Unit 2

Projectile Motion
Circular Motion
Simple Harmonic Motion
Universal Gravitation

Uniform Circular Motion

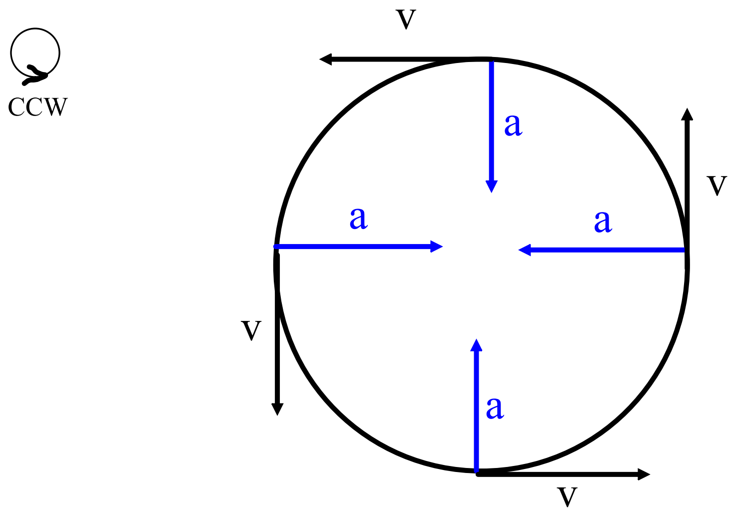
An object with uniform circular motion is an object that travels at constant speed in a circular path.

Although the object has the same speed at every point on the circular path, the direction of the object is constantly changing.

Because the direction of the object is continually changing, the object must be accelerating. The acceleration of an object travelling in a circular path is called centripetal acceleration.

Horizontal Circular Motion

Imagine you are looking down on a circular track with an object travelling counterclockwise.



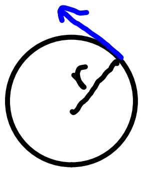
The object's speed is sometimes called the tangential speed - it is always drawn tangent to the circular path

Centripetal acceleration is always directed toward the center of the circular path.

centripetal -> center-seeking

Formulae for Horizontal Cir. Motion

$$v = \frac{d}{t}$$



Circumference = $2\pi r$

$$v = \frac{2\pi r}{T}$$

v -> speed (m/s)
r -> radius (m)
T -> period (s) (time for one revolution)

$$v = 2\pi r f$$

Remember: $T = \frac{1}{f}$

$$\frac{1}{T} = f$$

f = frequency (Hz)
the number of revolutions per second.

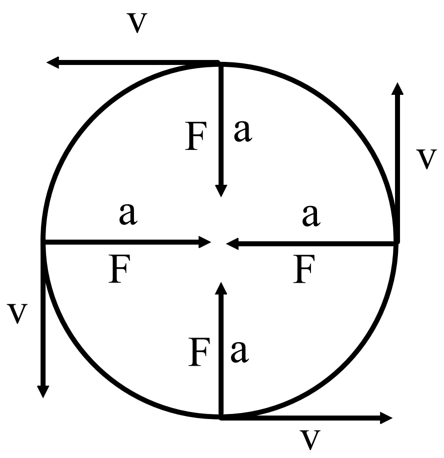
$$a_c = \frac{v^2}{r}$$

a_c -> centripetal acceleration (m/s^2)
v -> speed (m/s)
r -> radius (m)

*Sometimes its kinda nice to have everything as a function of the radius and period (or frequency):

Centripetal Force

Centripetal acceleration is due to a centripetal force. Centripetal force is the net force required to keep an object moving in a circular path. It may be a tension, force of friction, force of gravity or a combination of force components that point along the radial direction.



Centripetal force is always directed toward the center of the circular path.

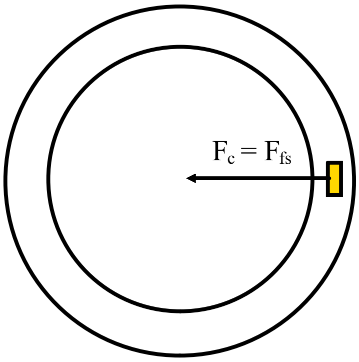
Formulas

$F_{net} = ma$ $F_c = ma_c$

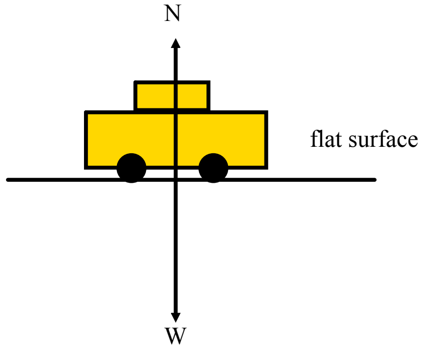
F_c -> centripetal force (N)
m -> mass (kg)
a_c -> centripetal accleration (m/s^2)

$$F_c = ma_c = \frac{mv^2}{r}$$

Unbanked Curves and Centripetal Force



When a car travels around a flat curve (unbanked curve), the centripetal force keeping the car on the curve comes from the static friction between the road and the tires. (It is static and not kinetic friction because the tires are not slipping with respect to the radial direction.) If static friction is insufficient given the speed and radius of the turn, the car will skid off the road.



$$F_c = F_{fs}$$

$$\frac{mv^2}{r} = \mu_s N$$

$$\frac{mv^2}{r} = \mu_s W$$

$$\frac{mv^2}{r} = \mu_s mg$$

$$v^2 = \mu_s gr$$

$$\boxed{v = \sqrt{rg\mu_s}}$$

v -> speed (m/s)
μ_s -> coefficient of static friction
r -> radius of curve (m)
g = 9.80 m/s²

$\frac{km}{h} \xrightarrow{\div 3.6} \frac{m}{s}$

Example:

If the maximum speed at which a car can safely navigate an unbanked turn of radius 50.0 m is 21.0 m/s, what is the coefficient of static friction? (μ_s = 0.900)

$v = 21 m/s$ $\quad \mu_s$

$r = 50 m$ $\quad v = \sqrt{rg\mu_s}$

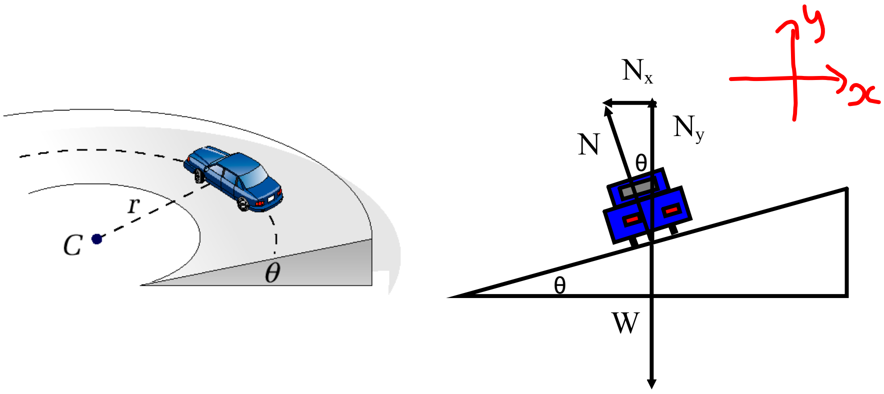
$g = 9.81$ $\quad (21)^2 = (50)(9.81)\mu_s$

$$\frac{21^2}{(50)(9.81)} = 0.9 = \mu_s$$

Banked Curves and Centripetal Force

The reliance on friction can be eliminated completely for a given speed if a curve is banked at an angle relative to the horizontal.

** Assume a friction-free banked curve.



We'll need two equations to derive a formula for this type of problem.

$F_c = N_x$

$$\frac{mv^2}{r} = N\sin\theta \quad (1)$$

$W = N_y$

$$mg = N\cos\theta \quad (2)$$

$$\frac{\dfrac{mv^2}{r} = N\sin\theta \quad (1)}{mg \quad N\cos\theta \quad (2)}$$

$$\frac{v^2}{gr} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{v^2}{gr} = \tan\theta$$

$$v^2 = gr\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$

Example:

The turns in a track have a maximum radius at the top of 316 m and are banked steeply. If a car travels at a speed of 43 m/s, what is the angle of the curve with respect to the horizontal? Assume the turn is frictionless. (31°)

Vertical Circular Motion -> Non-Uniform

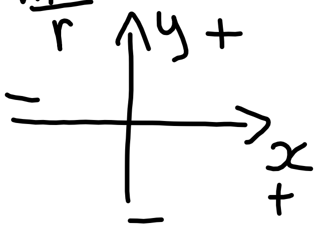
Let's look at the situation in which a motorcycle stunt driver drives his motorcycle around a vertical circular track. The speed varies in this stunt - the motion is non-uniform

As the cycle goes around, the normal force changes because the speed changes and the weight does not have the same effect at every point.

We'll look at four points along the circular path. At each point, F_c is the net sum of all the force components acting parallel to the radius of the circular path. The drawing shows the weight of the motorcycle and rider (imagine the rider) and the normal force pushing on the cycle. Braking and propulsion forces are omitted - they do not act toward the center of the path.

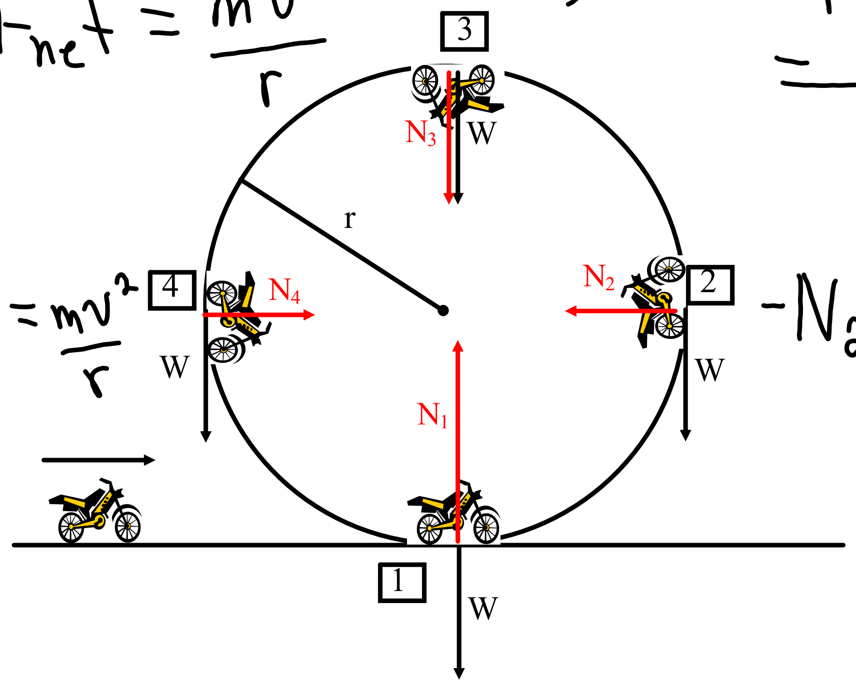
$$F_{net} = \frac{mv^2}{r}$$

$$-N_3 - W = -\frac{mv^2}{r}$$



$$N_4 = \frac{mv^2}{r}$$

$$-N_2 = -\frac{mv^2}{r}$$



$$N_1 - W = \frac{mv^2}{r}$$

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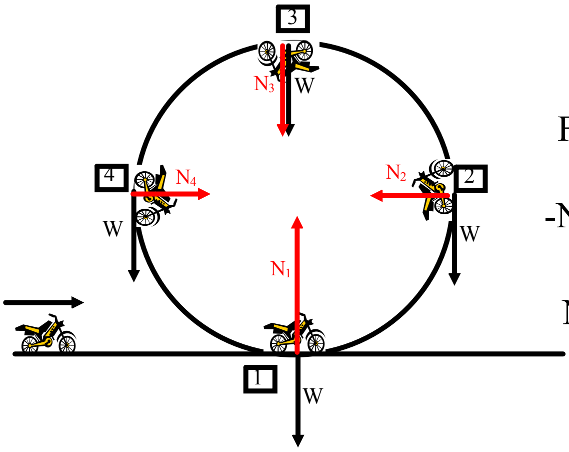
$$F_{c3} = \frac{mv_3^2}{r}$$

$$-N_3 + -W = \frac{-mv_3^2}{r}$$

$$N_3 + W = \frac{mv_3^2}{r}$$

$$F_{c4} = \frac{mv_4^2}{r}$$

$$N_4 = \frac{mv_4^2}{r}$$



$$F_{c2} = \frac{mv_2^2}{r}$$

$$-N_2 = \frac{-mv_2^2}{r}$$

$$N_2 = \frac{mv_2^2}{r}$$

$$F_{c1} = \frac{mv_1^2}{r}$$

$$N_1 - W = \frac{mv_1^2}{r}$$

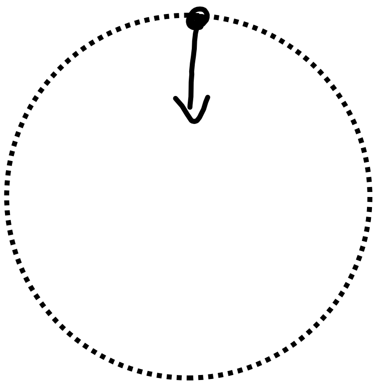
Riders must have at least a minimum speed at the top of the circle to remain on the track. This speed can be determined by considering the centripetal force at point 3. The v_3 is a minimum when N_3 is zero. At this speed the track does not exert a normal force to keep the motorcycle on the path because the weight provides all of the centripetal force. The rider experiences apparent weightlessness - rider and motorcycle are falling freely toward the center of the circle.

Mass Moving in Vertical Circular Path
(Object on a Rope)

An object is swung in a vertical circle with a radius of 0.75m. What is the minimum speed of the object to make it around the loop (2.7 m/s)

@ $v_{min} \Rightarrow N = 0$

$N + W = \frac{mv^2}{r}$



$$W = \frac{mv^2}{r}$$

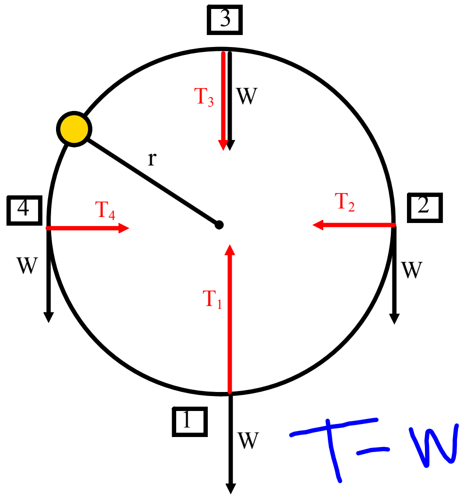
$$mg = \frac{mv^2}{r}$$

$$\sqrt{gr} = v$$

$$\sqrt{(9.81)(0.75)} = v_{min}$$

$$\boxed{2.71\ m/s = v_{min}}$$

Object on a String, Rope, Etc.



$$T - W = \frac{mv^2}{r}$$

A string requires 135 N force in order to break it. A 2.0 kg mass is tied to this string and whirled in a vertical circle with a radius of 1.10m. What is the maximum speed that this mass can be whirled without breaking the string? (8.0 m/s)

$T = 135\ N$

$r = 1.10\ m$

$m = 2.0\ kg$

$$T - W = \frac{mv^2}{r} \quad (W = F_g)$$

$$135 - (2)(9.81) = \frac{(2)(v^2)}{(1.10}$$

$$115.4 = \frac{2v^2}{1.1}$$

$$63.5 = v^2$$

$$8.0\ m/s = v$$

Problem Set Page 31- 34

Answers from MHR questions are on page 63 under Chapter 11.