7.2 Conservation of Total Energy

LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed, but it can be transformed from one form to another or transferred from one object to another. The total energy of an isolated system, including all forms of energy, always remains constant.

CONSERVATION OF TOTAL ENERGY

The work done by nonconservative forces is the difference of the final mechanical energy and the initial mechanical energy of a system.

$$W_{nc} = E_{final} - E_{initial}$$

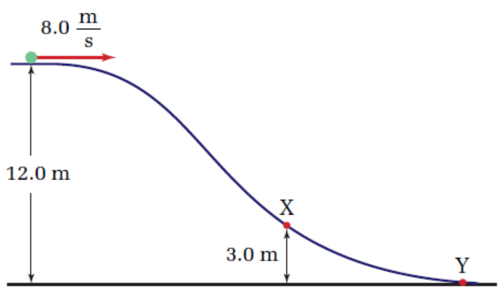
$W_{nc} = \Delta E_T$

Quantity	Symbol	SI unit
work done by nonconservative forces	W_{nc}	J (joule)
mechanical energy of the system after the process	E_{final}	J (joule)
mechanical energy of the system before the process	$E_{initial}$	J (joule)

Unit Analysis

All of the units are joules.

19. A sled at the top of a snowy hill is moving forward at 8.0 m/s, as shown in the diagram. The height of the hill is 12.0 m. The total mass of the sled and rider is 70.0 kg. Determine the speed of the sled at point X, which is 3.0 m above the base of the hill, if the sled does 1.22×10^3 J of work on the snow on the way to point X.



$W_{nc} = -1220\ J$

$W_{nc} = \Delta E_T$

↑ friction

$W_{nc} = \Delta E_{KT} + \Delta E_{gT}$

$W_{nc} = (E_{Kf} - E_{Ko}) + (E_{gf} - E_{go})$

$-1220 = \left(\frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2\right) + (mgh_f - mgh_o)$

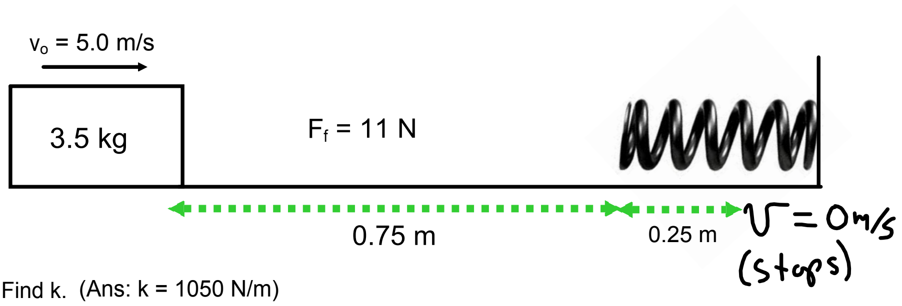
$\frac{-1220}{70} = \frac{1}{2}(70)v_f^2 - \frac{1}{2}(70)(8)^2 + (70)(9.81)(3) - (70)(9.81)(12)$

$-17.42 = \frac{v_f^2}{2} - 32 + 29.43 - 117.72$

$102.87 = \frac{v_f^2}{2} \rightarrow \sqrt{205.74} = v_f^2$

$14.3\ m/s = v_f$

Conservation of Total Mechanical Energy



a) Find k. (Ans: k = 1050 N/m)

$$W_{nc} = \Delta E_T \rightarrow W_{nc} = \Delta E_{KT} + \Delta E_{eT}$$

$$W_{nc} = (E_{Kf} - E_{Ko}) + (E_{ef} - E_{eo})$$

$$-F_f d = \tfrac{1}{2} m v_f^2 \,(\to 0) - \tfrac{1}{2} m v_o^2 + \tfrac{1}{2} k x_f^2 - \tfrac{1}{2} k x_o^2 \,(\to 0)$$

$$-(11)(1.0) = -\tfrac{1}{2}(3.5)(5)^2 + \tfrac{1}{2} k (0.25)^2$$

$$-11 = -43.75 + 0.03125k$$

$$32.75 = 0.03125k$$

$$\boxed{1048 \text{ N/m} = k}$$

b) How fast will the mass be moving as it leaves the spring? (4.14 m/s)

$$W_{nc} = \Delta E_T$$

$$-F_f d = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_o^2 \,(\to 0) + \tfrac{1}{2} k x_f^2 \,(\to 0) - \tfrac{1}{2} k x_o^2$$

$$-(11)(0.25) = \tfrac{1}{2}(3.5) v_f^2 - \tfrac{1}{2}(1048)(0.25)^2$$

$$-2.75 = 1.75 v_f^2 - 32.75$$

$$30 = 1.75 v_f^2$$

$$\sqrt{17.14} = v_f$$

$$\boxed{4.14 \text{ m/s} = v_f}$$