

Mathematical Equations: Modeling the Universe!

Position of a moving object: $\vec{d}_f = \vec{d}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$

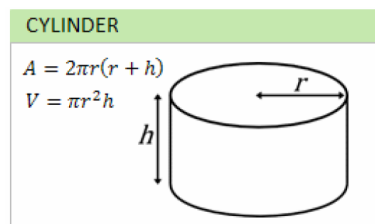
Max speed to take a corner: $v = \sqrt{r g \mu_s}$

Current decay: $I(t) = I_o e^{\frac{-t}{\tau}}$

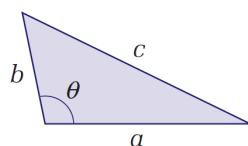
Force of gravity: $\vec{F}_g = G \frac{m_1 m_2}{r^2}$

Focal point of a lens: $\frac{1}{f} = \left(\frac{n_{lens}}{n_o} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$

Shapes:



Triangles:



Angle θ is contained between sides a and b.

The cosine law states $c^2 = a^2 + b^2 - 2ab \cos\theta$.

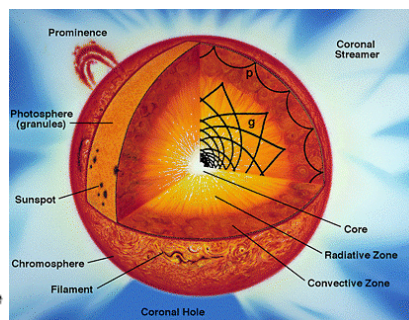
Reason why we are alive:

$$\frac{dR}{dM_r} = \frac{1}{4\pi R^2 \rho},$$

$$\frac{dL_r}{dM_r} = \epsilon - T \frac{dS}{dt},$$

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi R^4},$$

$$\frac{dT}{dM_r} = -\frac{GM_r T}{4\pi R^4 P} \nabla,$$



We will learn the general ways to setting up and solving equations.



Solving Equations...

Your mission
is to keep
everything
in balance!!



What ever you do to one side...
you must do to the other!!

Example 1 Modelling Equations with Variables on Both Sides

- a) Solve: $6x + 2 = 10 + 4x$
- b) Verify the solution.

$$6x + 2 = 10 + 4x$$

A Solution

- a) Rearrange the equation so that both terms containing the variable are on the same side of the equation. Then isolate the variable to solve the equation.

Draw balance scales.

On the left pan, draw masses to represent $6x + 2$.

On the right pan, draw masses to represent $10 + 4x$.

Remove four x masses from each pan.

Remove two 1-g masses from each pan.

Divide the masses in each pan into 2 equal groups. Each x -mass in the left pan corresponds to a group of 4 g in the right pan.

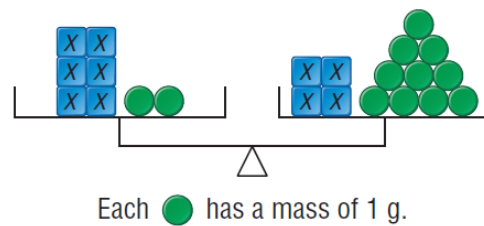
- b) Check: Substitute $x = 4$ in each side of the equation.

$$\begin{aligned} \text{Left side} &= 6x + 2 \\ &= 6(4) + 2 \\ &= 24 + 2 \\ &= 26 \end{aligned}$$

$$\begin{aligned} \text{Right side} &= 10 + 4x \\ &= 10 + 4(4) \\ &= 10 + 16 \\ &= 26 \end{aligned}$$

Since the left side equals the right side, $x = 4$ is correct.

Pictorial Solution



Algebraic Solution

$$6x + 2 = 10 + 4x$$

$$6x + 2 = 10 + 4x$$

$$\begin{aligned} 6x - 4x + 2 &= 10 + 4x - 4x \\ 2x + 2 &= 10 \end{aligned}$$

$$2x + 2 = 10$$

$$\begin{aligned} 2x + 2 - 2 &= 10 - 2 \\ 2x &= 8 \end{aligned}$$

$$2x = 8$$

$$\begin{aligned} \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

Equations in the form $ax + b = c$

$$5x + 723 = 15$$

$$5x + 723 - 723 = 15 - 723$$

$$5x = -708$$

$$| \quad \frac{\cancel{5}x}{\cancel{5}} = \frac{-708}{5}$$

$$x = -141.6$$

Warm up Challenge



What number, when multiplied by 3 and then subtracted from 13 is the same as 42 taken away from 2 times the number?

$$13 - 3x = 2x - 42$$

$$13 - 3x - 2x = \cancel{2x} - \cancel{2x} - 42$$

$$13 - 5x = -42$$

$$13 - 13 - 5x = -42 - 13$$

$$\cancel{-5x} = \frac{-55}{\cancel{-5}}$$

$$x = 11$$

1. $-2x + 51 = -3$
 $-2x + 51 - 51 = -3 - 51$
 $\frac{-2x}{-2} = \frac{-54}{-2}$
 $x = 27$

2. $17 = 17 - 32p$
 $17 - 17 = 17 - 32p - 17$
 $0 = \frac{-32p}{-32}$
 $0 = p$

3. $-39 = -18w - 21$
 $-39 + 21 = -18w - 21 + 21$
 $\frac{-18}{-18} = \frac{-18w}{-18}$
 $1 = w$

4. $-14t + 7 = 49$
 $-14t + 7 - 7 = 49 - 7$
 $-14t = 42$
 $\frac{-14t}{-14} = \frac{42}{-14}$
 $t = -3$

Warm Up!

For the relation: $y = -3x + 10$ what value of x gives results in y equaling 31?

$$31 = -3x + 10$$

$$31 - 10 = -3x + 10 - 10$$

$$\frac{21}{-3} = \frac{-3x}{-3}$$

$$\boxed{-7 = x}$$

When solving equations we have to keep our answers exact. That means fractions stay fractions :(

$$0 = \frac{r}{2} + 8 \quad -16$$

$$-8 = \frac{r}{2} + 8 - 8$$

$$-8 \times 2 = \frac{r}{\cancel{2}} \times \cancel{2}$$

$$\boxed{-16 = r}$$

$$\frac{3n}{5} - 6 = 0$$

$$\frac{3n}{5} - 6 + 6 = 0 + 6$$

$$\frac{\cancel{5}}{\cancel{3}} \times \frac{3n}{\cancel{5}} = 6 \times \frac{5}{3}$$

$$n = \frac{30}{3}$$

$$\boxed{n = 10}$$

$$20 = \frac{4n}{7} + 8$$

$$20 - 8 = \frac{4n}{7} + 8 - 8$$

$$12 = \frac{4n}{7}$$

$$12 \times \frac{7}{4} = \frac{4n}{7} \times \frac{7}{4}$$

$$\boxed{21 = n}$$

$$\frac{1}{6} = 10w + \frac{5}{6}$$

$$\frac{1}{6} \left(-\frac{5}{6}\right) = 10w + \frac{5}{6} \left(-\frac{5}{6}\right)$$

$$\frac{-4}{6} = \frac{10w}{1} \times \frac{1}{10}$$

$$\frac{-4}{6} \times \frac{1}{10} = \frac{-4}{60} = \frac{-2}{30} = \frac{-1}{15}$$

$$w = -\frac{1}{15}$$

Noah
Kazamer

A cell phone company offers two plans:

Plan A : 120 free minutes,
\$0.75 per additional
minute

Plan B : 30 free minutes,
\$0.25 per additional
minute

Plan A

$$0.75(x-120)$$

"x" equals the

Plan B

$$0.25(x-30)$$

number of
minutes you use.

Which time for calls will result in the same cost for both plans?

- Model the problem with an equation
- Solve the problem.
- Verify the problem

$$b) \quad 0.75(x-120) = 0.25(x-30)$$

$$0.75x - 90 = 0.25x - 7.5$$

$$0.75x - 0.25x - 90 = 0.25x - 7.5 - 0.25x$$

$$0.5x - 90 = -7.5$$

$$0.5x - 90 + 90 = -7.5 + 90$$

$$0.5x = 82.5$$

$$\frac{0.5x}{0.5} = \frac{82.5}{0.5}$$

$$x = 165 \text{ minutes}$$

HW: Page 281 #s 12 & 14

Solving Equations Requiring Distribution

$$4(y - 9) - 10y = 0$$

$$4y - 36 - 10y = 0$$

$$-6y - 36 = 0$$

$$-6y - 36 + 36 = 0 + 36$$

$$-6y = 36$$

$$\frac{-6y}{-6} = \frac{36}{-6}$$

$$y = -6$$

$$-3(z + 5) + 29 = -10$$

$$-3z - 15 + 29 = -10$$

$$-3z + 14 = -10$$

$$-3z + 14 - 14 = -10 - 14$$

$$-3z = -24$$

$$\frac{-3z}{-3} = \frac{-24}{-3}$$

$$z = +8$$

Solving Equations Part 3:
Variables on Both Sides*Equations
mini-test Wed.

$$7r - 5 = 2r + 5$$

$$7r - 2r - 5 = 2r - 2r + 5$$

$$5r - 5 = 5$$

$$5r - 5 + 5 = 5 + 5$$

$$5r = 10$$

$$\frac{5r}{5} = \frac{10}{5}$$

$$r = 2$$

$$4(2a - 2) = -2(1 - 5a)$$

$$8a - 8 = -2 + 10a$$

$$8a - 8a - 8 = -2 + 10a - 8a$$

$$-8 = -2 + 2a$$

$$-8 + 2 = -2 + 2 + 2a$$

$$-6 = 2a$$

$$\frac{-6}{2} = \frac{2a}{2}$$

$$-3 = a$$

$$\frac{3}{10}y = 19 - \frac{1}{5}y$$

$$\frac{2}{3}(6x + 9) = \frac{1}{2}(10x - 2)$$