

## James M. Hill

## Physics Handbook



Kinematics - Graphical Analysis

$>$ Slope at any point is the instantaneous velocity.
$>$ Sign of the slope indicates the direction the object is travelling.
$>$ Distance is the sum of the displacements in both directions. > Average velocity is the object's displacement divided by the time.
> Average speed is the object's distance divided by the time.
> Slope at any point is the instantaneous acceleration.
$>$ Sign of the slope indicates the direction of the acceleration not the object. $>$ Distance travelled during a time interval is the sum of the areas contained between the graph and the time-axis. $>$ Displacement is the total area of the top minus the total area of the bottom. > Average velocity is the object's displacement divided by the time. $>$ Average speed is the object's distance divided by the time.
**In the equations that follow all variables that are vectors could have a horizontal and vertical component. You have to remember to analyze each dimension independently if the problem warrants.**

## Mathematical Addition of Vectors


! Look very carefully at the trigonometry when calculating vector components.

## Kinematics - Mathematical Analysis \& Projectile Motion

| Symbol | Quantity (Unit) |  | Symbol | Quantity (Unit) |  | Symbol |  | Quantity (Unit) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| anything $_{f}$ | Final value |  | \|anything| | Magnitude |  | $d$ |  | Distance (m) |  |
| anythingo | Initial Value |  | $\vec{d}$ | Displacement (m) |  | $v_{s p}$ |  | Average Speed (m/s) |  |
| anything ${ }_{x}$ | Horizontal component |  | $\vec{v}_{\text {avg }}$ | Average velocity ( $\mathrm{m} / \mathrm{s}$ ) |  | $t$ |  | time (s; refers to a time interval) |  |
| anything $^{\text {a }}$ | Vertical component |  | $\stackrel{\rightharpoonup}{v}$ | Velocity ( $\mathrm{m} / \mathrm{s}$ ) |  | $\theta$ |  | Angle made with horizontal (degrees, ${ }^{\circ}$ ) |  |
| anything | Eastern component |  | $\vec{a}$ | Acceleration ( $\mathrm{m} / \mathrm{s}^{2}$ ) |  | $\Delta$ |  | Change in (final - initial) |  |
| anything $_{N}$ | Northern component |  | $\vec{g}$ | $9.81\left(\mathrm{~m} / \mathrm{s}^{2}\right.$; surface of the Earth) |  |  |  |  |  |
| $\vec{v}_{\text {avg }}=\frac{\vec{d}_{f}-\vec{d}_{o}}{t}$ | $v_{s p}=\frac{d}{t}$ |  | ${ }^{\text {a }}=\frac{\vec{v}_{f}+\vec{v}_{o}}{2}$ | $\vec{a}=\frac{\vec{v}_{f}-\vec{v}_{o}}{t}$ | $\vec{d}_{f}=\vec{d}_{o}+\vec{v}_{o} t+\frac{1}{2} \vec{a} t^{2}$ |  | $\vec{v}_{f}{ }^{2}=\vec{v}_{o}{ }^{2}+2 \vec{a}\left(\vec{d}_{f}-\vec{d}_{o}\right)$ |  |  |
| $v_{o x}=\|\vec{v}\| \cos \theta$ | $v_{o y}=\|\vec{v}\| \sin \theta$ |  | $v_{f x}{ }^{2}+v_{f y}{ }^{2}$ | $\theta=\tan ^{-1}\left\|\frac{v_{y}}{v_{x}}\right\|$ |  |  |  |  |  |

Dynamics - Forces, Impulse, Torque, Momentum, \& Circular Motion


## Work, Energy, \& Power



Current Electricity

| Symbol | Quantity (Unit) |  |  | Symbol |  | Symbol | Quantity (Unit) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I \quad$ Cur | Current (A; amperes) |  |  | $q$ | Charge (C; co | $t$ | Time (s) |
| $N$ Nu | Number of charges, resistors |  |  | $e$ | Elementary | $R$ | Resistance ( $\Omega$; Ohm) |
| $\rho$ $\operatorname{Re}$ | Resistivity ( $\Omega \cdot m$ ) |  |  | $L$ | Length (m) | $A$ | Cross-sectional area ( $\mathrm{m}^{2}$ ) |
| $V, \varepsilon$ Po | Potential Difference (V; volts) |  |  | $V_{T}$ | Voltage of p | $I_{T}$ | Current from power source (A) |
| $R_{\text {eqs }}$ Eq | Equivalent Resistance-series ( $\Omega$ ) |  |  | $R_{\text {eqp }}$ | Equivalent R | $P$ | Power (W) |
| $C_{\text {eqp }} \quad$Eq <br> ca | Equivalent capacitance for capacitors in parallel (F) |  |  | $C_{\text {eqs }}$ | Equivalent c capacitors in | $\tau$ | Discharge rate (s) |
| emf El | Electromotive force (V) |  |  | $r$ | Internal Resistance ( $\Omega$ ) | E | Electrical energy ( J ) |
| $I=\frac{\Delta q}{\Delta t}$ | $q=N e$ | $R=\rho \frac{L}{A}$ |  | $V=I R$ | $R_{\text {eqs }}=R_{1}$ | $\frac{1}{R_{\text {eqp }}}=$ | $\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}+\cdots+\frac{1}{R_{N}}$ |
| $R_{e q}=\frac{V_{T}}{I_{T}}$ | $P=I V$ | $\tau=R C$ |  | $=Q_{o} e^{\frac{-t}{\tau}}$ | $C_{\text {eqp }}=$ | $\frac{1}{C_{e q s}}=$ | $\frac{1}{C_{1}}+\frac{1}{C_{2}}+\frac{1}{C_{3}}+\cdots+\frac{1}{C_{N}}$ |
| $I(t)=I_{o} e^{\frac{-t}{\tau}}$ | $V_{\text {terminal }}=e m f-I r$ |  | Efficiency $=\frac{E_{\text {out }}}{E_{\text {in }}}=\frac{\text { Work }}{} E_{\text {in }}$ |  |  |  |  |

## Gravitational \& Electrical Fields

| Symbol | Quantity (Unit) |  | Symbol | Quantity (Unit) |  | Symbol | Quantity (Unit) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\vec{F}_{q} \quad$ Electr | Electrostatic Force (N) |  | $q$ | Electric Charge (C) |  | $k$ |  | $0^{9}\left(\frac{N \cdot m^{2}}{C^{2}}\right),$ | ulomb's |
| Dis | Distance from centres (m) |  | $\vec{E}$ | Electric field intensity $\left(\frac{N}{C}\right)$ |  | $\vec{g}$ | Gra | tional Fie | Intensity ( $\left.\frac{N}{\mathrm{~kg}}\right)$ |
| $\vec{F}_{g} \quad$ For | Force of gravity ( N ) |  | $E_{g}$ | Gravitational potential energy (J) |  | $E_{q}$ | Ele | poten | nergy (J) |
| $V \quad$ Ele | Electric potential difference (V) |  | ) m | Mass of object (kg) |  | G | 6.67 grav | $\begin{aligned} & \times 10^{-11}\left(\frac{N}{k}\right. \\ & \text { ational cor } \end{aligned}$ | Universal ant |
| W Wor | Work (J) |  | $\varepsilon_{o}$ | $8.85 \times 10^{-12}(\mathrm{~F} / \mathrm{m})$ <br> Permittivity of free space. |  | C | Cap | ance (F | led farads) |
| $Q \quad$ Ch | Charge stored in a capacitor (C) |  | $\kappa$ | Dielectric constant (no unit) |  | $d$ | Plate separation (m) |  |  |
| $A \quad$ Pla | Plate area ( $\mathrm{m}^{2}$ ) |  | $E_{C}$ | Potential energy in a capacitor (J) |  |  |  |  |  |
| $\stackrel{\rightharpoonup}{F}_{g}=G \frac{m_{1} m_{2}}{r^{2}}$ | $\vec{g}=\frac{\vec{F}_{g}}{m}$ | $\|\vec{g}\|=G \frac{m}{r^{2}}$ | $G \frac{m}{r}=v^{2}$ | $\frac{G m}{4 \pi^{2}}=\frac{r^{3}}{T^{2}}$ | $E_{g}=-G \frac{m_{1} m_{2}}{r}$ | $\stackrel{\rightharpoonup}{F}_{q}=k \frac{q_{1} q_{2}}{r^{2}}$ |  | $\vec{E}=\frac{\vec{F}_{q}}{q}$ | $\|\vec{E}\|=k \frac{q}{r^{2}}$ |
| $V=\frac{E_{q}}{q}$ | $V=k \frac{q}{r}$ | $\Delta E_{q}=q \Delta V$ | $Q=C V$ | $C=\frac{\kappa \varepsilon_{o} A}{d}$ | $E_{C}=\frac{1}{2} C V^{2}$ | $\stackrel{\rightharpoonup}{E}=\frac{V}{d}$ | $E_{q}=k \frac{q_{1} q_{2}}{r}$ |  |  |

## General Waves \& Sound Waves

| Symbol | Quantity (Unit) |  |  | Symbol | Quantity (Unit) |  | Symbol | Quantity (Unit) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $T$ | Period (s) |  |  | $f$ | Frequency ( Hz ) |  | $v$ | Wave speed ( $\mathrm{m} / \mathrm{s}$ ) |  |  |
| $\lambda$ | Wavelength (m) |  |  | $T_{\text {air }}$ | Temp. of air ( ${ }^{\circ} \mathrm{Celsius}$ ) |  | $V_{\text {sound }}$ | Sound speed ( $\mathrm{m} / \mathrm{s}$ ) |  |  |
| Vsrc | Source speed (m/s) |  |  | Vobs | Observer's speed (m/s) |  | $f_{\text {obs }}$ | Observed frequency ( Hz ) |  |  |
| $f_{s r c}$ | Source frequency ( Hz ) |  |  | $F_{T}$ | Force of Tension (N) |  | $\mu$ | Mass/unit length (kg/m) |  |  |
| $T=\frac{\Delta t}{\# w a}$ |  | $f=\frac{\# \text { waves }}{\Delta t}$ | $T=\frac{1}{f}$ | $f=\frac{1}{T}$ | $v=f \lambda$ | $v=\sqrt{\frac{F_{T}}{\mu}}$ | $v_{\text {sound }}=3$ | +0.59T ${ }_{\text {air }}$ | $f_{\text {obs }}=f_{s r}$ | $\left(\frac{v_{\text {sound }} \pm v_{\text {obs }}}{v_{\text {sound }} \overline{\overline{1}} v_{\text {src }}}\right)$ |

## Refraction



Table 9.2 Index of Refraction of Various Substances*

| Substance | Index of Refraction $(n)$ |
| :--- | :---: |
| vacuum | 1.00000 |
| gases at $0^{\circ} \mathrm{C}, 1.013 \times 10^{5} \mathrm{~Pa}$ |  |
| hydrogen | 1.00014 |
| oxygen | 1.00027 |
| air | 1.00029 |
| carbon dioxide | 1.00045 |
| water | 1.333 |
| ethyl alcohol | 1.362 |
| glycerin | 1.470 |
| carbon disulfide | 1.632 |

* Measured using yellow light, with a wavelength of 589 nm in a vacuum.

| Substance |  |
| :--- | :--- |
| solids at $20^{\circ} \mathrm{C}$ |  |
| ice (at $0^{\circ} \mathrm{C}$ ) | 1.31 |
| quartz (fused) | 1.46 |
| optical fibre (cladding) | 1.47 |
| optical fibre (core) | 1.50 |
| Plexiglas $^{\mathrm{TM}}$ or Lucite ${ }^{\mathrm{TM}}$ | 1.51 |
| glass (crown) | 1.52 |
| sodium chloride | 1.54 |
| glass (crystal) | 1.54 |
| ruby | 1.54 |
| glass (flint) | 1.65 |
| zircon | 1.92 |
| diamond | 2.42 |

Optics (Spherical Mirrors \& Lenses)

| Symbol | Quantity (Unit) | Symbol | Quantity (Unit) | Symbol | Quantity (Unit) |
| :---: | :--- | :---: | :--- | :--- | :--- |
| $f$ | Focal length ( $\mathrm{m}, \mathrm{cm}, \mathrm{mm}$ ) | $d_{o}$ | Object distance $(\mathrm{m}, \mathrm{cm}, \mathrm{mm})$ | $d_{i}$ | Image distance ( $\mathrm{m}, \mathrm{cm}, \mathrm{mm}$ ) |
| $h_{i}$ | Image height $(\mathrm{m}, \mathrm{cm}, \mathrm{mm})$ | $h_{o}$ | Object height $(\mathrm{m}, \mathrm{cm}, \mathrm{mm})$ | $M$ | Magnification |
| $n_{\text {lens }}$ | Lens refractive index | $n_{o}$ | Surrounding medium refractive index | $R$ | Radius of curvature ( $\mathrm{m}, \mathrm{cm}, \mathrm{mm}$ ) |
| $\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f}$ | $M=\frac{h_{i}}{h_{o}}$ | $M=-\frac{d_{i}}{d_{o}}$ | $\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}}$ | $\frac{1}{f}=\left(\frac{n_{\text {lens }}}{n_{o}}-1\right)\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)$ |  |
|  |  |  |  |  |  |

Types of Lenses


| Symbol | Quantity (Unit) | Symbol | Quantity (Unit) | Symbol | Quantity (Unit) |
| :---: | :--- | :---: | :--- | :---: | :---: |
| $m$ | Maximum \# (integer) | $\lambda$ | wavelength $(m)$ | $d$ | separation of slits (m) |
| $W_{m}$ | Distance of maximum <br> from centre $(m)$ | $L$ | Distance from screen (m) | $\theta$ | angle (degrees) |
| $m \lambda=\frac{d W_{m}}{L}$ | $m \lambda=d \sin \theta$ | Above is for thin film interference. <br> $\mathrm{n}=$ refractive index of the film |  |  |  |


| Solar System Data |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Quantity | Sun | Mercury | Venus | Earth | Mars | Jupiter | Saturn | Uranus | Neptune |
| Distance from Sun | N/A | $5.8 \times 10^{10}$ | $1.1 \times 10^{11}$ | $1.5 \times 10^{11}$ | $2.3 \times 10^{11}$ | $7.8 \times 10^{11}$ | $1.4 \times 10^{12}$ | $2.9 \times 10^{12}$ | $4.5 \times 10^{12}$ |
| Radius | $7.0 \times 10^{8}$ | $2.5 \times 10^{6}$ | $6.1 \times 10^{6}$ | $6.4 \times 10^{6}$ | $3.4 \times 10^{6}$ | $7.1 \times 10^{7}$ | $6.0 \times 10^{7}$ | $2.6 \times 10^{7}$ | $2.5 \times 10^{7}$ |
| Mass | $2.0 \times 10^{30}$ | $3.3 \times 10^{23}$ | $4.9 \times 10^{24}$ | $6.0 \times 10^{24}$ | $6.4 \times 10^{23}$ | $2.0 \times 10^{27}$ | $5.7 \times 10^{26}$ | $8.7 \times 10^{25}$ | $1.0 \times 10^{26}$ |
| Revolution | N/A | 88d | 225d | 365d | 1.88 y | 11.9 y | 29 y | $84 y$ | $164 y$ |
| Rotation | Varies | 58d | 243d | 24h | 24h | 10h | 11h | 17h | 16h |

Table 4.4 Free-Fall Accelerations Due to Gravity in the Solar System

| Location | Acceleration due to gravity $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: |
| Earth | 9.81 |
| Moon | 1.64 |
| Mars | 3.72 |
| Jupiter | 25.9 |

Table 4.3 Free-Fall Accelerations Due to Gravity on Earth

| Location | Acceleration <br> due to gravity <br> $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | Altitude <br> $(\mathrm{m})$ | Distance from <br> Earth's centre <br> $(\mathrm{km})$ |
| :--- | :---: | :---: | :---: |
| North Pole | 9.8322 | 0 <br> (sea level) | 6357 |
| equator | 9.7805 | 0 <br> (sea level) | 6378 |
| Mt. Everest (peak) | 9.7647 | 8850 | 6387 |
| Mariana Ocean <br> Trench* (bottom) | 9.8331 | 11034 <br> (below <br> sea level) | 6367 |
| International <br> Space Station* | 9.0795 | 250000 | 6628 |
| T |  |  |  |

[^0]
## Definition of the Quadratic Formula

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form $a x^{2}+b x+c=0$, where $a, b$, and $c$ are real numbers and $a \neq 0$, the roots of it can be found using

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Statistical Analysis

In science, data are collected until a trend is observed. Three statistical tools that assist in determining if a trend is developing are mean, median, and mode.

## Trigonometric Ratios

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine ( $\sin$ ), cosine ( $\cos$ ), and tangent (tan).


$$
\begin{aligned}
\sin \theta & =\frac{\text { opposite }}{\text { hypotenuse }} \\
\sin \theta & =\frac{a}{c} \\
\cos \theta & =\frac{\text { adjacent }}{\text { hypotenuse }} \\
\cos \theta & =\frac{b}{c} \\
\tan \theta & =\frac{\text { opposite }}{\text { adjacent }} \\
\tan \theta & =\frac{a}{b}
\end{aligned}
$$

The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the $90^{\circ}$ angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the opposite side. The side that you could touch and is not the hypotenuse is the adjacent side.

Mean: The sum of the numbers divided by the number of values. It is also called the "average."
Median: When a set of numbers is organized in order of size, the median is the middle number. When the data set contains an even number of values, the median is the average of the two middle numbers.
Mode: The number that occurs most often in a set of numbers. Some data sets will have more than one mode.

See examples of these on the following page.


A scientific calculator or trigonometry tables can be used to obtain an angle value from the ratio result. Your calculator performs a complex calculation (Maclaurin series summation) when the $\sin ^{-1}$, or $\cos ^{-1}$, or $\tan ^{-1}$ operation is used to determine the angle value. $\operatorname{Sin}^{-1}$ is not simply a $1 /$ sin operation.

## Definition of the Pythagorean Theorem

The Pythagorean theorem is used to determine side lengths of a right-angle ( $90^{\circ}$ ) triangle. Given a right-angle triangle ABC, the Pythagorean theorem states

$$
c^{2}=a^{2}+b^{2}
$$

| $\quad$ Quantity | Symbol | SI unit |
| :--- | :---: | :---: |
| hypotenuse side is <br> opposite the $90^{\circ}$ angle | $c$ | m (metres) |
| side $a$ | $a$ | m (metres) |
| side $b$ | $b$ | m (metres) |

Note: The hypotenuse is always the side across from the right $\left(90^{\circ}\right)$ angle. The Pythagorean theorem is a special case of a more general mathematical law called the "cosine law." The cosine law works for all triangles.

## Definition of the Cosine Law

The cosine law is useful when

- determining the length of an unknown side given two side lengths and the contained angle between them
- determining an unknown angle given all side lengths


Angle $\theta$ is contained between sides $a$ and $b$.

The cosine law states $\boldsymbol{c}^{2}=\boldsymbol{a}^{2}+\boldsymbol{b}^{2}-2 \boldsymbol{a} \boldsymbol{b} \cos \theta$.

| Quantity | Symbol | SI unit |
| :--- | :---: | ---: |
| unknown length side $c$ |  |  |
| opposite angle $\theta$ | $c$ | m (metres) |
| length side $a$ | $a$ | m (metres) |
| length side $b$ | $b$ | m (metres) |
| angle $\theta$ opposite unknown side $c$ | $\theta$ | (radians) |

Note: Applying the cosine law to a right angle triangle, setting $\theta=90^{\circ}$, yields the special case of the Pythagorean theorem.

## Definition of the Sine Law

The sine law is useful when

- two angles and any one side length are known
- two side lengths and any one angle are known


Given any triangle ABC the sine law states

$$
\frac{\sin \mathrm{A}}{a}=\frac{\sin \mathrm{B}}{b}=\frac{\sin \mathrm{C}}{c}
$$

## Quantity

length side a opposite angle A
length side $b$ opposite angle $B$ length side $c$ opposite angle C angle A opposite side a angle $B$ opposite side $b$ angle $C$ opposite side $c$
Symbol
$a$
$b$
$c$
A
B
C

## SI unit

 m (metres) m (metres) m (metres) (radians) (radians) (radians)Note: The sine law generates ambiguous results in some situations because it does not discriminate between obtuse and acute triangles. An example of the ambiguous case is shown below.

## Example

Use the sine law to solve for $\theta$.


Clearly, angle $\theta$ is much greater than $30^{\circ}$. In this case, the supplementary angle is required $\left(180^{\circ}-30^{\circ}=150^{\circ}\right)$. It is important to recognize when dealing with obtuse angles ( $>90^{\circ}$ ) that the supplementary angle might be required.
Application of the cosine law in these situations will help reduce the potential for error.

## Algebra

In some situations, it might be preferable to use algebraic manipulation of equations to solve for a specific variable before substituting numbers. Algebraic manipulation of variables follows the same rules that are used to solve equations after substituting values. In both cases, to maintain equality, whatever is done to one side must be done to the other.

## TRIGONOMETRIC IDENTITIES

- Reciprocal identities

$$
\begin{array}{ll}
\sin u=\frac{1}{\csc u} & \cos u=\frac{1}{\sec u} \\
\tan u=\frac{1}{\cot u} & \cot u=\frac{1}{\tan u} \\
\csc u=\frac{1}{\sin u} & \sec u=\frac{1}{\cos u}
\end{array}
$$

- Pythagorean Identities

$$
\begin{aligned}
& \sin ^{2} u+\cos ^{2} u=1 \\
& 1+\tan ^{2} u=\sec ^{2} u \\
& 1+\cot ^{2} u=\csc ^{2} u
\end{aligned}
$$

- Quotient Identities

$$
\tan u=\frac{\sin u}{\cos u} \quad \cot u=\frac{\cos u}{\sin u}
$$

## - Co-Function Identities

$$
\begin{array}{ll}
\sin \left(\frac{\pi}{2}-u\right)=\cos u & \cos \left(\frac{\pi}{2}-u\right)=\sin u \\
\tan \left(\frac{\pi}{2}-u\right)=\cot u & \cot \left(\frac{\pi}{2}-u\right)=\tan u \\
\csc \left(\frac{\pi}{2}-u\right)=\sec u & \sec \left(\frac{\pi}{2}-u\right)=\csc u
\end{array}
$$

- Parity Identities (Even \& Odd)

$$
\begin{array}{ll}
\sin (-u)=-\sin u & \cos (-u)=\cos u \\
\tan (-u)=-\tan u & \cot (-u)=-\cot u \\
\csc (-u)=-\csc u & \sec (-u)=\sec u
\end{array}
$$

- Sum \& Difference Formulas

$$
\begin{aligned}
& \sin (u \pm v)=\sin u \cos v \pm \cos u \sin v \\
& \cos (u \pm v)=\cos u \cos v \mp \sin u \sin v \\
& \tan (u \pm v)=\frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}
\end{aligned}
$$

## - Double Angle Formulas

$$
\begin{aligned}
\sin (2 u) & =2 \sin u \cos u \\
\cos (2 u) & =\cos ^{2} u-\sin ^{2} u \\
& =2 \cos ^{2} u-1 \\
& =1-2 \sin ^{2} u \\
\tan (2 u) & =\frac{2 \tan u}{1-\tan ^{2} u}
\end{aligned}
$$

- Power-Reducing/Half Angle Formulas

$$
\begin{aligned}
\sin ^{2} u & =\frac{1-\cos (2 u)}{2} \\
\cos ^{2} u & =\frac{1+\cos (2 u)}{2} \\
\tan ^{2} u & =\frac{1-\cos (2 u)}{1+\cos (2 u)}
\end{aligned}
$$

## - Sum-to-Product Formulas

$\sin u+\sin v=2 \sin \left(\frac{u+v}{2}\right) \cos \left(\frac{u-v}{2}\right)$
$\sin u-\sin v=2 \cos \left(\frac{u+v}{2}\right) \sin \left(\frac{u-v}{2}\right)$
$\cos u+\cos v=2 \cos \left(\frac{u+v}{2}\right) \cos \left(\frac{u-v}{2}\right)$
$\cos u-\cos v=-2 \sin \left(\frac{u+v}{2}\right) \sin \left(\frac{u-v}{2}\right)$

## - Product-to-Sum Formulas

$\sin u \sin v=\frac{1}{2}[\cos (u-v)-\cos (u+v)]$
$\cos u \cos v=\frac{1}{2}[\cos (u-v)+\cos (u+v)]$
$\sin u \cos v=\frac{1}{2}[\sin (u+v)+\sin (u-v)]$
$\cos u \sin v=\frac{1}{2}[\sin (u+v)-\sin (u-v)]$

## GEOMETRY SHAPESAND SOLIDS

## SQUARE

$$
\begin{aligned}
& P=4 s \\
& A=s^{2}
\end{aligned}
$$



## TRIANGLE

$P=a+b+c$
$A=\frac{1}{2} b h$


PYTHAGOREAN THEOREM


TRAPEZOID


## CUBE

$A=6 l^{2}$
$V=l^{3}$


RECTANGLE

## $P=2 a+2 b$ <br> $A=a b$ <br> 

## PARALLELOGRAM

## $P=2 a+2 b$ <br> $A=b h$ <br> 

## CIRCULAR RING

$$
A=\pi\left(R^{2}-r^{2}\right)
$$



## RECTANGULAR BOX

$A=2 a b+2 a c+2 b c$
$V=a b c$


## CYLINDER



## eCalc.com

The Best Online Calculator

- Unit Converter
- RPN and Algebraic Mode
- Constants Library
- Decimal to Fraction
- Polynomial Root Solver
- Simultaneous Equation Solver
- Complex Numbers
- Free Online and Downloadable


## CIRCLE

$$
\begin{aligned}
& P=2 \pi r \\
& A=\pi r^{2}
\end{aligned}
$$



## CIRCULAR SECTOR

$L=\pi r \frac{\theta}{180^{\circ}}$


## SPHERE

$S=4 \pi r^{2}$
$V=\frac{4 \pi r^{3}}{3}$


## RIGHT CIRCULAR CONE

$A=\pi r^{2}+\pi r s$
$s=\sqrt{r^{2}+h^{2}}$
$V=\frac{1}{3} \pi r^{2} h$


## FRUSTUM OF A CONE

$V=\frac{1}{3} \pi h\left(r^{2}+r R+R^{2}\right)$


## Appendix A

## The Metric System: Fundamental and Derived Units

## Metric System Prefixes

| Prefix | Symbol | Factor |
| :---: | :---: | :---: |
| tera | T | $1000000000000=10^{12}$ |
| giga | G | $1000000000=10^{9}$ |
| mega | M | $1000000=10^{6}$ |
| kilo | k | $1000=10^{3}$ |
| hecto | h | $100=10^{2}$ |
| deca | da | $10=10^{1}$ |
|  |  | $1=10^{0}$ |
| deci | d | $0.1=10^{-1}$ |
| centi | c | $0.01=10^{-2}$ |
| milli | m | $0.001=10^{-3}$ |
| micro | $\mu$ | $0.000001=10^{-6}$ |
| nano | n | $0.000000001=10^{-9}$ |
| pico | p | $0.000000000001=10^{-12}$ |
| femto | f | $0.000000000000001=10^{-15}$ |
| atto | a | $0.000000000000000001=10^{-18}$ |

## Fundamental Physical Quantities and Their SI Units

| Quantity | Symbol | Unit | Symbol |
| :--- | :---: | :--- | :---: |
| length | $l$ | metre | m |
| mass | $m$ | kilogram | kg |
| time | $t$ | second | s |
| absolute temperature | $T$ | Kelvin | K |
| electric current | $I$ | ampère | A |
| amount of substance | mol | (amp) |  |
| mole | mol |  |  |

## Derived SI Units

| Quantity | Quantity symbol | Unit | Unit symbol | Equivalent unit(s) |
| :---: | :---: | :---: | :---: | :---: |
| ```area volume velocity acceleration force work energy power density pressure frequency period wavelength electric charge electric potential resistance magnetic field intensity magnetic flux radioactivity radiation dose temperature (Celsius)``` | $A$ $V$ $V$ $a$ $F$ $W$ $E$ $P$ $\rho$ $p$ $f$ $T$ $\lambda$ $Q$ $V$ $R$ $B$ $\Phi$ $\Delta N / \Delta t$ $T$ | square metre <br> cubic metre <br> metre per second <br> metre per second <br> per second <br> newton <br> joule <br> joule <br> watt <br> kilogram per cubic metre <br> pascal <br> hertz <br> second <br> metre <br> coulomb <br> volt <br> ohm <br> tesla <br> weber <br> becquerel <br> gray <br> degree Celsius <br> atomic mass unit <br> electron volt | $\mathrm{m}^{2}$ $\mathrm{~m}^{3}$ $\mathrm{~m} / \mathrm{s}$ $\mathrm{m} / \mathrm{s}^{2}$ N J J W $\mathrm{~kg} / \mathrm{m}^{3}$ Pa Hz s m C V $\Omega$ T T Wb Bq Gy ${ }^{\circ} \mathrm{C}$ u eV | $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$ <br> $\mathrm{N} \cdot \mathrm{m}, \mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}$ <br> $\mathrm{N} \cdot \mathrm{m}, \mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}$ <br> $\mathrm{J} / \mathrm{s}, \mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{3}$ $\underset{\mathrm{s}^{-1}}{\mathrm{~N} / \mathrm{m}^{2}, \mathrm{~kg} /\left(\mathrm{m} \cdot \mathrm{~s}^{2}\right)}$ <br> A.s <br> W/A, J/C, <br> $\mathrm{kg} \cdot \mathrm{m}^{2} /\left(\mathrm{C} \cdot \mathrm{s}^{2}\right)$ <br> V/A, <br> $\mathrm{kg} \cdot \mathrm{m}^{2} /\left(\mathrm{C}^{2} \cdot \mathrm{~s}\right)$ <br> $\mathrm{N} \cdot \mathrm{s} /(\mathrm{C} \cdot \mathrm{m}), \mathrm{N} /(\mathrm{A} \cdot \mathrm{m})$ <br> $\mathrm{V} \cdot \mathrm{s}, \mathrm{T} \cdot \mathrm{m}^{2}, \mathrm{~m}^{2} \cdot \mathrm{~kg} /(\mathrm{C} \cdot \mathrm{s})$ $\mathrm{s}^{-1}$ <br> $\mathrm{J} / \mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}$ <br> $T^{\circ} \mathrm{C}=(\mathrm{T}+273.15) \mathrm{K}$ <br> $1 \mathrm{u}=1.660566 \times 10^{-27} \mathrm{~kg}$ <br> $1 \mathrm{eV}=1.602 \times 10^{-19} \mathrm{~J}$ |

## Appendix B

## Physical Constants and Data

## Fundamental Physical Constants

| Ouantity | Symbol | Accepted value |
| :--- | :---: | :--- |
| speed of light in a vacuum | $c$ | $2.998 \times 10^{8} \mathrm{~m} / \mathrm{s}$ |
| gravitational constant | $G$ | $6.673 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}$ |
| Coulomb's constant | $k$ | $8.988 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$ |
| charge on an electron | $e$ | $1.602 \times 10^{-19} \mathrm{C}$ |
| rest mass of an electron | $m_{\mathrm{e}}$ | $9.109 \times 10^{-31} \mathrm{~kg}$ |
| rest mass of a proton | $m_{\mathrm{p}}$ | $1.673 \times 10^{-27} \mathrm{~kg}$ |
| rest mass of a neutron | $m_{\mathrm{n}}$ | $1.675 \times 10^{-27} \mathrm{~kg}$ |
| Planck's constant | $h$ | $6.626 \times 10^{-34} \mathrm{~J} \cdot \mathrm{~s}$ |

Electric Circuit Symbols


## Other Physical Data

| Quantity | Symbol | Accepted value |
| :---: | :---: | :---: |
| standard atmospheric pressure <br> speed of sound in air <br> water: density ( $4^{\circ} \mathrm{C}$ ) <br> latent heat of fusion <br> latent heat of vaporization <br> specific heat capacity $\left(15^{\circ} \mathrm{C}\right)$ <br> kilowatt hour <br> acceleration due to Earth's gravity <br> mass of Earth <br> mean radius of Earth <br> mean radius of Earth's orbit <br> period of Earth's orbit <br> mass of Moon <br> mean radius of Moon <br> mean radius of Moon's orbit <br> period of Moon's orbit <br> mass of Sun <br> radius of Sun | P <br> E <br> g <br> $m_{E}$ <br> $r_{\mathrm{E}}$ <br> $R_{\mathrm{E}}$ <br> $T_{\mathrm{E}}$ <br> $m_{\mathrm{M}}$ <br> $r_{\mathrm{M}}$ <br> $R_{\mathrm{M}}$ <br> $T_{\mathrm{M}}$ <br> $m_{\text {s }}$ <br> $r_{\mathrm{s}}$ | $\begin{aligned} & 1.013 \times 10^{5} \mathrm{~Pa} \\ & 343 \mathrm{~m} / \mathrm{s}\left(\text { at } 20^{\circ} \mathrm{C}\right) \\ & 1.000 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3} \\ & 3.34 \times 10^{5} \mathrm{~J} / \mathrm{kg} \\ & 2.26 \times 10^{6} \mathrm{~J} / \mathrm{kg} \\ & 4186 \mathrm{~J} /\left(\mathrm{kg}^{\circ} \mathrm{C}\right) \\ & 3.6 \times 10^{6} \mathrm{~J} \\ & 9.81 \mathrm{~m} / \mathrm{s}^{2}(\text { standard value; at sea level }) \\ & 5.98 \times 10^{24} \mathrm{~kg} \\ & 6.38 \times 10^{6} \mathrm{~m} \\ & 1.49 \times 10^{11} \mathrm{~m} \\ & 365.25 \text { days or } 3.16 \times 10^{7} \mathrm{~s} \\ & 7.36 \times 10^{22} \mathrm{~kg} \\ & 1.74 \times 10^{6} \mathrm{~m} \\ & 3.84 \times 10^{8} \mathrm{~m} \\ & 27.3 \text { days or } 2.36 \times 10^{6} \mathrm{~s} \\ & 1.99 \times 10^{30} \mathrm{~kg} \\ & 6.96 \times 10^{8} \mathrm{~m} \end{aligned}$ |

## Resistor Colour Codes

| Colour | Digit <br> represented | Multiplier | Tolerance |
| :--- | :---: | :---: | :---: |
| $\square$ black | 0 | $\times 1$ |  |
| $\square$ brown | 1 | $\times 1.0 \times 10^{1}$ |  |
| red | 2 | $\times 1.0 \times 10^{2}$ |  |
| orange | 3 | $\times 1.0 \times 10^{3}$ |  |
| yellow | 4 | $\times 1.0 \times 10^{4}$ |  |
| green | 5 | $\times 1.0 \times 10^{5}$ |  |
| blue | 6 | $\times 1.0 \times 10^{6}$ |  |
| violet | 7 | $\times 1.0 \times 10^{7}$ |  |
| gray | 8 | $\times 1.0 \times 10^{8}$ |  |
| $\square$ white | 9 | $\times 1.0 \times 10^{9}$ |  |
| gold |  | $\times 1.0 \times 10^{-1}$ | $5 \%$ |
| silver |  | $\times 1.0 \times 10^{-2}$ | $10 \%$ |
| no colour |  |  | $20 \%$ |




[^0]:    *These values are calculated.

