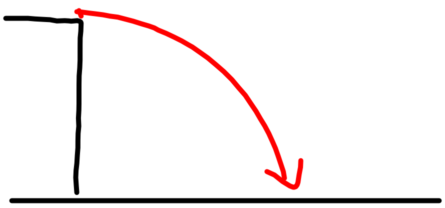
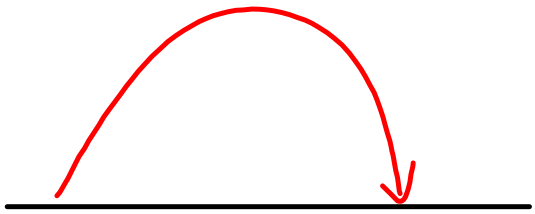
Projectile Motion

An object that is launched into the air and then comes under the influence of gravity moves in two dimensions (up/down and forward) and is called a projectile. The path taken by the projectile is called a trajectory.

Horizontally Launched Projectile



Projectile Launched At An Angle



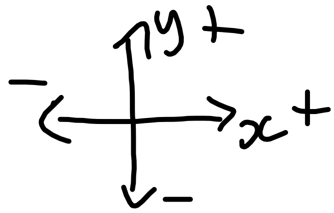
The vertical and horizontal motion of a projectile are independent of one another.

Horizontal Motion -> The horizontal velocity of a projectile is constant (ignoring air resistance).

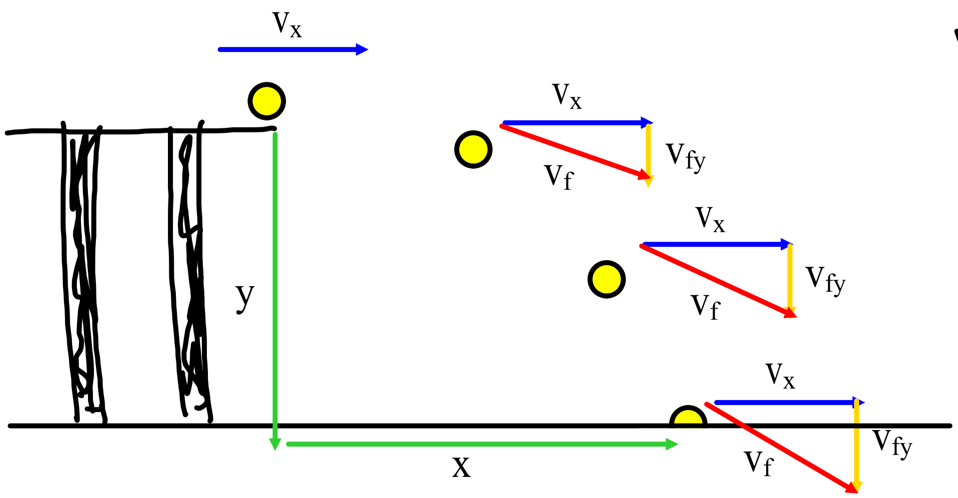
Vertical Motion-> The vertical velocity of a projectile is constantly changing due to gravity.

Projectile Fired Horizontally

Imagine the trajectory of a ball launched horizontally from the top of a cliff.



$v_{oy} = 0$ m/s



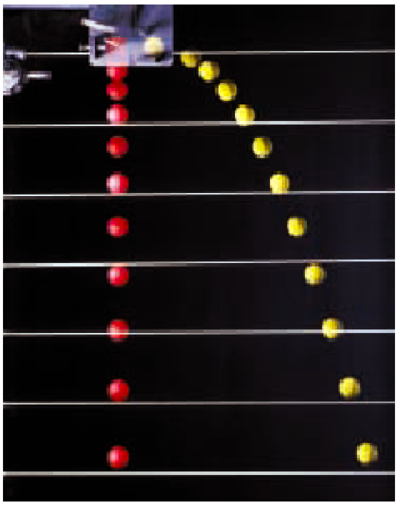


Figure 11.2 You can see that the balls are accelerating downward, because the distances they have travelled between flashes of the strobe light are increasing. If you inspected the horizontal motion of the ball on the right, you would find that it travelled the same horizontal distance between each flash of the strobe light.

<u>Example:</u> A projectile is fired horizontally from a height of 44.1 m at a speed of 50.0 m/s.

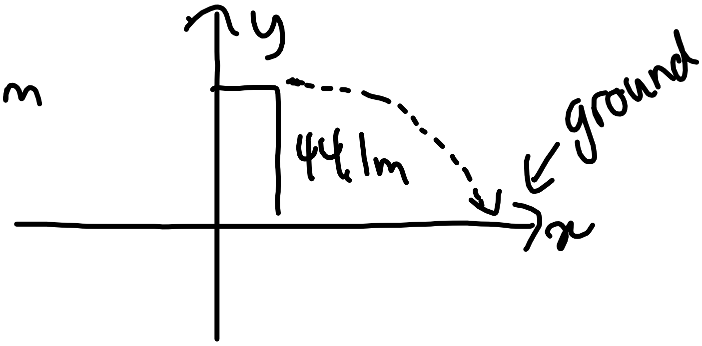
a) How long after it was fired, did the projectile hit the ground? (3.00 s)

b) How far forward did the projectile travel? (150 m)

$d_{oy} = 44.1\,m$ $d_{fy} = 0\,m$

$v_x = 50.0\,m/s$

$g = -9.81\,m/s^2$ $(\vec{a})$



$v_{oy} = 0\,m/s$

$t = ?$

a) $d_{fy} = d_{oy} + v_{oy}t + \frac{1}{2}gt^2$

$0 = 44.1 + (0)t + \frac{1}{2}(-9.81)t^2$

$0 = 44.1 - 4.9t^2$

$\sqrt{\frac{-44.1}{-4.9}} = t$

$3.0\,s = t$

b) $d_{fx} = ?$

$t = 3.0\,s$

$v_x = 50\,m/s$

$d_{fx} = v_x t$

$= (50)(3)$

$d_{fx} = 150\,m$

A projectile is fired with a horizontal velocity of 330 m/s from the top of a cliff 80 m high. With what velocity will it strike the ground? 1

x-dir

$v_x = 330 \text{ m/s}$

$d_{ox} = 0 \text{ m/s}$

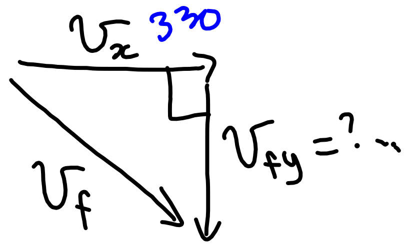
y-dir

$v_{oy} = 0 \text{ m/s}$

$g = -9.81 \text{ m/s}^2$

$d_{oy} = 80 \text{ m}$

$d_{fy} = 0 \text{ m}$



Find v_{fy}

$$v_{fy}^2 = v_{oy}^2 + 2g(d_f - d_o)$$

$$v_{fy}^2 = 0^2 + 2(-9.81)(0 - 80)$$

$$v_{fy}^2 = 1569.6 \rightarrow v_{fy} = \pm\sqrt{1569.6}$$

$$v_{fy} = -39.6 \text{ m/s}$$

$$v_f = \sqrt{v_{fy}^2 + v_x^2}$$

$$= \sqrt{(1570) + (330)^2}$$

$$v_f = 332 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_x}\right)$$

$$\tan^{-1}\left(\frac{39.6}{330}\right) =$$

$$\theta = 6.8^\circ$$

$v_f = 332 \text{ m/s}$
6.8° down from x-axis

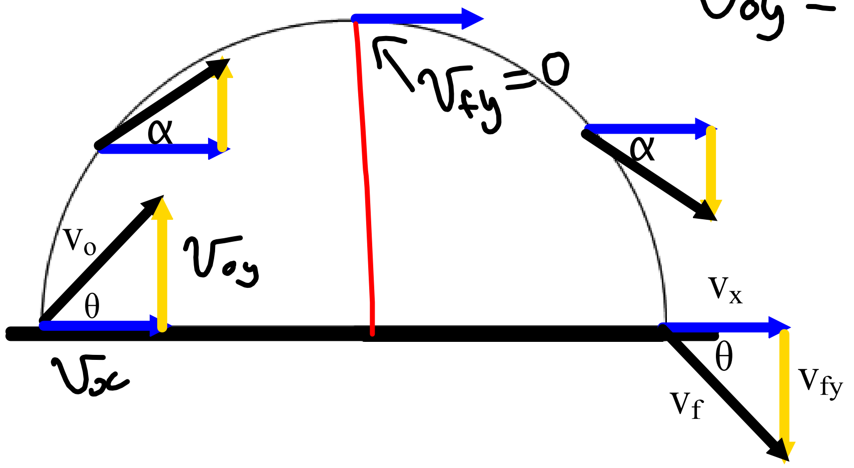
Projectiles Fired At An Angle

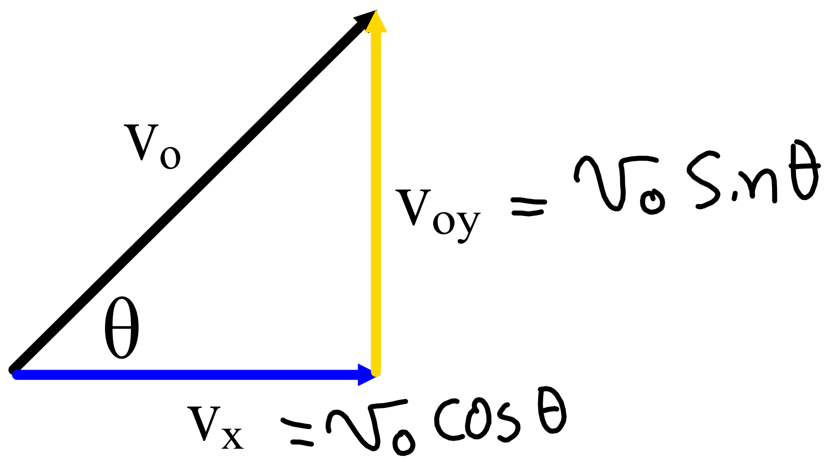
horizontal velocity -> **constant**
vertical velocity -> **changes**

$V_x = V_0 \cos\theta$

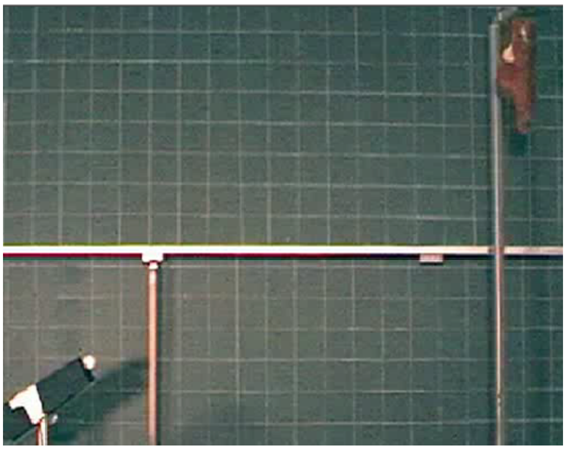
$V_{oy} = V_0 \sin\theta$

Trajectory





The Monkey and the Hunter



Example: An arrow is shot at an angle of 30.0° with the ground. It has a speed of 49 m/s. Assuming the arrow is shot from ground level and it lands on the ground, answer the following questions. $v_0 = 49\,m/s$ $\theta = 30°$

a) How high will the arrow go? (31 m)

b) Assuming the arrow lands on the ground, what is its range? (d_{fx}) (2.1 x 10^2 m)

$v_x = v_0 \cos\theta$
$= 49 \cos 30$
$v_x = 42.4\,m/s$

$v_{oy} = v_0 \sin\theta$
$v_{oy} = 49 \sin 30$
$v_{oy} = 24.5\,m/s$

a)

x-dir

$v_x = 42.4\,m/s$

y-dir

$v_{oy} = 24.5\,m/s$
$v_{fy} = 0\,m/s$
$g = -9.81\,m/s^2$
$d_{oy} = 0\,m$
$d_{fy} = ?$

$$v_f^2 = v_{oy}^2 + 2g(d_{fy} - d_{oy})$$
$$0 = 24.5^2 + 2(-9.81)(d_{fy} - 0)$$
$$0 = 600.25 - 19.62 d_{fy}$$
$$\frac{-600.25}{-19.62} = d_{fy}$$
$$\boxed{30.6\,m = d_{fy}}$$

b) $d_{fx} = ?$
$v_x = 42.4$
$d_{fx} = v_x t$

y-dir

$v_{oy} = 24.5\,m/s$
$d_{oy} = 0\,m$
$d_{fy} = 0\,m$
$g = -9.81\,m/s^2$
$t = ?$

$v_{fy} = -24.5\,m/s$ * because @ same vertical position

$$d_{fy} = d_{oy} + v_{oy} t + \tfrac{1}{2} a t^2$$
$$0 = 0 + 24.5t - 4.9t^2$$
$$0 = t(24.5 - 4.9t)$$
$$t = 0\,s$$
$$0 = 24.5 - 4.9t$$
$$\frac{-24.5}{-4.9} = t$$
$$5.0\,s = t$$

$$d_{fx} = v_x t$$
$$= (42.4)(5)$$
$$\boxed{= 212.5\,m}$$

A golfer standing on a fairway hits a golf ball to a green that is elevated 5.50 m above the point where she is standing. If the ball leaves the club with a velocity of 46.0 m/s, 35.0° above the ground, find the time the ball spends in the air. (5.15 s)

$V_{oy} = 46 \sin 35$
$= 26.4 \text{ m/s}$

$V_{ox} = 46 \cos 35$
$= 37.7 \text{ m/s}$

x-dir
$V_{ox} = 37.3$

y-dir
$V_{oy} = 26.4$
$g = -9.81$
$d_{oy} = 0 \text{ m}$
$d_{fy} = 5.5 \text{ m}$
$t = ?$

$$d_{fy} = d_{oy} + V_{oy}t + \frac{1}{2}at^2$$

$$5.5 = 0 + 26.4t - 4.9t^2$$

$$4.9t^2 - 26.4t + 5.5 = 0$$

$$t = \frac{-(-26.4) \pm \sqrt{(-26.4)^2 - 4(4.9)(5.5)}}{2(4.9)}$$

$$t = \frac{26.4 \pm 24.3}{9.81}$$

$t = 5.2\text{s}$ and 0.2s

↑ on the way down

↑ on way up

Home Run Question

Suppose the left-centre field wall is 3 m high off the ground and located 105 m from home plate. A batter connects the ball 1.0 m above the ground at an angle of 55° and the ball has a speed of 32 m/s. Will this be a home run?

(i.e when the ball has travelled 105 m horizontally is it higher than 2 m above the initial point of contact?)

- Break up the initial velocity into its components.
- Determine the time it takes to travel 105 m horizontally.
- Determine the height above the point of contact using the found time.
- If y > 2 m then it is a home run.

Suppose the left-centre field wall is 4.1 m high and located 100.8 m from home plate. A batter connects the ball 1.0 m above the ground at an angle of 45^{o} and the ball has a speed of 31 m/s. Will this be a home run?

With what minimum speed does a baseball have to come off the bat to clear the Green Monster at Fenway Park? The wall is 115.5 m from home plate and 11.3 m high. Assume the batter makes contact 1.0 m off the ground at an angle of 65°.

x-dir

$v_x = v\cos 65$

$d_{fx} = 115.5\,m$

$d_{fx} = v_x t$

$115.5 = (v\cos 65)t$

↑ Solve for t —sub in→

$\frac{115.5}{v\cos 65} = t$

y-dir

$g = -9.81\,m/s^2$

$d_{oy} = 1\,m$

$d_{fy} = 11.3\,m$

$v_{oy} = v\sin 65$

$d_{fy} = d_{oy} + v_{oy}t + \frac{1}{2}at^2$

$11.3 = 1 + (v\sin 65)t - 4.9t^2$

$11.3 = 1 + (v\sin 65)\left(\frac{115.5}{v\cos 65}\right) - 4.9\left(\frac{115.5}{v\cos 65}\right)^2$ ←

$10.3 = 115.5\tan 65 - \frac{365985}{v^2}$

$10.3 = 247.7 - \frac{365985}{v^2}$

$-237.4 = -\frac{365985}{v^2}$

$v^2 = \frac{-365985}{-237.4} = 1541.7$

$v = 39.3\,m/s$

* ignore neg root, we want $|v|$

In an archery skills competition an archer stands next to her target. A bell rings and the target begins to move away from the archer at 3.5 m/s but the archer must wait a certain amount of time before shooting. This archer releases the arrow 8.0 s after the bell has rung in an attempt to hit the moving target. Assuming the launch angle is 45° and the arrow is launched at the same height as the target, determine the initial speed of the arrow to successfully hit the target.

- Find an expression for the horizontal distance covered by the arrow.
- Write the expression for V_x using d_{fx}/t and remember that $V_x = V\cos 45^o$.
- Combine the above to have a formula for V.
- Write $V_{oy} = V\sin 45^o$.
- Write the formula for vertical displacement, d_{fy}.
- In the above substitute $V\sin 45^o$ for V_{oy}.
- Now substitute the formula for V into the vertical displacement expression.
- Solve for t, then find V.

A target is 525 m from an archer. The archer is using a bow that launches the arrow at 75 m/s. What angle(s) must the archer make to hit the target? Assume the target is hit at the same height the arrow is launched.

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