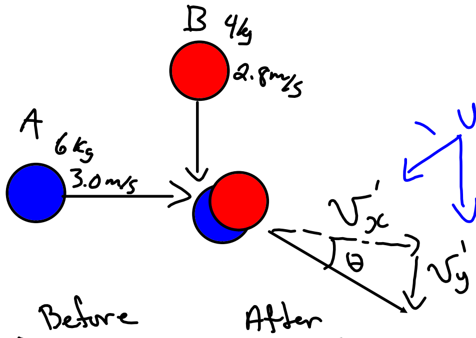
Two Dimensional (2D) Collisions

In order to solve two dimensional collision problems, write a conservation of momentum equation for the horizontal components of the momenta and a conservation of momentum equation for the vertical components of the momenta.

A 4.0 kg object is travelling south at a speed of 2.8 m/s when it collides with a 6.0 kg object travelling east at a velocity of 3.0 m/s. If these objects stick together upon collision, at what velocity do the combined masses move? 2.1 m/s [E 32° S]



Use: momentum in each direction to solve for v_x' and v_y'

Before

$m_A = 6$ kg

$v_A = 3.0$ m/s [E]

$m_B = 4$ kg

$v_B = 2.8$ m/s [S]

After

$v_A' = ?$

$v_B' = ?$ } v' *stick together

x - Dir

$$m_A v_{Ax} + m_B v_{Bx} = m_A v_{Ax}' + m_B v_{Bx}'$$

$$(6)(3) + (4)(0) = (6+4) v_x'$$

$$18 = 10 v_x'$$

$$1.8 = v_x'$$

y - dir

$$m_A v_{Ay} + m_B v_{By} = m_A v_{Ay}' + m_B v_{By}'$$

$$(6)(0) + 4(-2.8) = (6+4) v_y'$$

$$-11.2 = 10 v_y'$$

$$-1.12 = v_y'$$

Solve for v'

$$v' = \sqrt{(v_x')^2 + (v_y')^2}$$

$$= \sqrt{(1.8)^2 + (-1.12)^2}$$

$$= 2.1 \text{ m/s}$$

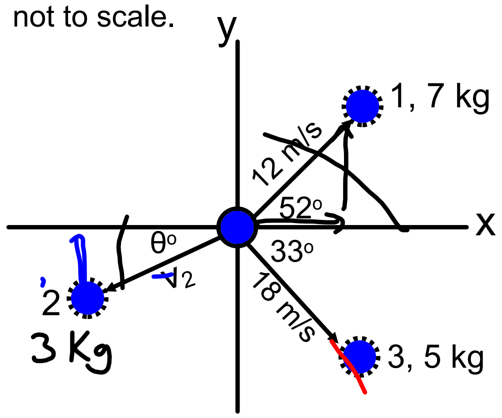
$$\theta = \tan^{-1}\left|\frac{v_y'}{v_x'}\right|$$

$$\theta = \tan^{-1}\left(\frac{1.12}{1.8}\right)$$

$$\theta = 32^\circ$$

$$v' = 2.1 \text{ m/s [E } 32^\circ \text{ S]}$$

A 15 kg object explodes in to three pieces numbered 1, 2, and 3. The velocities of piece 1 and 3 are labeled below. Calculate the velocity of piece 2. Diagram is not to scale.



P_{TOT} Before = 0 kg·m/s

$v_{1x}' = 12\cos 52$
$= 7.39$ m/s

$v_{1y}' = 12\sin 52$
$= 9.46$ m/s

$v_{3x}' = 18\cos 33$
$= 15.1$ m/s

$v_{3y}' = -18\sin 33$
$= 9.80$ m/s

x-dir

$0 = m_1 v_{1x}' + m_2 v_{2x}' + m_3 v_{3x}'$

$0 = (7)(7.39) + (3)v_{2x}' + (5)(15.1)$

$0 = 51.73 + 3v_{2x}' + 75.5$

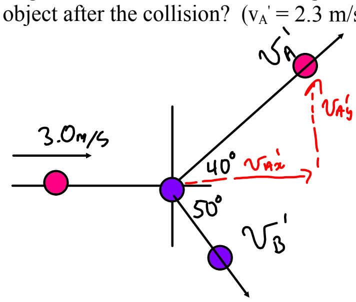
$\frac{-127.23}{3} = v_{2x}'$

$-42.41 \text{ m/s} = v_{2x}'$

$v_{2y}' = -5.74$ m/s @ 187.7°

or 7.7° south of west

Example: A 6.0 kg object, A, moving at a velocity of 3.0 m/s east collides with a 6.0 kg object, B, at rest. After the collision, A moves off in a direction 40.0° to the left of its original direction. B moves off in a direction 50.0° to the right of A's original direction. What is the magnitude of the velocity of each object after the collision? ($v_A' = 2.3$ m/s, $v_B' = 1.9$ m/s)



Before

$v_{Ax} = 3.0$ m/s

$v_{Bx} = 0$ m/s

$v_{Ay} = 0$ m/s

$v_{By} = 0$ m/s

After

$v_{Ax}' = v_A' \cos 40°$ $\qquad v_{Ay}' = v_A' \sin 40°$

$v_{Bx}' = v_B' \cos 50°$ $\qquad v_{By}' = -v_B' \sin 50°$

x-dir

$$v_{Ax} + v_{Bx} = v_{Ax}' + v_{Bx}'$$

$$3 = v_{Ax}' + v_{Bx}' \quad ⊗$$

y-dir

$$v_{Ay} + v_{By} = v_{Ay}' + v_{By}'$$

$$0 = v_{Ay}' + v_{By}' \quad ⊗$$

from x | from y

$3 = v_{Ax}' + v_{Bx}'$ | $0 = v_{Ay}' + v_{By}'$

$$3 = v_A' \cos 40 + v_B' \cos 50 \quad (1)$$

$$0 = v_A' \sin 40 + (-v_B' \sin 50) \quad (2)$$

Solve (2) for v_A'

$$v_B' \sin 50 = v_A' \sin 40$$

$$\frac{v_B' \sin 50}{\sin 40} = v_A' \leftarrow \text{sub into (1)}$$

$$3 = \left(\frac{v_B' \sin 50}{\sin 40}\right) \cos 40 + v_B' \cos 50$$

$$3 = 0.9129\, v_B' + 0.6428\, v_B'$$

$$3 = 1.5557\, v_B'$$

$$1.93 \text{ m/s} = v_B' \quad \leftarrow \text{Sub in (2)}$$

$$(2) \Rightarrow \frac{v_B' \sin 50}{\sin 40} = v_A'$$

$$\frac{(1.93)(\sin 50)}{\sin 40} = v_A'$$

$$\boxed{2.3 \text{ m/s} = v_A', \quad v_B' = 1.93 \text{ m/s}}$$