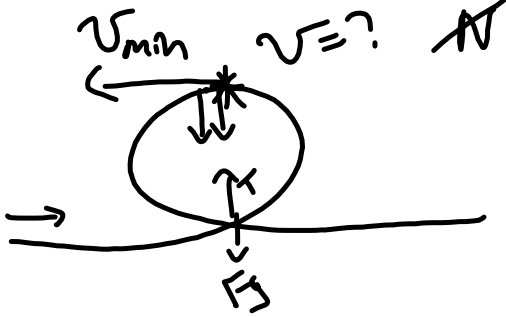
26. A motorcycle stunt rider wants to do a loop-the-loop within a vertical circular track. If the radius of the circular track is 10.0 m, what minimum speed must the motorcyclist maintain to stay on the track?



$\cancel{N}^{0} + F_g = F_c$

$F_g = F_c$

$\cancel{m}g = \frac{\cancel{m}v^2}{r}$

$9.81 = \frac{v^2}{10}$

$98.1 = v^2$

$9.9\ m/s = v$

Universal Gravitation

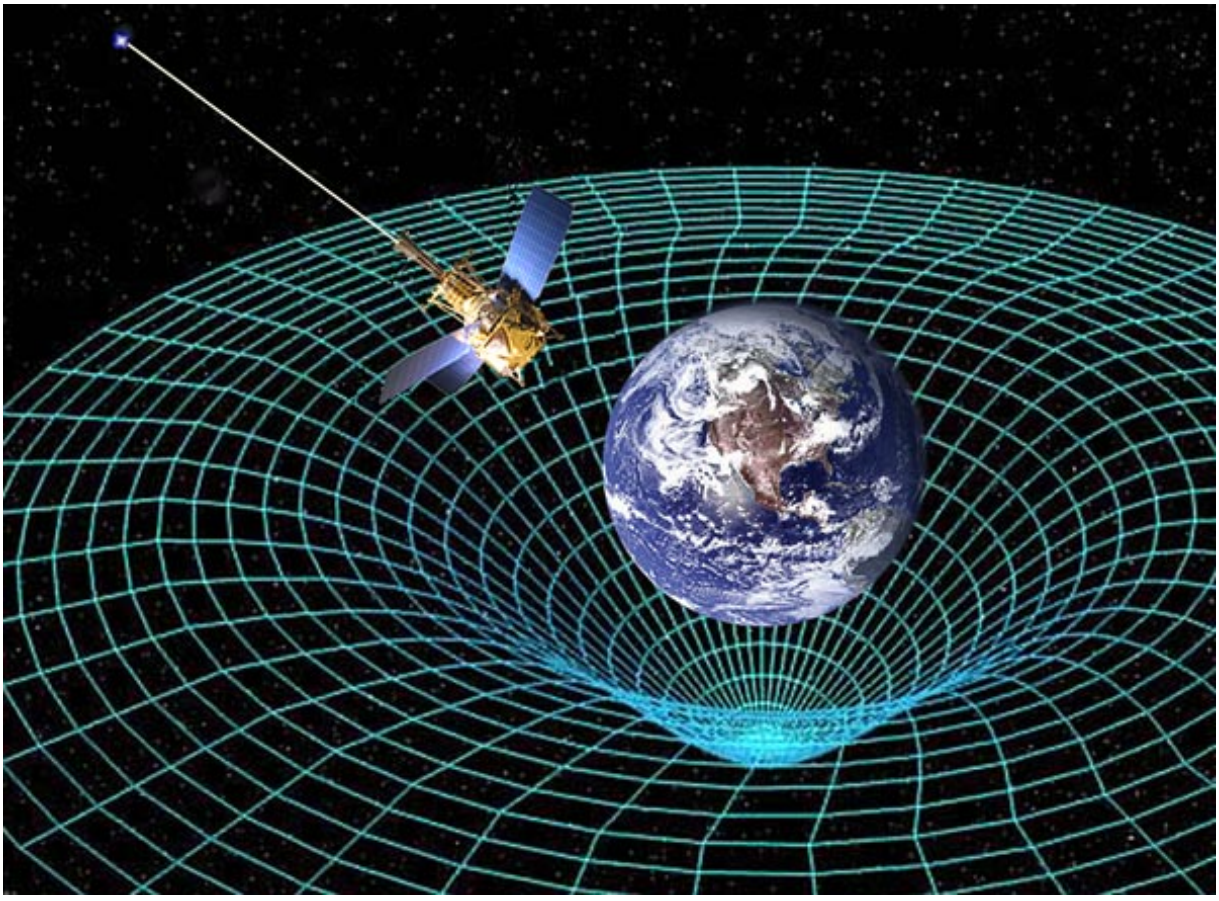
Theories of Planetary Motion

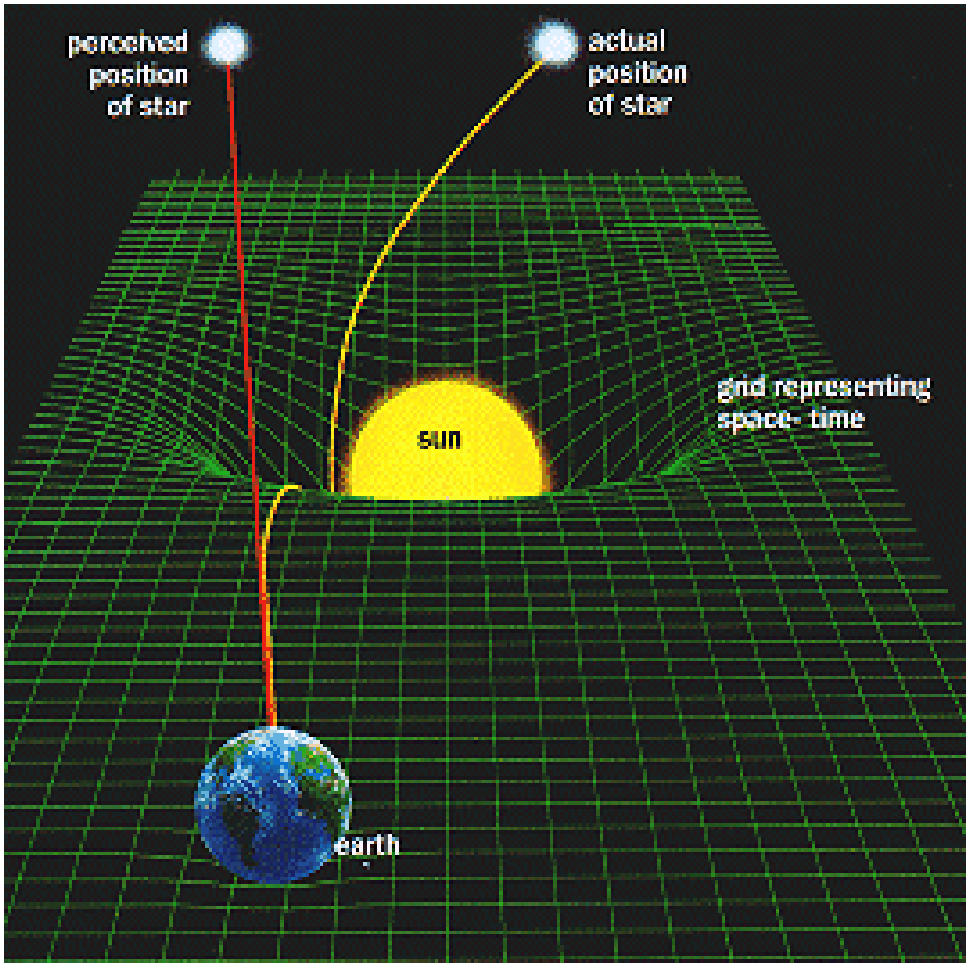
1. Tycho Brahe (1546-1601)

   Brahe believed in the geocentric theory of planetary motion - Earth as the center of the universe.

2. Johannes Kepler (1571-1630)

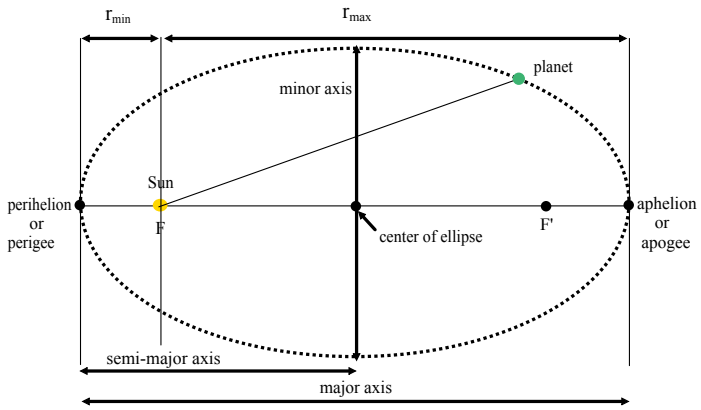
   Kepler believed in the heliocentric theory of planetary motion - Sun as the center of the universe.




perceived position of star
actual position of star
grid representing space- time
sun
earth

Kepler's Three Laws of Planetary Motion

1. Law of Orbits - The paths of the planets are ellipses with the center of the Sun at one focus.



eccentricity - a quantity indicating how non-circular an ellipse is
- based on a scale from 0 to 1 where a value nearer 1 implies a higher degree of non-circularity

2. Law of Areas - An imaginary line from the sun to a planet sweeps out equal areas in equal time intervals. Therefore, planets move faster when closest to the sun and slowest when farthest away.

Simulation

3. Law of Periods - The ratio of the square of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun.

T_A -> period of object A (s, min, h, etc)
T_B -> period of object B (s, min, h, etc)
r_A -> mean orbital radius (m, km, etc)
r_B -> mean orbital radius (m, km, etc)

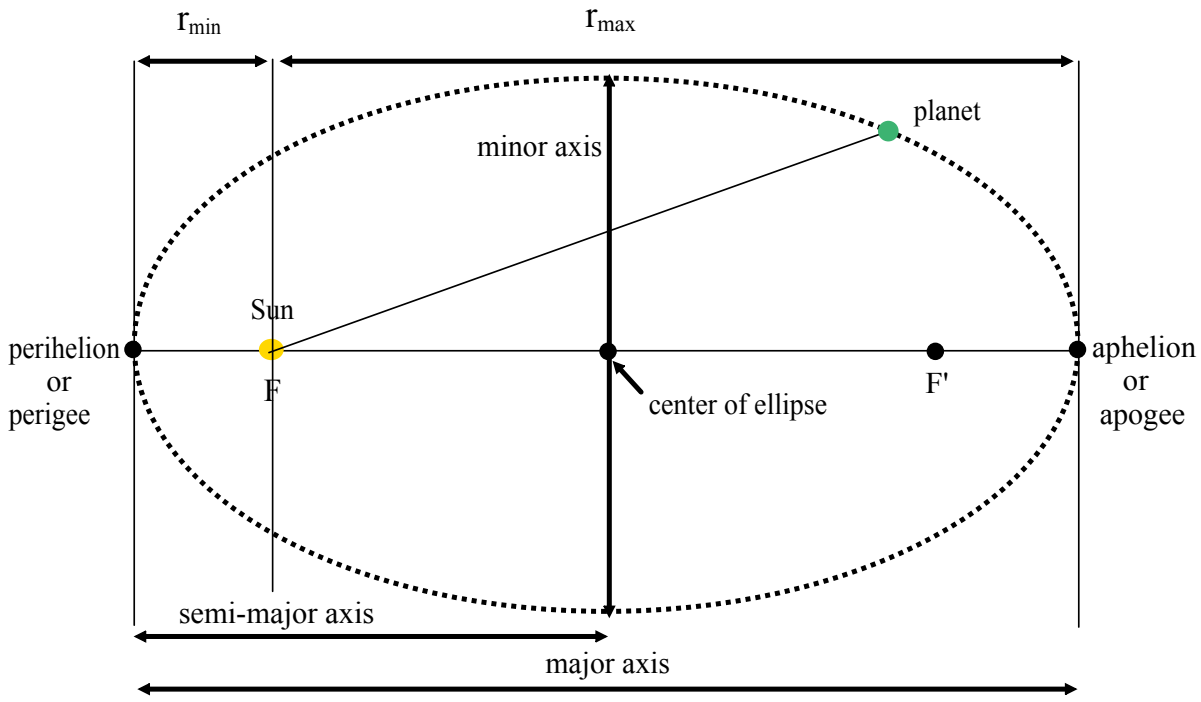
NOTE: The first two laws apply to each planet, satellite or moon. The third relates to the motion of two objects which are orbiting the same central body. The central body does not have to be the sun.

NOTE: One astronomical unit (AU) is the average distance from Earth to the Sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{m}$$

NOTE: Physical data can be found on Page 573 and 955.

Elliptical Orbit



Universal Law of Gravitation

Newton -> explained the motion of the planets
-> a force of attraction between the sun and planets keep the planets traveling along their elliptical paths

1. $F \propto m_1 m_2$ ← proportional to

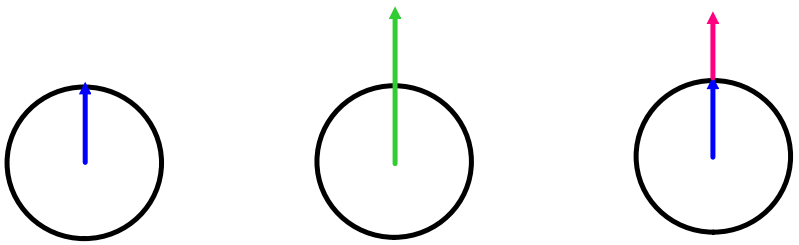
* Force of attraction is directly proportional to the product of the masses of the objects.

2. $F \propto 1/r^2$

* Gravitation force is inversely proportional to the distance squared.

~~=> $F = Gm_1m_2/r^2$~~ $\vec{F}_g = \frac{G m_1 m_2}{r^2}$

F = gravitational force (N)
m_1 = mass of object 1 (kg)
m_2 = mass of object 2 (kg)
r = distance between masses (m)
G = universal gravitational constant (6.672×10^{-11} Nm^2/kg^2)



radius | orbital radius | radius + distance above surface

Proportionality Questions

By what factor will the force of gravity on a person change if the mass of this person doubles?

$$F_g = \frac{G m_1 m_2}{r^2}$$

By what factor will the force of gravity on a person change if the person orbits a distance three times the Earth's radius?

$$F = \frac{G M_1 M_2}{r^2}$$

Sun's Mass
1.989×10^{30} kg
Radius $= 6.96 \times 10^8$ m

$M_E = 5.974 \times 10^{24}$ kg

$r = 1.5 \times 10^{11}$ m $\Rightarrow$ (1 AU)

M_{Earth} M_{sun}

$$F = \frac{6.672 \times 10^{-11}(5.974 \times 10^{24})(1.989 \times 10^{30})}{(1.5 \times 10^{11})^2}$$

$$F = \frac{7.928 \times 10^{44}}{(1.5 \times 10^{11})^2}$$

$$= 3.52 \times 10^{22} \text{ N}$$

Moon: 7.349×10^{22} kg

Diameter: 3476 km $= 3.476 \times 10^{6}$ m

Earth-Moon Dist = 384 400 Km

$= 3.844 \times 10^{8}$ m

Sun Mass $= 1.989 \times 10^{30}$ kg

Sun Radius $= 6.96 \times 10^{8}$ m

Proton Mass $= 1.673 \times 10^{-27}$ kg

Electron Mass $= 9.109 \times 10^{-31}$ kg

Force of Gravity between Earth & Sun: 3.52×10^{22} N

" " " " Earth & Moon: 1.97×10^{20} N

Tidal Force: Change in gravitational force with distance (Force Gradient)

-4.70×10^{11} N/m Sun $F = G M_E M_P (r^{-2})$

-1.03×10^{12} N/m Moon $\frac{dF}{dr} = -\frac{2 M_E M_P G}{r^3}$

Tides are caused by the Moon because the change in force is greater per unit distance than that of the Sun.