$$

$$T \sin 12 = 319$$

$$T = \frac{319}{\sin 12} = \boxed{1534\text{ N}}$$



A sign that weighs 168 N is supported by two ropes, A and B, that make 22.5° angles with the horizontal. Determine the tension along the ropes.

* First label the diagram to view the components of each rope's tension.

Determine y-component of tension:

Fnety = 0 N

Fnety = 2Ty + Fg

0 N = 2Ty - 168 N

Ty = 84 N

Determine tension in each rope (remember they are the same if the angles are the same):

T = (Ty)/(sinθ°)

T = 84 N/sin 22.5°

T = 220 N

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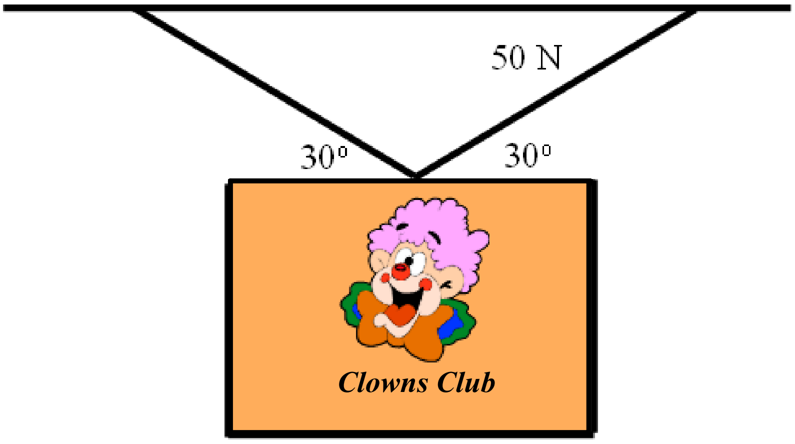
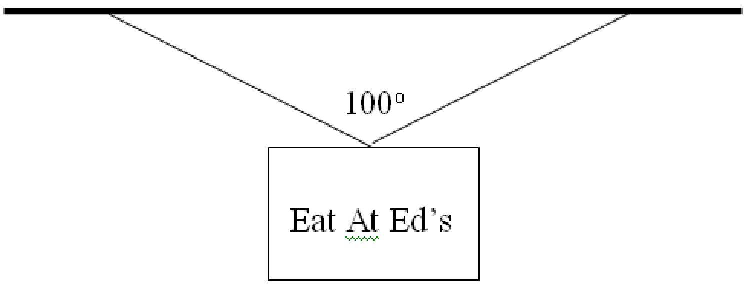
Handout - Static Equlibrium -Hanging Signs

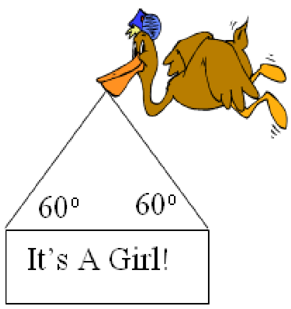
1. Find the magnitude of the weight of the clown's picture. (50 N)



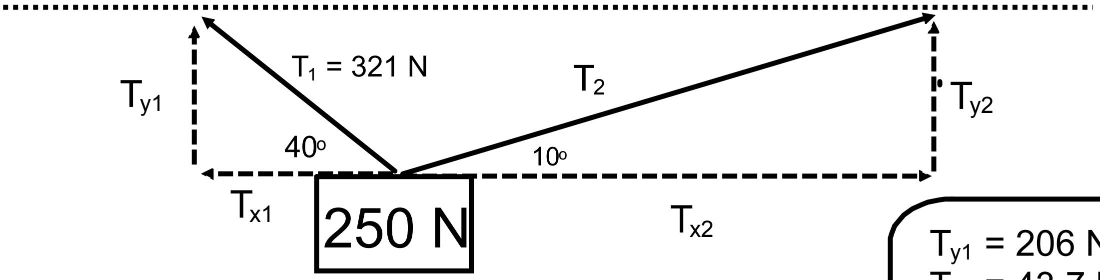
2. If the sign has a mass of 5.00 kg, what is the tension in the cables? (38 N)



3. The infamous stork announces good news. If the sign has a mass of 10 kg, then what is the force of tension in each cable? (57 N)



Determine T_{y1}, T_{y2}, T_2 , T_{x1}, and T_{x2} in the following sketch.



T_{y1} = 206 N
T_{y2} = 43.7 N
T_2 = 251 N
T_{x1} = 247 N (left)
T_{x2} = 247 N (right)

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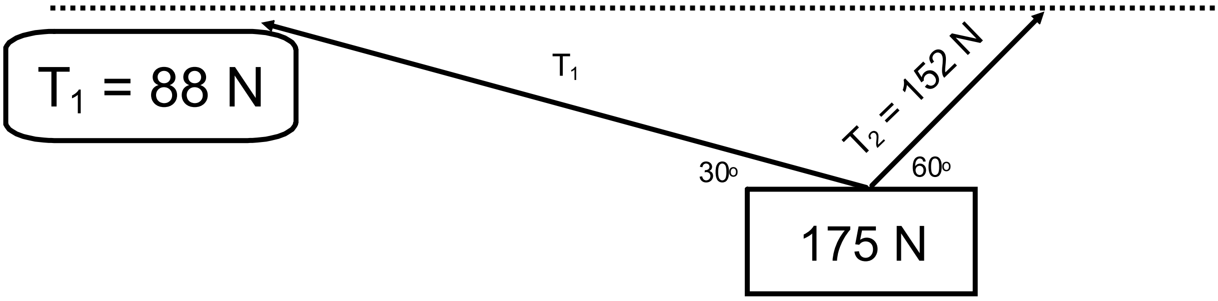
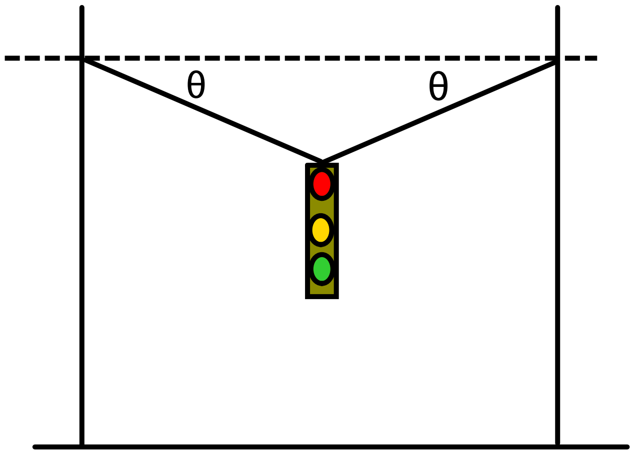
Determine T_1 in the following sketch.



A traffic light is to be hung like in the diagram to the right (both angles are the same). The cable being used will break if their tension reaches 2100 N. What is the smallest angle that can be made if the lights have a mass of 110 kg? (Answer: 15°)



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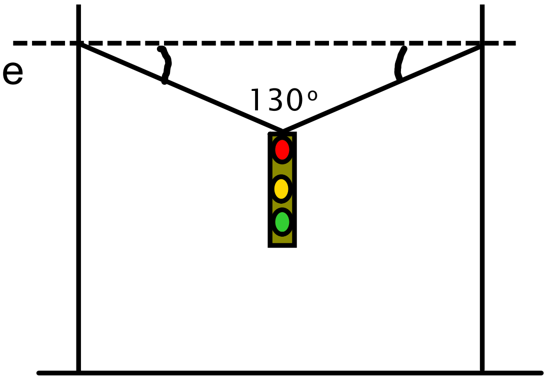
A traffic light is to be hung like in the diagram to the right. The cable being used will break if its tension reaches 1750 N. What is the largest mass that can be hung? (Answer: 151 kg)



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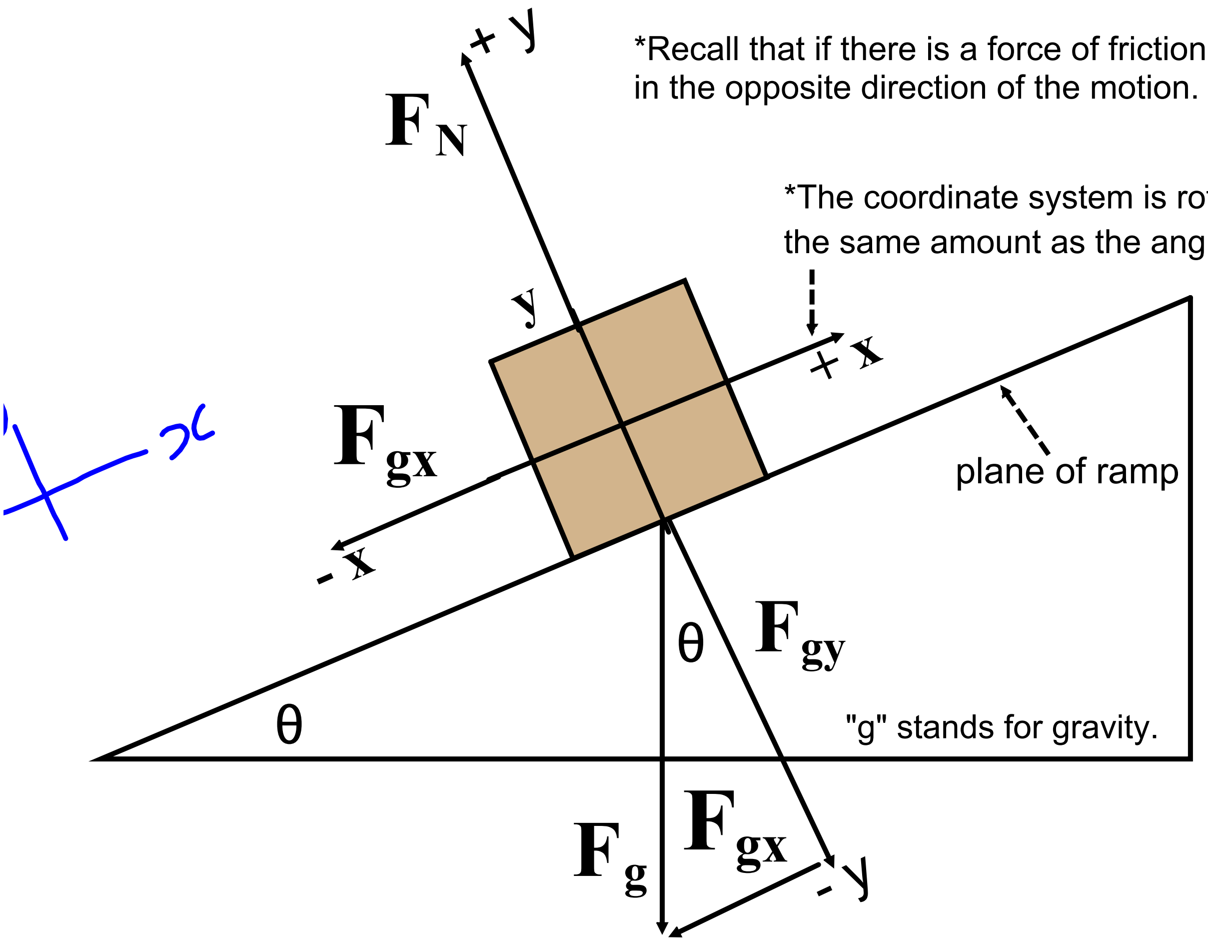
Type III - Inclined Planes, Hills, Ramps

(printed copy for students)

*Recall that if there is a force of friction it acts in the opposite direction of the motion.

*The coordinate system is rotated the same amount as the angle θ.



F_{gy} and F_g are separated by θ because of two similar triangles.

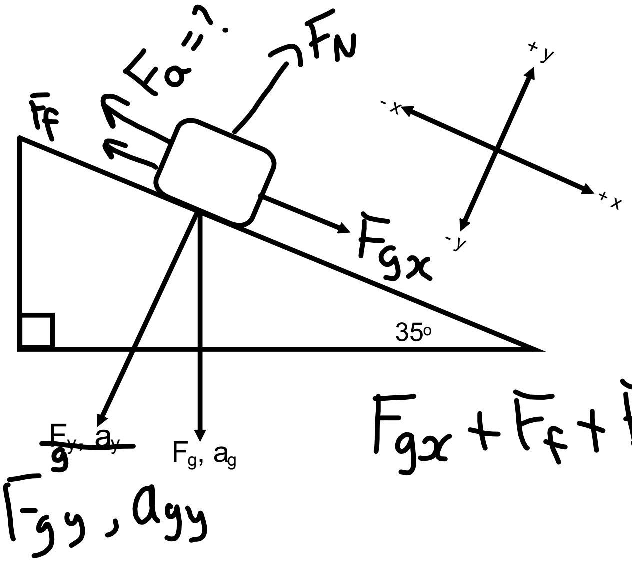
$F_{gx} = F_g \sin\theta$ ← component parallel to the plane.

$F_{gy} = F_g \cos\theta$ ← component perpendicular to the plane.

NOTE! The *sin* and *cos* have switched places. This will only happen when dealing with objects on a ramp.

NOTE FURTHER! Every ***F*** in the above diagram can be replaced with an ***a*** for acceleration.

1. A 55 kg block is sliding down an incline. The coefficient of kinetic friction is 0.13 and the incline makes an angle of 35 ° with the ground. What applied force up the ramp is necessary so the block accelerates with a magnitude of 0.83 m/s² down the ramp ?



$F_{net} = ma$

$F_{net} = \sum$ Forces along the ramp

$F_{gx} + F_f + F_a = m a_x$

$F_{gx} = F_g \sin\theta = (55)(9.81)\sin 35$
$= +309.5\ N$

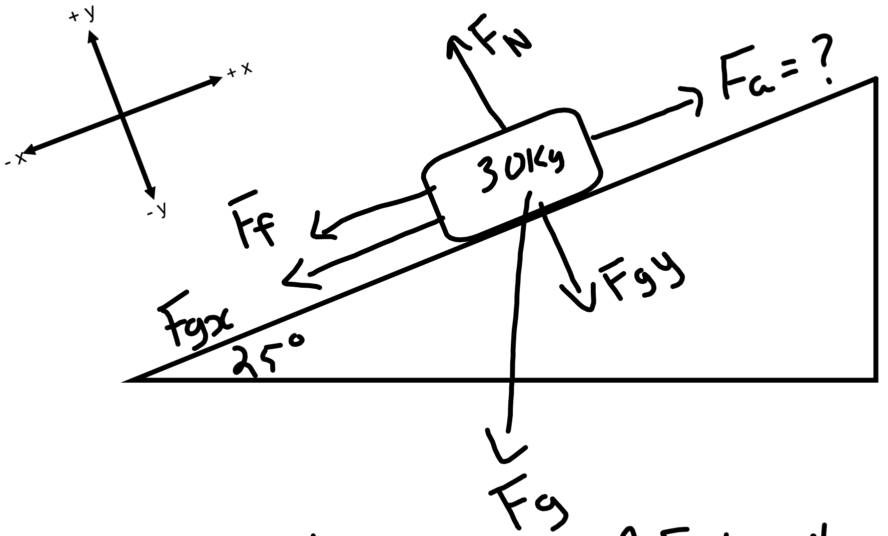
$F_N = |F_{gy}|$

$F_f = \mu F_N = (0.13)\ F_{gy} = 0.13\ F_g \cos\theta$
$= (0.13)(55)(9.81)\cos 35$
$= 57.5\ N \leftarrow$ up the ramp
so $F_f = -57.5\ N$

Thusly, $309.5 + (-57.5) + F_a = (55)(0.83)$
$252 + F_a = 45.65$

$\boxed{F_a = -206\ N}$

2. What applied force is necessary for a person to pull a 30 kg object up a ramp at a constant velocity? The ramp makes an angle of 25° with the ground and the coefficient of kinetic friction is 0.12.



$F_{net} = 0$ N because velocity is constant

$F_a = ?$

$|F_{gx}| = F_g \sin\theta = (30kg)(9.81)\sin 25° = 124.4$ N

$|F_f| = \mu F_N$, $|F_N| = |F_{gy}| = (30)(9.81)\cos 25$

$|F_f| = (0.12)(266.7) = 32$ N

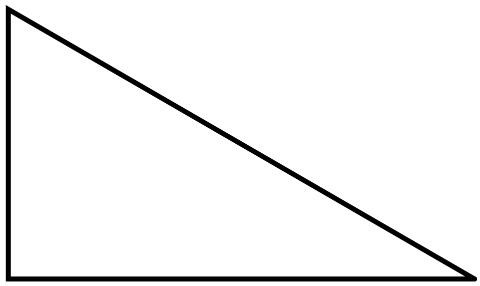
$F_{net_x} = \sum_x Forces$

$F_{net_x} = F_{gx} + F_f + F_a$

$0 = (-124.4) + (-32) + F_a$

$156.4\text{ N} = F_a$

What applied force is necessary for a person to pull a 55 kg object up a ramp at a constant velocity? The ramp makes an angle of 60 ^{o} with the ground and the coefficient of kinetic friction is 0.25.



A 25 kg box is placed on an incline 33° to the horizontal. The coefficient of kinetic friction is 0.38. Find the acceleration of the box. (2.2 m/s^2 down the ramp)

3. An inclined ramp is to be used to slide down an object at a constant speed. The coefficient of kinetic friction is 0.16. What angle should the ramp make with the ground for this to happen?

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Pg 16

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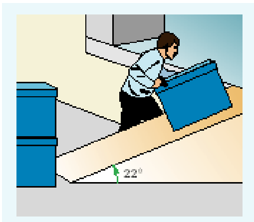
Sample Problems - Inclined Planes

Handout

1. **A trunk weighing 562 N is resting on a plane inclined at** 30.0° from the horizontal. Find the components of the trunk's weight parallel and perpendicular to the plane.

2. A 562 N trunk is placed on a frictionless plane inclined at 30.0° from the horizontal. Find the magnitude and direction of the trunk's acceleration.

3. A worker places a large plastic waste container with a mass of 84 kg on the ramp of a loading dock. The ramp makes an angle of 22° with the horizontal. The worker turns to pick up another container before pushing the first one up the ramp. If the coefficient of static friction is 0.47, will the crate slide down the ramp?



4. A 1975 kg car is rolling down a hill inclined at an angle of 15° What is the acceleration of the car? Neglect friction.

5. A skier coasts down a 3.5° slope at a constant speed. Find the coefficient of kinetic friction between the skis and the snow covering the slope.

6. You slide a 325 N trunk up a 20.0° inclined plane with a constant velocity by exerting a force of 211 N parallel to the inclined plane.

   a) What is the sum of your applied force, friction and the parallel component of the trunk's weight? Justify your answer.

   b) What is the magnitude and direction of the force of friction?

   c) What is the coefficient of friction?

Inclined Plane - Solutions

1. $F_{gx} = 281\ N$, $F_{gy} = 487\ N$

2. $a_{gx} = 4.905\ m/s^2$ down the ramp

3. No, $F_f > F_g$ $F_f = 359\ N$; $F_{gx} = 309\ N$

4. $a_x = 2.53\ m/s^2$

5. $\mu = 0.061$

6. a) $F_{netx} = 0\ N$

   b) $|F_f| = 100\ N$ down the ramp

   c) $\mu = 0.32$

Handout

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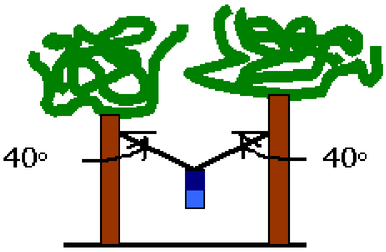
Handout: Problems I, II and III

1. On a camping trip you stretch a rope between two trees and hang your backpack from the middle of it to keep it safe from bears. The mass of your backpack is 36.0 kg and each half of the rope makes an angle of 40.0° with the horizontal.
   a) Find the amount of weight supported by each half of the rope.
   b) Find the magnitude of the tension in each rope.



2. A 2.5 kg brick is pulled at a constant speed across a table by a cord that makes an angle of 20° with the horizontal. There is 7.0 N of force in the cord.
   a) Calculate the force of friction between the brick and the table.
   b) Calculate the normal force.

3. Joey moves a 26 kg wagon at a constant speed by pushing on the handle that makes an angle, theta, with the horizontal. Joey exerts a force of 54 N on the handle and the force of friction on the wagon is 34 N.
   a) Calculate the angle the handle of the wagon makes with the horizontal.
   b) What is the magnitude of the normal force acting on the wagon?

4. A 10 N block is held motionless on a frictionless inclined plane which makes an angle of 30° with the horizontal. What force would be needed to hold the block in position?

5. An object weighing 600 N is pulled up a frictionless incline at a constant speed using a rope. If the incline makes an angle of 42.0° with the horizontal, what is the magnitude of the force that is applied to the rope?

6. A 10 kg object, starting from rest, slides down a frictionless incline with a constant acceleration of 2.0 m/s^2. What angle does the incline make with the horizontal?

7. An object with a mass of 7.2 kg is allowed to slide from rest down an inclined plane. The plane makes an angle of 30° with the horizontal and is 65 m long. The coefficient of friction between the plane and the object is 0.45. What is the velocity of the object at the bottom of the plane?

8. A piano is accelerating down a ramp that is inclined at an angle of 38.5° above the horizontal. The acceleration is 4.62 m/s^2. What is the coefficient of friction between the piano and the ramp?

Answers

1. a)Each half of the rope supports half of the weight of the backpack, 176 N.
   b)The tension in each rope is 274 N.

2. a)The force of friction is 6.6 N, in a direction opposite to the motion of the brick.
   b)The magnitude of the normal force is 22 N.

3. a)The handle makes an angle of 51° with the horizontal.
   b)The normal force is 3.0×10^2 N, up.

4. A 5.0 N force exerted up the incline would be needed.

5. It is 401 N.

6. The incline makes an angle of 12°.

7. The velocity of the object is –12 m/s.

8. The coefficient of friction is 0.19.

Reciprocal identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

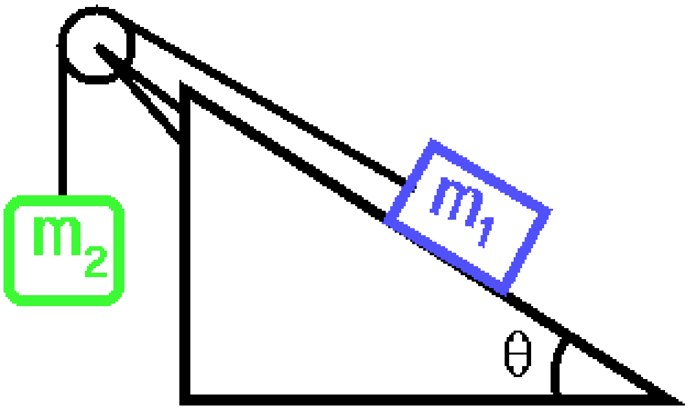
Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \quad 1 + \tan^2 u = \sec^2 u \quad 1 + \cot^2 u = \csc^2 u$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Objects Connected at an Angle



We approach this problem like the others:

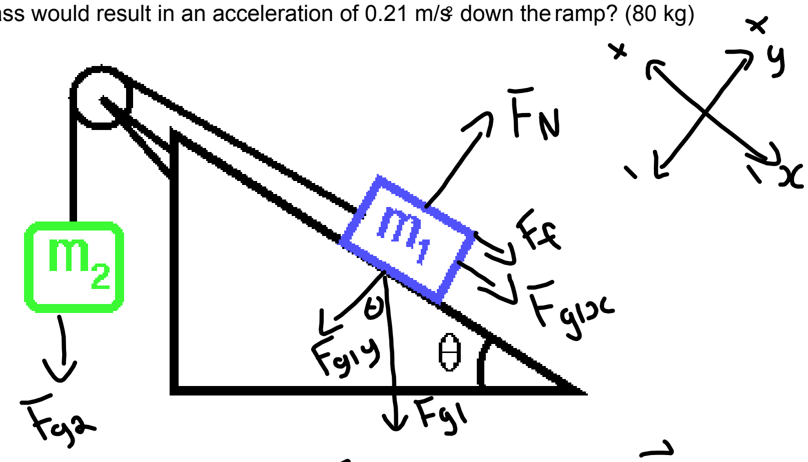
- Picture the motion in one dimension.
- Apply Newton's second law.

Physics 122: Newton's 2nd Law in 2D

A counterweight is used to slide an object up an inclined plane that makes a 40° angle with the horizontal. The counterweight has a massof 35 kg and is suspended with a massless string and a friction lesspulley. The coefficient of kinetic friction on the plane is 0.23.

a) For the acceleration of the object not to exceed 0.42 m/s² up the ramp, what must be the minimum mass of the object? (39 kg)

b) What mass would result in an acceleration of 0.21 m/s² down the ramp? (80 kg)



a) $\sum \text{Forces} = \sum \text{masses} \times \vec{a}$

$F_{g1x} = -F_{g1}\sin\theta = -m_1(9.81)\sin 40 = -6.31 m_1$

$F_{g2} = +(35)(9.81) = 343.35\ N$

$F_f = \mu F_N = \mu F_{g1y} = -(0.23)(m_1)(9.81)\cos 40°$

$F_f = -1.73 m_1$

$F_{g1x} + F_{g2} + F_f = (m_1 + m_2)(a)$

$-6.31 m_1 + 343.35 - 1.73 m_1 = (m_1 + 35)(0.42)$

$-8.04 m_1 + 343.35 = 0.42 m_1 + 14.7$

$-8.46 m_1 = -328.65$

$\boxed{m_1 = 39\ Kg}$

Physics 122: Newton's 2nd Law in 2D

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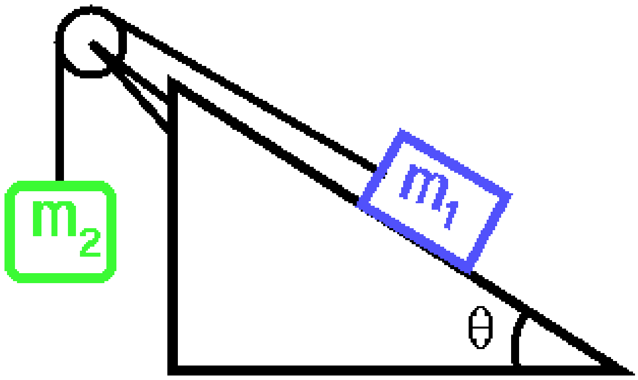
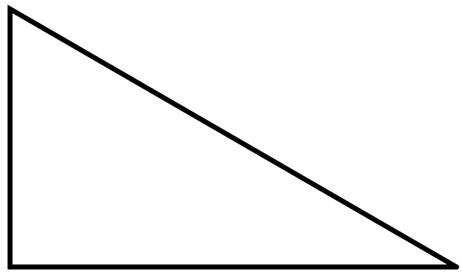


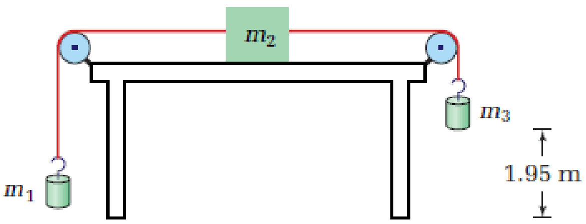
A counterweight is used to slide an object up an inclined plane that makes a 42^{o} angle with the horizontal. The counterweight has a mass of 40 kg and is suspended with a massless string and a friction less pulley. The coefficient of kinetic friction on the plane is 0.33. For the acceleration of the object not to exceed 0.22 m/s^2 up the ramp, what must be the minimum mass of the object?



A counterweight is used to slide an object up an inclined plane of 20 ^{o}. The counterweight has a mass of 25 kg and is suspended with a massless string and a friction less pulley. The coefficient of friction on the plane is 0.19. What is the acceleration of a 16 kg object?

Questions 27 & 28 only

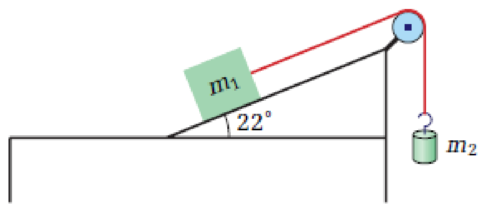
26. The objects in the diagram have the following masses: $m_1 = 228$ g, $m_2 = 615$ g, and $m_3 = 455$ g. The coefficient of kinetic friction between the block and the table is 0.260. Mass m_3 is 1.95 m above the floor. What will be the time interval between the instant that the masses start to move and the instant when mass m_3 hits the floor?



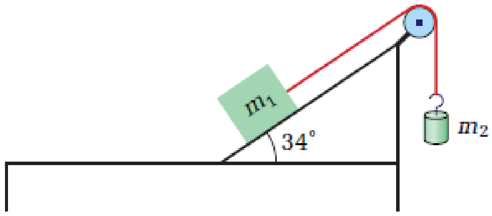
27. The block in the diagram has a mass of 145 g and the freely hanging object has a mass of 85 g. The coefficient of kinetic friction between the block and the ramp is 0.18. The ramp makes an angle of 22° with the horizontal.

(a) What will be the speed of the masses 2.5 s after they just start to move?

(b) What is the tension in the string while they are moving?



28. The block in the diagram has a mass of 725 g, and the hanging object has a mass of 595 g. The coefficient of static friction between the block and the inclined plane is 0.47, and the coefficient of kinetic friction is 0.12. The inclined plane makes an angle of 34° with the horizontal.



(a) What force directed up the incline would you have to apply to the block, to make the objects start to move?

(b) After the objects start to move, what will be their acceleration?

(c) What will be the tension in the string when the objects are moving?