

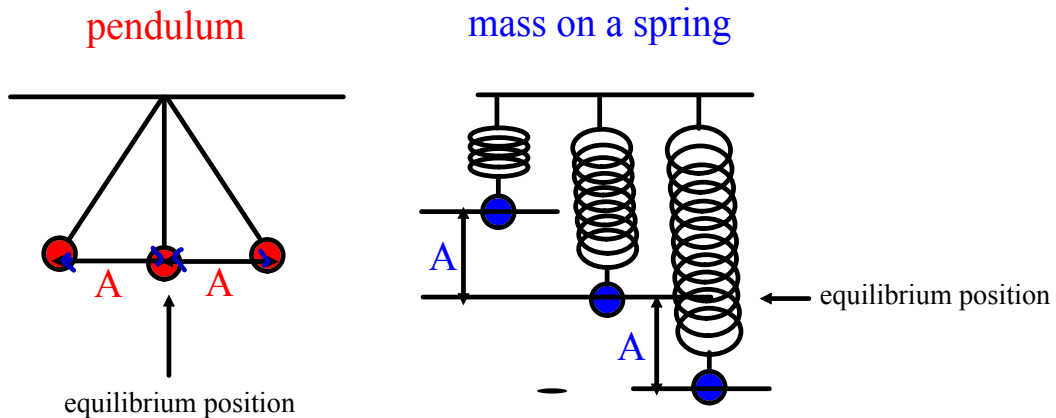
Work, ~~Power~~, and Energy

Simple Harmonic Motion (SHM)

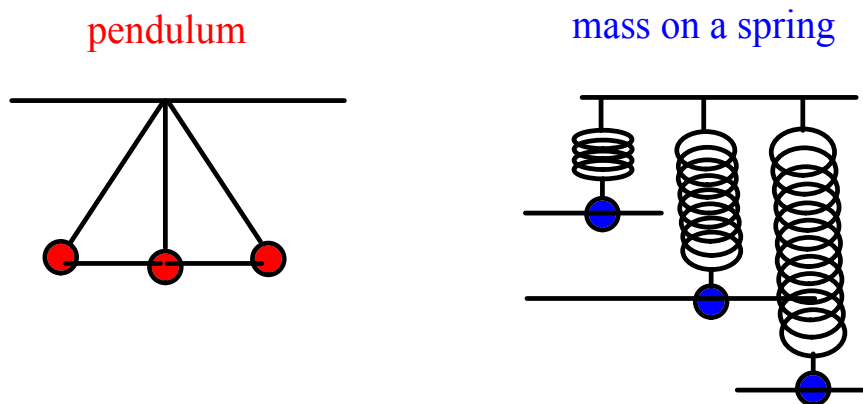
Many motions take the form of vibrations or oscillations. These motions take place in a straight line - a body or parts of the body move back and forth over and over again.

In describing vibrational motion, we generally refer to the object's amplitude, its period and its frequency.

amplitude - maximum displacement from the object's equilibrium position (cm or m)



period - time for one complete vibration (s)



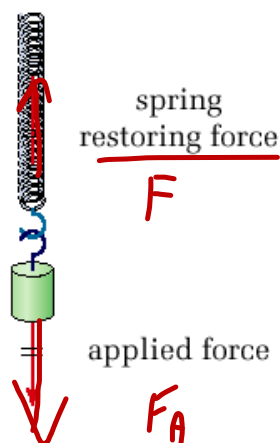
frequency - number of vibrations per unit time (Hz or s^{-1})

There are two requirements for SHM.

- 1) The acceleration of the object is proportional to the displacement of the body from its equilibrium position.
- 2) The acceleration of the object is directed toward its equilibrium position.

Mass on a Spring and Hookes' Law

Physics
McGraw-Hill
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Hooke's Law - Applied Force

$$F_A = kx$$

F_A -> applied force (N)
 k -> spring constant (N/m)
 x -> elongation or compression (m)

Hooke's Law - Restoring Force

$$F = -kx$$

F -> restoring force (N)
 k -> spring constant (N/m)
 x -> elongation or compression (m)

Potential energy is the energy stored by an object due to its position or condition.

For all forms of potential energy, there is no absolute zero position or condition. Only changes in potential energy are measured. You must assign a reference position (or establish a reference line or zero line) to determine potential energy.

Elastic Potential Energy

Many objects can stretch, compress, bend or change shape in some way. If an object can return to its original condition, it is said to be elastic. Since the object can undergo motion when the force causing the change in condition or state is removed, there must be stored energy due to its condition. This form of stored energy is called elastic potential energy.

Restoring Force vs Extension (or Compression)

The area under a Hooke's Law graph (restoring force vs. extension or compression) gives the amount of elastic potential energy stored in a spring (or any elastic substance).

ELASTIC POTENTIAL ENERGY

The elastic potential energy of a perfectly elastic material is one half the product of the spring constant and the square of the length of extension or compression.

$$E_e = \frac{1}{2}kx^2$$

Quantity	Symbol	SI unit
elastic potential energy	E_e	J (joules)
spring constant	k	$\frac{\text{N}}{\text{m}}$ (newtons per metre)
length of extension or compression	x	m (metres)

Unit Analysis

$$\text{joule} = \frac{\text{newton}}{\text{metre}} \text{metre}^2 \quad \text{J} = \left(\frac{\text{N}}{\text{m}}\right)\text{m}^2 = \text{N} \cdot \text{m} = \text{J}$$

MODEL PROBLEM

Elastic Potential Energy of a Spring

A spring with spring constant of 75 N/m is resting on a table.

- If the spring is compressed a distance of 28 cm, what is the increase in its potential energy?
- What force must be applied to hold the spring in this position?

Potential Energy
(Page 247)

Gravitational Potential Energy

Gravitational potential energy is the potential energy an object has because of its position above Earth's surface.

$$\Delta E_g = mg\Delta h$$

$$E_g = mgh$$

ΔE_g -> change in gravitational potential energy (J)

m -> mass (kg)

g -> acceleration due to gravity (m/s^2)

Δh -> change in height (m)

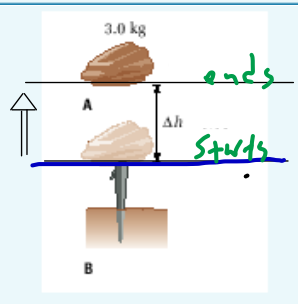
Chapter 6 Work, Power, and Efficiency • MHR 249

MODEL PROBLEM

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Calculating Gravitational Potential Energy

You are about to drop a 3.0 kg rock onto a tent peg. Calculate the gravitational potential energy of the rock after you lift it to a height of 0.68 m above the tent peg.



reference level -> $E_g = 0$ J, $h = 0$ m

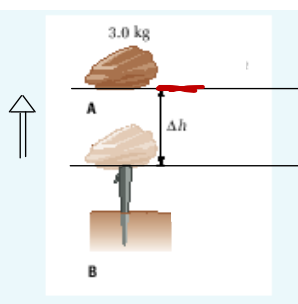
must be stated

$$E_g = mg\Delta h$$

$$E_g = mg(h_f - h_i)$$

$$E_g = (3.0 \text{ kg})(9.80 \text{ m/s}^2)(0.68 \text{ m} - 0 \text{ m})$$

$$E_g = 20 \text{ J}$$



reference level -> $E_g = 0$ J, $h = 0$ m

Kinetic Energy

McGraw-Hill: Page 236

Reminder: Kinetic energy is energy due to motion.

$$E_k = \frac{1}{2}mv^2$$

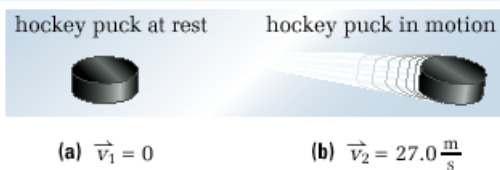
E_k -> kinetic energy (J)
 m -> mass (kg)
 v -> velocity (m/s)

NOTE: When velocity is squared, it is no longer a vector so no vector notation is used in the kinetic energy equation.

MODEL PROBLEM

Calculating Kinetic Energy

A 0.200 kg hockey puck, initially at rest, is accelerated to 27.0 m/s. Calculate the kinetic energy of the hockey puck (a) at rest and (b) in motion.

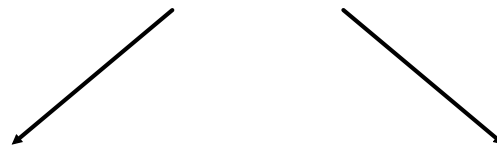


$$E_k = 0 \text{ J}$$

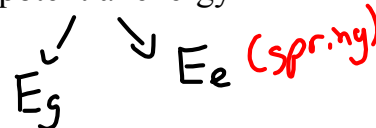
$$E_k = 72.9 \text{ J}$$

Work and Energy

Types of Energy



mechanical energy = kinetic energy + potential energy

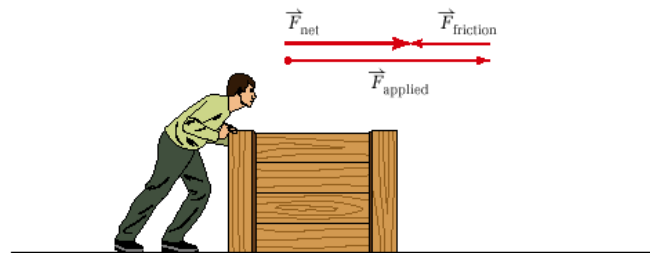


Work

Work is a transfer of energy. Work is always done on an object and results in a change in that object. A force does work on an object if it causes the object to move.

Work is the product of the magnitude of an individual force acting on an object (not the net force acting on the object) and the magnitude of the displacement of the object. The force and displacement must be parallel to one another.

Figure 6.3 When you were determining the motion of objects in Chapter 4, you used the net force acting on the object. The net force is really the vector sum of all of the forces acting on the object. When calculating work, you determine the work done by one specific force, not the net force.



$W = F_{\parallel} \Delta d$

→

$W = Fd$

W -> work

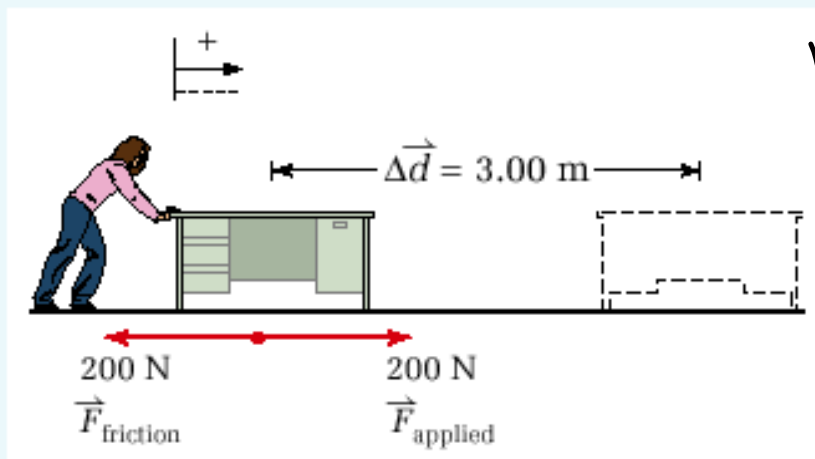
F_{\parallel} -> magnitude of individual force

Δd -> magnitude of displacement

NOTE: Force and displacement are vectors. Work is a scalar.

Determining the Amount of Work Done

A physics student is rearranging her room. She decides to move her desk across the room, a total distance of 3.00 m. She moves the desk at a constant velocity by exerting a horizontal force of 2.00×10^2 N. Calculate the amount of work the student did on the desk in moving it across the room.



F_{applied}

$$W = \vec{F} \vec{d}$$
$$W = (200)(3)$$
$$W = 600 \text{ J}$$

Three Cases
When No Work is Done
 (Page 222)

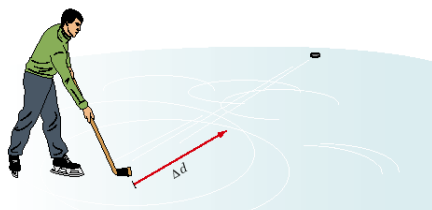
Case 1: Applying a Force That Does Not Cause Motion

Consider the energy that you could expend trying to move a house. Although you are pushing on the house with a great deal of force, it does not move. Therefore, the work done on the house, according to the equation for work, is zero (see Figure 6.4). In this case, your muscles feel as though they did work; however, they did no work on the house. The work equation describes work done by a force that moves the object on which the force is applied. Recall that work is a transfer of energy to an object. In this example, the *condition* of the house has not changed; therefore, no work could have been done on the house.



Case 2: Uniform Motion in the Absence of a Force

Recall from Chapter 5 that Newton's first law of motion predicts that an object in motion will continue in motion unless acted on by an *external* force. A hockey puck sliding on a frictionless surface at constant speed is moving and yet the work done is still zero (see Figure 6.5). Work was done to start the puck moving, but because the surface is frictionless, a force is not required to keep it moving; therefore, no work is done on the puck to keep it moving.



Case 3: Applying a Force That Is Perpendicular to the Motion

Assume that you are carrying your physics textbook down the hall, at constant velocity, on your way to class. Your hand applies a force directly upward to your textbook as you move along the hallway. When considering the work done on the textbook by your hand, you can see that the upward force is perpendicular (i.e., at 90°) to the displacement. In this case, the work done by your hand on the textbook is zero (see Figure 6.6). It is important to note that your hand does do work on the textbook to accelerate it when you begin to move, but once you and the textbook are moving at a constant velocity, you are no longer doing work on the book.

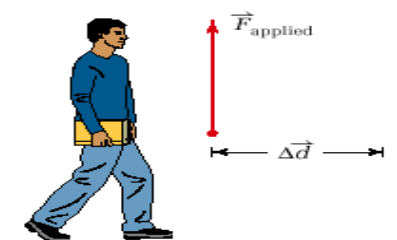


Figure 6.6 You are exerting an upward force (against gravity) on your book to prevent it from falling. However, since this force is perpendicular to the motion of the book, it does no work on the book.

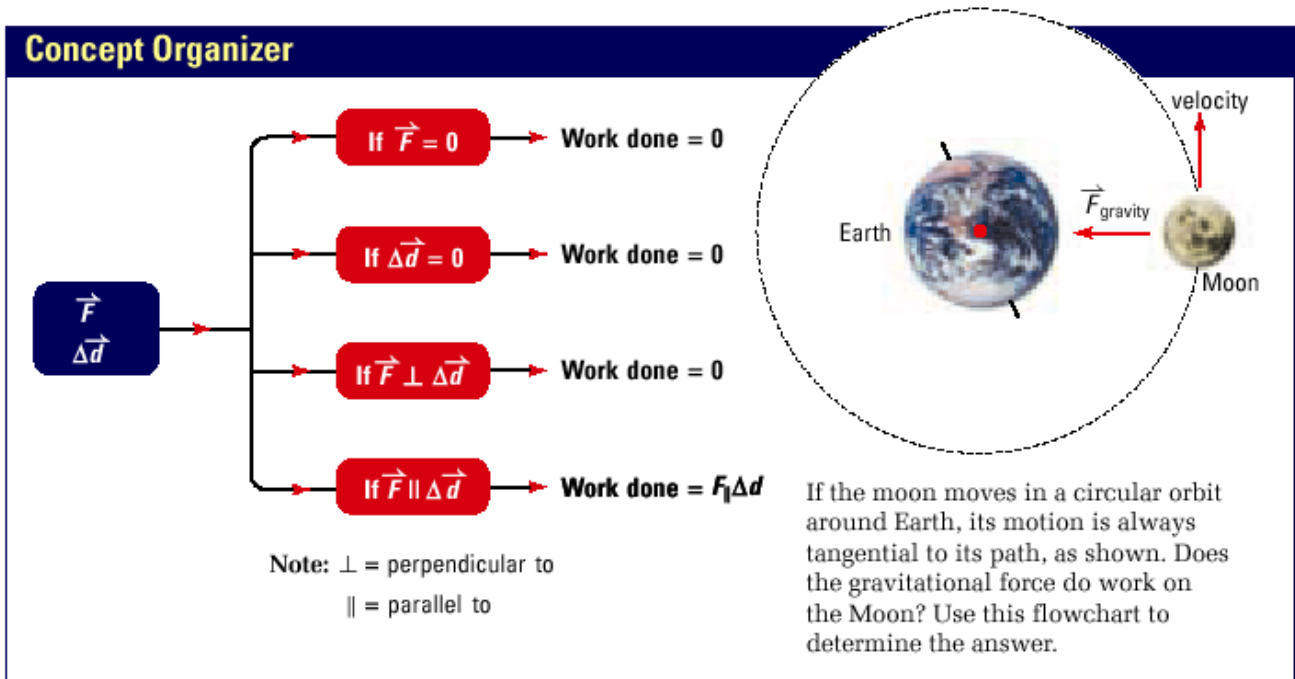
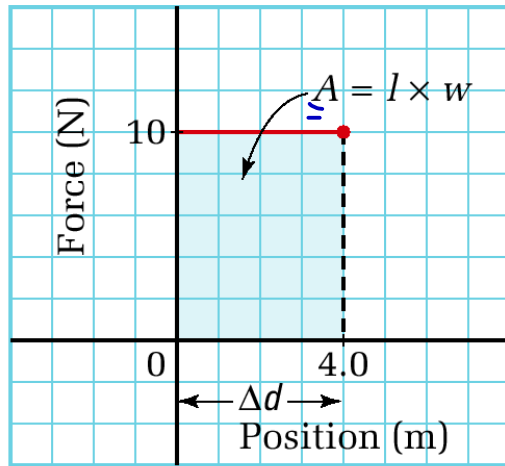


Figure 6.7 Making decisions about work done.

Work Done by Forces
(Page 225)

Force vs. Position



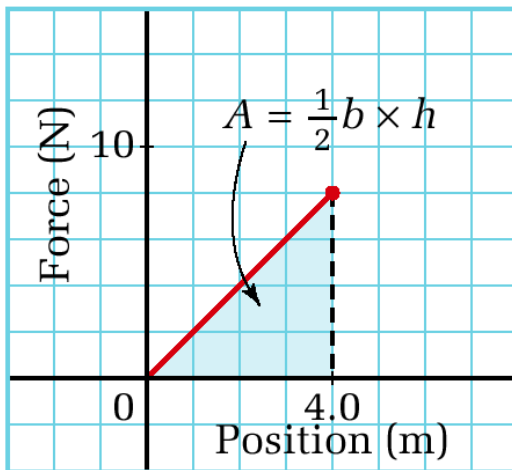
Constant Force

Area = $l \times w$

Area = Fd

Area = Work

Force vs. Position



Force Not Constant

(increasing steadily)

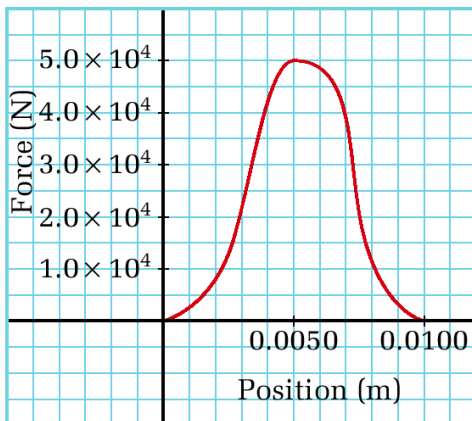
$A = \frac{1}{2} b \times h$

$A = \frac{1}{2} (dF)$ or $A = \frac{1}{2} Fd$

$W = \frac{1}{2} Fd$

* Average force is used.

Force vs. Position



Force Not Constant

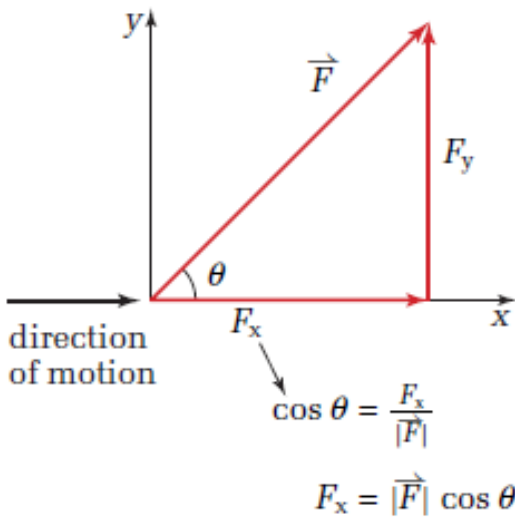
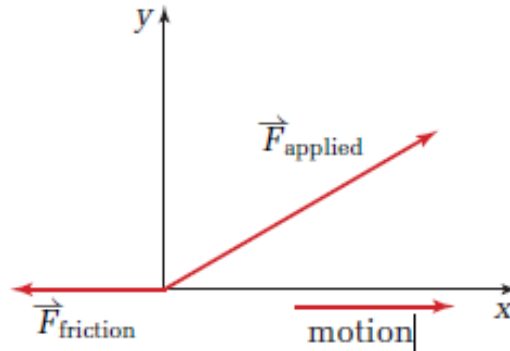
Force changes. It reaches a maximum then falls back to zero.

Area = Work Done

Use Calculus to find work.

OR

To find the work done, count the number of squares and estimate the area of the partial squares.



WORK

Work done when the force and displacement are not parallel and pointing in the same direction

$$W = F \Delta d \cos \theta$$

θ is the angle between the force and displacement vectors. Note: Since work is a scalar quantity and only the magnitudes of the force and displacement affect the value of the work done, vector notations have been omitted.

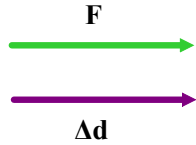
Positive and Negative Work
(Page 233)

Positive work is done when the force causing the displacement is in the **same** direction as the displacement. Positive work **adds** energy to an object.

Negative work is done when the force causing the displacement is in a direction **opposite** that of the displacement. Negative work **removes** energy from an object.

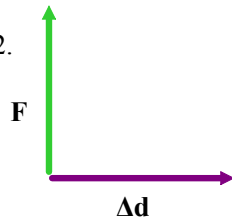
Examples

1.



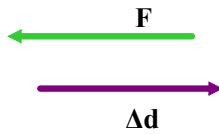
Maximum Positive Work

2.



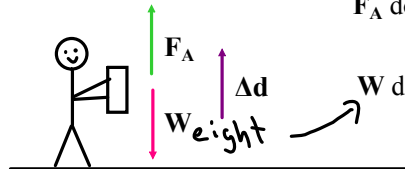
No Work is Done

3.



Maximum Negative Work

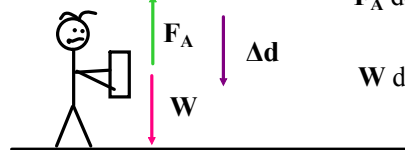
4.



F_A does positive work.

W does negative work.

5.



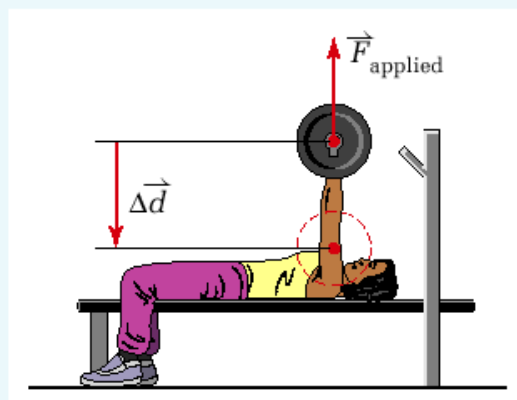
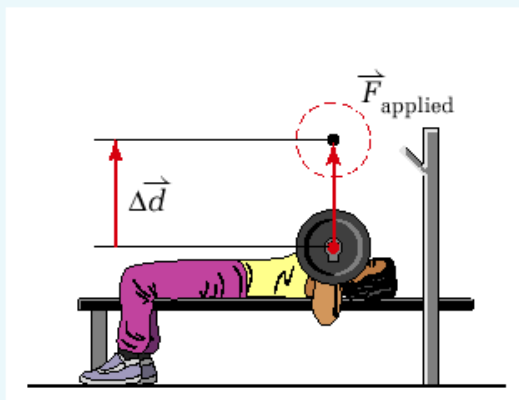
F_A does negative work.

W does positive work.

MODEL PROBLEM

Doing Positive and Negative Work

Consider a weight lifter bench-pressing a barbell weighing $6.50 \times 10^2 \text{ N}$ through a height of 0.55 m. There are two distinct motions: (1) when the barbell is lifted up and (2) when the barbell is lowered back down. Calculate the work that the weight lifter does on the barbell during each of the two motions.



Lifting

$$W = Fd$$

$$W = (6.50 \times 10^2 \text{ N})(0.55 \text{ m})$$

$$W = 3.2 \times 10^2 \text{ J}$$



$$W = 3.2 \times 10^2 \text{ J}$$

Lowering

$$W = Fd$$

$$W = (6.50 \times 10^2 \text{ N})(0.55 \text{ m})$$

$$W = 3.2 \times 10^2 \text{ J}$$



$$W = - 3.2 \times 10^2 \text{ J}$$

Work-Kinetic Energy Theorem
(Page 239)

The special relationship between doing work on an object and the resulting kinetic energy of the object is given by the work-kinetic energy theorem.

Assumptions:

- all work done on a system gives the system only kinetic energy
- a constant force gives the system a constant acceleration
- directions of force and displacement are parallel
- the object moves in a straight line

$$W_{net} = F\Delta x = ma\Delta x$$

When you studied one-dimensional motion, you learned that the following relationship holds when an object undergoes constant acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x$$
$$a\Delta x = \frac{v_f^2 - v_i^2}{2}$$

Substituting this result into the equation $W_{net} = ma\Delta x$ gives

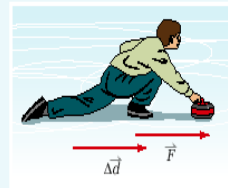
$$W_{net} = m\left(\frac{v_f^2 - v_i^2}{2}\right)$$
$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The equation describes how doing work on an object can change the object's kinetic energy.

MODEL PROBLEMS

Applying the Work-Kinetic Energy Theorem

1. A physics student does work on a 2.5 kg ~~curling~~ stone by exerting 4.0×10^1 N of force horizontally over a distance of 1.5 m.
- (a) Calculate the work done by the student on the ~~curling~~ stone.
- (b) Assuming that the stone started from rest, calculate the velocity of the stone at the point of release. (Consider the ice surface to be effectively frictionless.)



- a) 60 J
b) +6.9 m/s

$$a) W = Fd$$

$$W = (40)(1.5) = 60 \text{ J.}$$

$$b) W = \Delta E_K$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2$$

$$60 = \frac{1}{2} (2.5) (v_f^2)$$

$$\frac{120}{2.5} = v_f^2$$

$$48 = v_f^2$$

$$6.9 \text{ m/s} = v_f$$

2. A 75 kg skateboarder (including the board), initially moving at 8.0 m/s, exerts an average force of 2.0×10^2 N by pushing on the ground, over a distance of 5.0 m. Find the new kinetic energy of the skateboarder if the trip is completely horizontal.

$$W = Fd$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 = Fd$$

$$E_{Kf} = Fd + \underbrace{\frac{1}{2} m v_0^2}_{E_{K0} \text{ Initial Kinetic Energy}}$$

$$E_{Kf} = (200)(5) + \frac{1}{2} (75)(8)^2$$

$$E_{Kf} = 3400 \text{ J}$$

Work-Gravitational Potential Energy Theorem

$$W = \Delta E_g$$

$$W = E_{gf} - E_{gi}$$

$$W = mgh_f - mgh_i$$

$$Fd = mgh_f - mgh_i$$