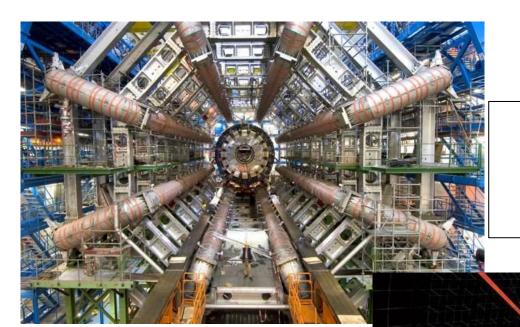




# James M. Hill Physics Handbook

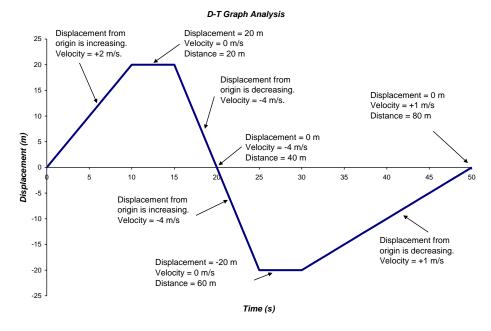


- Equations, Variables, and Units
- Physical Constants
- Solar System Data
- Metric System
- Last Edited: Fall 2014

Mr. P. MacDonald

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# Kinematics - Graphical Analysis

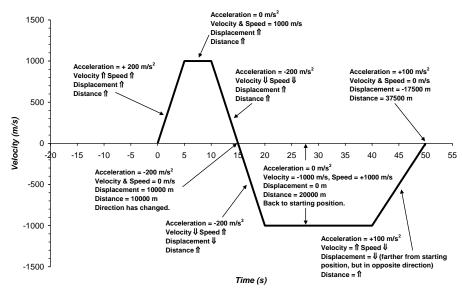


Slope at any point is the instantaneous velocity.

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- > Sign of the slope indicates the direction the object is travelling.
- Distance is the sum of the displacements in both directions.
- Average velocity is the object's displacement divided by the time.
- Average speed is the object's distance divided by the time.

#### V-T Graph Analysis



- > Slope at any point is the instantaneous acceleration.
- > Sign of the slope indicates the direction of the acceleration *not* the object.
- > Distance travelled during a time interval is the sum of the areas contained between the graph and the time-axis.
- > Displacement is the total area of the top minus the total area of the bottom.
- Average velocity is the object's displacement divided by the time.
- Average speed is the object's distance divided by the time.

\*\*In the equations that follow all variables that are vectors could have a horizontal and vertical component. You have to remember to analyze each dimension independently if the problem warrants.\*\*

#### Mathematical Addition of Vectors

Symbol	Quantity (Unit)	Symbol	Quantity (Unit)			
$\vec{R}$	Resultant	$\vec{A}$ , $\vec{B}$	Arbitrary Vectors			
$R_E$ or $R_x$	Horizontal component of resultant (E = Eastern)	$A_E$ or $A_x$	Horizontal component of arbitrary vectors (E = Eastern)			
$R_N$ or $R_y$	Vertical component of resultant (N = Northern)	$A_N$ or $A_y$	Vertical component of arbitrary vectors (N = Northern)			
$\theta$	Angle made with horizonta	d (°)				
$R_x = A_x + B_x$	$+ \cdots \qquad R_y = A_y + B_y + \cdots$	$ \vec{R}  = \sqrt{R_x^2}$	$\frac{1}{H_{y}^{2}} \qquad \theta = \tan^{-1} \left  \frac{R_{y}}{R_{x}} \right $			

! Look very carefully at the trigonometry when calculating vector components.

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# Kinematics - Mathematical Analysis & Projectile Motion

Symbol	Quantity (Unit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)
$anything_f$	Final value	anything	Magnitude	d	Distance (m)
$anything_o$	Initial Value	$\vec{d}$	Displacement (m)	$v_{sp}$	Average Speed (m/s)
$anything_x$	Horizontal component	$ec{v}_{avg}$	Average velocity (m/s)	t	time (s; refers to a time interval)
$anything_y$	Vertical component	$\vec{v}$	Velocity (m/s)	$\theta$	Angle made with horizontal (degrees, °)
$anything_E$	Eastern component	$\vec{a}$	Acceleration (m/s²)	Δ	Change in (final - initial)
$anything_N$	Northern component	$ec{g}$	9.81 (m/s²; surface of the Earth)		
$\vec{v}_{avg} = \frac{\vec{d}_f - \vec{d}_o}{t}$	$v_{sp} = \frac{d}{t}$	$\vec{v}_{avg} = \frac{\vec{v}_f + \vec{v}_o}{2}$	$\vec{a} = \frac{\vec{v}_f - \vec{v}_o}{t} \qquad \vec{d}_f = \vec{d}_o + \vec{v}_o t + $	$-\frac{1}{2}\vec{a}t^2$	$\ddot{v_f}^2 = \ddot{v_o}^2 + 2\vec{a}(\vec{d_f} - \vec{d_o})$
$v_{ox} =  \vec{v}  \cos \theta$	$v_{oy} =  \vec{v}  \sin \theta \qquad  \vec{v}  =$	$\sqrt{{v_{fx}}^2 + {v_{fy}}^2}$	$\theta = \tan^{-1} \left  \frac{v_y}{v_x} \right $		

## Dynamics - Forces, Impulse, Torque, Momentum, & Circular Motion

Symbol	Quantity (Unit) Symbol			ol	Quantity (Unit)			Quantity (Unit)			
$ec{F}_{net}$	Net for	ce (N)		$\vec{F}_T$	Force o	Force of Tension (N)			Impulse (N·s)		
$ec{F}_{A}$	Force applied (N) $\vec{F}_N$			Normal	Force (N)		$\vec{p}$	Momentum (kg·m/s)			
$ec{F}_g$	Force of	gravity (N)		$ec{F_c}$	Centrip	etal Force (N)		$ec{p}_{oT}$	Initial total momentums (kg	g·m/s	
$\vec{F}_f$	Force of	friction (N)		m	Mass (k	g)		$ec{p}_{fT}$	Final total momentums (kg-	m/s)	
$\vec{F}_{s}$	Restorin	ig Force (N)		μ	Coeffic	ient of friction	(no units)	f	Frequency (Hz)		
$\vec{a}_c$	Centripe	tal Acceleration (m/s	s²)	T	Period (	(s)		Г	Circular & Orbital Radius (r	m)	
τ	Torque (	$(N \cdot m)$		$F_{\perp}$	Perpend	Perpendicular Component of Force (N)			Circular Speed (m/s)		
k	Spring C	Constant (N/m)									
$\vec{F}_{net} = \sum$	$\vec{F}_{net} = \sum Forces$ $\sum \vec{F} = \sum m \times \vec{a}$ $\vec{F}_{net}$		$\vec{F}_{net}$	$= m\vec{a}$	$\vec{F}_g = m\vec{g}$	$\vec{F}_f = \mu \vec{F}_N$	$F_S = -kx$	$\vec{p} = m\vec{v}$	$ \vec{J} = \Delta \vec{p}  \vec{F}t = m\Delta \vec{v}                                   $		
$v = \frac{1}{2}$	$v = \frac{2\pi r}{T} \qquad a_c = \frac{v^2}{r} \qquad F_c =$		$=\frac{mv^2}{r}$	$f = \frac{1}{T}$	$v = \sqrt{rg\mu_s}$	$v = \sqrt{rg\tan\theta}$	$\tau = rF_{\perp}$	$ \vec{ au}_{net} = \sum Torques $	_		
$\vec{J} = Are$	$\vec{J} = \vec{F}t$ a under F	- t curve $T = 2\pi$	$\sqrt{\frac{m}{k}}$								

# Work, Energy, & Power

Symbol	Quai	ntity (Unit)		S	ymbol	Quantity (Unit)						
W	Work (J)				P	Power (W; Watts)						
$W_{nc}$	Work by non-conservative force (J)				h	Hei	ght above ret	ference (m)				
$E_k$	$E_k$ Kinetic energy (J)				d	Dist	tance (m)					
$E_g$	Gravitational potential energy (J)				r	Radius of circle (m)						
$E_e$	Elastic potential	energy (J)			X	Stretch/compression of spring (m)						
$E_T$	Total mechanica	l energy (J)							_			
$W = F_{\parallel} \Delta d$ $W = \text{Area under F} - \text{d curve}$ $E_g = mgh$ $E_k = \frac{1}{2}$		$v^2$	$E_e = \frac{1}{2}kx$	x <sup>2</sup>	$W = \Delta E_k$	$W = \Delta E_g$	$W = \Delta E_e$	$\Delta E_T = 0$	$W_{nc} = \Delta E_T$	$P = \frac{W}{t}$		

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# Current Electricity

Symbol	Quantity	y (Unit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)
I	Current (A; ampe	res)	q	Charge (C; coulombs)	t	Time (s)
N	Number of charge	ımber of charges, resistors		Elementary Charge (C)	R	Resistance ( $\Omega$ ; Ohm)
ρ	Resistivity $(\Omega \cdot m)$	)	L	Length (m)	A	Cross-sectional area (m²)
<i>V, ε</i>	Potential Differe	nce (V; volts)	$V_T$	Voltage of power source (V)	$I_T$	Current from power source (A)
$R_{eqs}$	Equivalent Resista	ance-series (Ω	) R <sub>eqp</sub>	Equivalent Resistance-parallel ( $\Omega$ )	Р	Power (W)
$C_{eqp}$	Equivalent capacit		Ceqs	Equivalent capacitance for capacitors in series (F)	τ	Discharge rate (s)
emf	Electromotive for	rce (V)	r	Internal Resistance ( $\Omega$ )	Е	Electrical energy (J)
$I = \frac{\Delta q}{\Delta t}$	q = Ne	$R = \rho \frac{L}{A}$	V = IR	$R_{eqs} = R_1 + R_2 + R_3 + \dots + R_N$	$\frac{1}{R_{eqp}} =$	$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$
$R_{eq} = \frac{V}{I}$	P = IV	$\tau = RC$	$Q(t) = Q_o e^{\frac{-t}{\tau}}$	$C_{eqp} = C_1 + C_2 + C_3 + \dots + C_N$	$\frac{1}{C_{eqs}} =$	$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$
$I(t) = I_0 \epsilon$	$v_{terminal} = e^{\frac{-t}{\tau}}$	emf — Ir E	$Efficiency = \frac{E}{f}$	$\frac{E_{out}}{E_{in}} = \frac{Work}{E_{in}}$		

#### Gravitational & Electrical Fields

Symbol	Qua	ntity (Unit)	Symbol	Quanti	ty (Unit)	Symbol	Quanti	ty (Unit)
$ec{F_q}$	Electrostatio	Force (N)	q	Electric Chai	rge (C)	k	$9.0 \times 10^9 \left(\frac{N \cdot m^2}{c^2}\right)$ , constant	Coulomb's
r	Distance from	n centres (m)	$ec{E}$	Electric field	d intensity $\left(\frac{N}{c}\right)$	$ec{g}$	Gravitational Fie	Id Intensity $\left(\frac{N}{kg}\right)$
$ec{F_g}$	Force of grav	rity (N)	$E_g$	Gravitational energy (J)	l potential	$E_q$	Electric potential energy (J)	
V	Electric pote	ntial difference ('	V) m	Mass of object (kg)		G	$6.672 \times 10^{-11} \left( \frac{N \cdot m^2}{kg^2} \right)$ , Universal gravitational constant	
W	Work (J)	ork (J) $\varepsilon_o$			8.85 x 10 <sup>-12</sup> (F/m) Permittivity of free space.		Capacitance (F, c	called farads)
Q	Charge store	d in a capacitor (C	<b>к</b>	Dielectric co unit)	nstant (no	d	Plate separation	(m)
A	Plate area (m²)		$E_C$	Potential ene	31			
$\vec{F}_g = G \frac{m}{m}$	$\left  \frac{\vec{r}_1 m_2}{r^2} \right   \vec{g} = \frac{\vec{F}_g}{m}$	$ \vec{g}  = G \frac{m}{r^2}$	$G\frac{m}{r} = v^2$	$\frac{Gm}{4\pi^2} = \frac{r^3}{T^2}$	$E_g = -G \frac{\overline{m_1 m_2}}{r}$	$ \vec{F}_q  = 1$	$k\frac{q_1q_2}{r^2} \qquad \vec{E} = \frac{\vec{F}_q}{q}$	$\left \vec{E}\right  = k \frac{q}{r^2}$
$V = \frac{E}{c_i}$	$V = k \frac{q}{q}$	$\frac{d}{dx} \Delta E_q = q \Delta V$	Q = CV	$C = \frac{\kappa \varepsilon_o A}{d}$	$E_C = \frac{1}{2}CV^2$	$\vec{E} = \frac{V}{d}$	$E_q = k \frac{q_1 q_2}{r}$	

#### General Waves & Sound Waves

Symbol	Quantity (Unit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)	
T	Period (s)	f	Frequency (Hz)	ν	Wave speed (m/s)	
λ	Wavelength (m)	$T_{air}$	Temp. of air (° Celsius)	Vsound	Sound speed (m/s)	
V <sub>src</sub>	Source speed (m/s)	Vobs	Observer's speed (m/s)	$f_{obs}$	Observed frequency (Hz)	
$f_{src}$	Source frequency (H	z) $F_T$	Force of Tension (N)	μ	Mass/unit length (kg/m)	
$T = \frac{\Delta t}{\#wa}$	7 =	$T = \frac{1}{f} \qquad f = \frac{1}{T}$	$v = f\lambda  v = \sqrt{\frac{F_T}{\mu}}$	$v_{sound} = 33$	$f_{obs} = f_{src} \left( \frac{v_{so}}{v_{so}} \right)$	$\frac{v_{ound} \pm v_{obs}}{v_{ound} \mp v_{src}}$

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#### Refraction

Symbol	Quantity (Unit) Sy		Sym	bol	Quantity (Unit)		Symbol	Quantity (Unit)
С	3.00 x 108 (m/s), speed of light in a vacuum		V		Speed of light in material (m/s)		n	Index of refraction
ni	Incident medium inde	ex n <sub>R</sub>		?	Refractive medium index		$\theta_i$	Incident angle (degrees)
$\theta_{\!\scriptscriptstyle R}$	Refractive angle (degrees)		$\theta_c$		Critical Angle (degrees)		$ heta_{max}$	Max angle refraction (degrees)
$n = \frac{c}{v}$	$n_i \sin \theta_i = n_R \sin \theta_R$	$n_i \sin  heta_i$	$c = n_R$	$n_i =$	$n_R \sin  heta_{max}$			

Table 9.2 Index of Refraction of Various Substances\*

Substance	Index of Refraction ( <i>n</i> )								
vacuum	1.00000								
gases at 0°C, 1.013 × 10 <sup>5</sup> Pa									
hydrogen	1.00014								
oxygen	1.00027								
air	1.00029								
carbon dioxide	1.00045								
liquid	s at 20°C								
water	1.333								
ethyl alcohol	1.362								
glycerin	1.470								
carbon disulfide	1.632								

<sup>\*</sup> Measured using yellow light, with a wavelength of 589 nm in a vacuum.

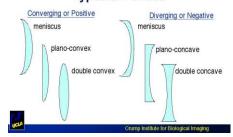
Substance	Index of Refraction (n)
solids	at 20°C
ice (at 0°C)	1.31
quartz (fused)	1.46
optical fibre (cladding)	1.47
optical fibre (core)	1.50
Plexiglas™ or Lucite™	1.51
glass (crown)	1.52
sodium chloride	1.54
glass (crystal)	1.54
ruby	1.54
glass (flint)	1.65
zircon	1.92
diamond	2.42

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#### Optics (Spherical Mirrors & Lenses)

Symbol	Quantity (l	Jnit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)
f	Focal length (m, cm, mm) $d_o$		$d_o$	Object distance (m, cm, mm)	$d_i$	Image distance (m, cm, mm)
hi	$h_i$ Image height (m, cm, mm) $h_o$		$h_o$	Object height (m, cm, mm)	М	Magnification
n <sub>lens</sub>	n <sub>lens</sub> Lens refractive index		$n_o$	Surrounding medium refractive index	R	Radius of curvature (m, cm, mm)
$\frac{1}{d_o} + \frac{1}{d_i} =$	$=\frac{1}{f}$ $M=\frac{h_i}{h_o}$	$M = -\frac{d}{d}$	$\frac{l_i}{h_o} = \frac{h_i}{h_o} =$	$-\frac{d_i}{d_o} \qquad \frac{1}{f} = \left(\frac{n_{lens}}{n_o} - 1\right) \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$		

#### Types of Lenses



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# Diffraction

Symbol	Quantity (Unit)	Symbol	Quantity (Unit)	Symbol	Quantity (Unit)
m	Maximum # (integer)	λ	wavelength (m)	d	separation of slits (m)
W <sub>m</sub>	Distance of maximum from centre (m)	L	Distance from screen (m)	θ	angle (degrees)

1147		$m\lambda = 2nd$
$m\lambda = \frac{dW_m}{I}$	$m\lambda = d\sin\theta$	Above is for thin film interference.
L		n = refractive index of the film

Solar System Data									
Quantity	Sun	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from Sun	N/A	$5.8 \times 10^{10}$	$1.1 \times 10^{11}$	$1.5 \times 10^{11}$	$2.3 \times 10^{11}$	$7.8 \times 10^{11}$	$1.4 \times 10^{12}$	$2.9 \times 10^{12}$	$4.5 \times 10^{12}$
Radius	$7.0 \times 10^{8}$	$2.5 \times 10^{6}$	$6.1 \times 10^{6}$	$6.4 \times 10^{6}$	$3.4 \times 10^{6}$	$7.1 \times 10^{7}$	$6.0 \times 10^{7}$	$2.6 \times 10^{7}$	$2.5 \times 10^{7}$
Mass	$2.0 \times 10^{30}$	$3.3 \times 10^{23}$	$4.9 \times 10^{24}$	$6.0 \times 10^{24}$	$6.4 \times 10^{23}$	$2.0 \times 10^{27}$	$5.7 \times 10^{26}$	$8.7 \times 10^{25}$	$1.0 \times 10^{26}$
Revolution	N/A	88d	225d	365d	1.88y	11.9y	29y	84y	164y
Rotation	Varies	58d	243d	24h	24h	10h	11h	17h	16h

Table 4.4 Free-Fall Accelerations Due to Gravity in the Solar System

Location	Acceleration due to gravity (m/s²)
Earth	9.81
Moon	1.64
Mars	3.72
Jupiter	25.9

Table 4.3 Free-Fall Accelerations Due to Gravity on Earth

Location	Acceleration due to gravity (m/s²)	Altitude (m)	Distance from Earth's centre (km)
North Pole	9.8322	0 (sea level)	6357
equator	9.7805	0 (sea level)	6378
Mt. Everest (peak)	9.7647	8850	6387
Mariana Ocean Trench* (bottom)	9.8331	11 034 (below sea level)	6367
International Space Station*	9.0795	250 000	6628

<sup>\*</sup>These values are calculated.

Mean:

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The sum of the numbers divided by the number of values. It is also called the

"average."

**Median:** When a set of numbers is organized in

order of size, the median is the middle number. When the data set contains an even number of values, the median is the average of the two middle numbers.

Mode: The number that occurs most often in a set of numbers. Some data sets will have

more than one mode.

See examples of these on the following page.

#### **Definition of the Quadratic Formula**

The quadratic equation is used to solve for the roots of a quadratic function. Given a quadratic equation in the form  $ax^2 + bx + c = 0$ , where a, b, and c are real numbers and  $a \neq 0$ , the roots of it can be found using

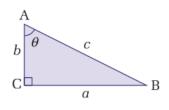
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# **Statistical Analysis**

In science, data are collected until a trend is observed. Three statistical tools that assist in determining if a trend is developing are *mean*, *median*, and *mode*.

# **Trigonometric Ratios**

The ratios of side lengths from a right-angle triangle can be used to define the basic trigonometric function sine (sin), cosine (cos), and tangent (tan).



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

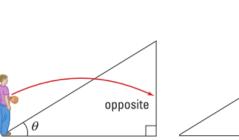
$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{b}{c}$$
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

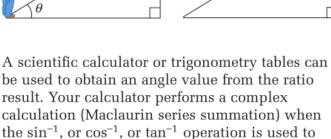
 $\tan \theta = \frac{a}{b}$ 

The angle selected determines which side will be called the opposite side and which the adjacent side. The hypotenuse is always the side across from the 90° angle. Picture yourself standing on top of the angle you select. The side that is directly across from your position is called the *opposite* side. The side that you could touch and is not the hypotenuse is the *adjacent* side.



determine the angle value. Sin-1 is not simply a

1/sin operation.



adiacent

#### **Definition of the Pythagorean Theorem**

The Pythagorean theorem is used to determine side lengths of a right-angle (90°) triangle. Given a right-angle triangle ABC, the Pythagorean theorem states

$$c^2 = a^2 + b^2$$

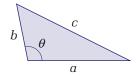
Quantity	Symbol	SI unit
hypotenuse side is		
opposite the 90° angle	С	m (metres)
side a	а	m (metres)
side b	b	m (metres)

**Note:** The hypotenuse is always the side across from the right (90°) angle. The Pythagorean theorem is a special case of a more general mathematical law called the "cosine law." The cosine law works for all triangles.

#### **Definition of the Cosine Law**

The cosine law is useful when

- determining the length of an unknown side given two side lengths and the contained angle between them
- determining an unknown angle given all side lengths



Angle  $\theta$  is contained between sides a and b.

The cosine law states  $c^2 = a^2 + b^2 - 2ab \cos\theta$ .

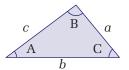
Quantity	Symbol	SI unit
unknown length side $\emph{c}$		
opposite angle $ heta$	С	m (metres)
length side a	а	m (metres)
length side b	b	m (metres)
angle $ heta$ opposite unknown side $c$	$\theta$	(radians)

**Note:** Applying the cosine law to a right angle triangle, setting  $\theta = 90^{\circ}$ , yields the special case of the Pythagorean theorem.

#### **Definition of the Sine Law**

The sine law is useful when

- two angles and any one side length are
- two side lengths and any one angle are known



Given any triangle ABC the sine law states

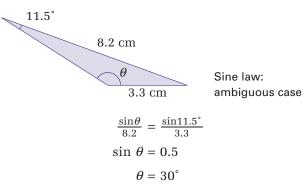
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Quantity	Symbol	SI unit
length side a opposite angle A	а	m (metres)
length side b opposite angle B	b	m (metres)
length side $\emph{c}$ opposite angle $\emph{C}$	С	m (metres)
angle A opposite side a	Α	(radians)
angle B opposite side b	В	(radians)
angle ${\sf C}$ opposite side $c$	С	(radians)

Note: The sine law generates ambiguous results in some situations because it does not discriminate between obtuse and acute triangles. An example of the ambiguous case is shown below.

#### Example

Use the sine law to solve for  $\theta$ .



Clearly, angle  $\theta$  is much greater than 30°. In this case, the supplementary angle is required  $(180^{\circ} - 30^{\circ} = 150^{\circ})$ . It is important to recognize when dealing with obtuse angles (> 90°) that the supplementary angle might be required. Application of the cosine law in these situations will help reduce the potential for error.

#### Algebra

In some situations, it might be preferable to use algebraic manipulation of equations to solve for a specific variable before substituting numbers. Algebraic manipulation of variables follows the same rules that are used to solve equations after substituting values. In both cases, to maintain equality, whatever is done to one side must be done to the other.

#### TRIGONOMETRIC IDENTITIES

#### • Reciprocal identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u} \quad \cot u = \frac{1}{\tan u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u}$$

#### • Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

#### • Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

#### • Co-Function Identities

$$\sin(\frac{\pi}{2} - u) = \cos u \quad \cos(\frac{\pi}{2} - u) = \sin u$$

$$\tan(\frac{\pi}{2} - u) = \cot u \quad \cot(\frac{\pi}{2} - u) = \tan u$$

$$\csc(\frac{\pi}{2} - u) = \sec u \quad \sec(\frac{\pi}{2} - u) = \csc u$$

#### • Parity Identities (Even & Odd)

$$\sin(-u) = -\sin u \quad \cos(-u) = \cos u$$
  

$$\tan(-u) = -\tan u \quad \cot(-u) = -\cot u$$
  

$$\csc(-u) = -\csc u \quad \sec(-u) = \sec u$$

#### • Sum & Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

#### • Double Angle Formulas

$$\sin(2u) = 2\sin u \cos u$$

$$\cos(2u) = \cos^2 u - \sin^2 u$$

$$= 2\cos^2 u - 1$$

$$= 1 - 2\sin^2 u$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

# • Power-Reducing/Half Angle Formulas

$$\sin^{2} u = \frac{1 - \cos(2u)}{2}$$
$$\cos^{2} u = \frac{1 + \cos(2u)}{2}$$
$$\tan^{2} u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

#### • Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

#### • Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} \left[ \cos(u - v) - \cos(u + v) \right]$$

$$\cos u \cos v = \frac{1}{2} \left[ \cos(u - v) + \cos(u + v) \right]$$

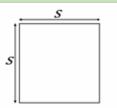
$$\sin u \cos v = \frac{1}{2} \left[ \sin(u + v) + \sin(u - v) \right]$$

$$\cos u \sin v = \frac{1}{2} \left[ \sin(u + v) - \sin(u - v) \right]$$

# GEOMETRY SHAPES AND SOLIDS

#### SQUARE

$$P = 4s$$
$$A = s^2$$



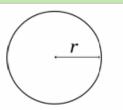
#### RECTANGLE

$$P = 2a + 2b$$
$$A = ab$$



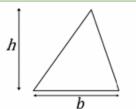
#### CIRCLE

$$P = 2\pi r$$
 $A = \pi r^2$ 



#### TRIANGLE

$$P = a + b + c$$
$$A = \frac{1}{2}bh$$



#### **PARALLELOGRAM**

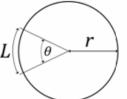
$$P = 2a + 2b$$

$$A = bh$$

$$h$$

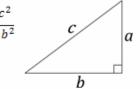
#### CIRCULAR SECTOR

$$L = \pi r \frac{\theta}{180^{\circ}}$$
$$A = \pi r^2 \frac{\theta}{360^{\circ}}$$



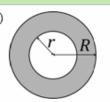
#### PYTHAGOREAN THEOREM

$$a^2 + b^2 = c^2$$
$$c = \sqrt{a^2 + b^2}$$



#### CIRCULAR RING

$$A = \pi (R^2 - r^2)$$



#### **SPHERE**

$$S = 4\pi r^2$$

$$V = \frac{4\pi r^3}{3}$$



#### **TRAPEZOID**

$$P = a + b + c + d \qquad a$$

$$A = h \frac{a+b}{2} \qquad d \qquad c$$

#### **RECTANGULAR BOX**

$$A = 2ab + 2ac + 2bc$$

$$V = abc$$

$$C$$

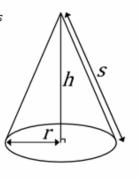
$$D$$

#### RIGHT CIRCULAR CONE

$$A = \pi r^2 + \pi rs$$

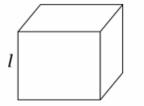
$$s = \sqrt{r^2 + h^2}$$

$$V = \frac{1}{3}\pi r^2 h$$



#### CUBE

$$A = 6l^2$$
$$V = l^3$$



b

#### **CYLINDER**

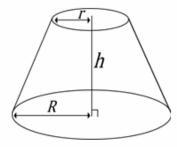
$$A = 2\pi r(r+h)$$

$$V = \pi r^2 h$$

$$h$$

#### FRUSTUM OF A CONE

$$V=\frac{1}{3}\pi h(r^2+rR+R^2)$$



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# The Metric System: Fundamental and Derived Units

# **Metric System Prefixes**

Prefix	Symbol	Factor
tera	Т	$1\ 000\ 000\ 000\ 000 = 10^{12}$
giga	G	$1\ 000\ 000\ 000 = 10^9$
mega	M	$1\ 000\ 000 = 10^6$
kilo	k	$1000 = 10^3$
hecto	h	$100 = 10^2$
deca	da	$10 = 10^1$
		$1 = 10^0$
deci	d	$0.1 = 10^{-1}$
centi	С	$0.01 = 10^{-2}$
milli	m	$0.001 = 10^{-3}$
micro	μ	$0.000\ 001 = 10^{-6}$
nano	n	$0.000\ 000\ 001 = 10^{-9}$
pico	p	$0.000\ 000\ 000\ 001 = 10^{-12}$
femto	f	$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$
atto	a	$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$

### **Fundamental Physical Quantities** and Their SI Units

Quantity	Symbol	Unit	Symbol
length	1	metre	m
mass	m	kilogram	kg
time	t	second	s
absolute temperature	T	Kelvin	K
electric current	I	ampère	Α
		(amp)	
amount of substance	mol	mole	mol

#### **Derived SI Units**

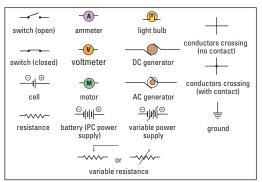
Quantity	Quantity symbol	Unit	Unit symbol	Equivalent unit(s)
area	A	square metre	$m^2$	
volume	V	cubic metre	$\mathrm{m}^3$	
velocity	V	metre per second	m/s	
acceleration	а	metre per second		
		per second	$m/s^2$	
force	F	newton	N	kg ⋅ m/s <sup>2</sup>
work	W	joule	J	$N \cdot m$ , kg · m <sup>2</sup> /s <sup>2</sup>
energy	E	joule	J	$N \cdot m$ , kg · m <sup>2</sup> /s <sup>2</sup>
power	P	watt	W	J/s, kg $\cdot$ m <sup>2</sup> /s <sup>3</sup>
density	ρ	kilogram per cubic metre	kg/m³	
pressure	p	pascal	Pa	$N/m^2$ , kg/(m · s <sup>2</sup> )
frequency	f	hertz	Hz	$s^{-1}$
period	T	second	s	
wavelength	λ	metre	m	
electric charge	Q	coulomb	C	A·s
electric potential	V	volt	V	W/A, J/C,
				$kg \cdot m^2/(C \cdot s^2)$
resistance	R	ohm	Ω	V/A,
				$kg \cdot m^2/(C^2 \cdot s)$
magnetic field intensity	B	tesla	T	$N \cdot s/(C \cdot m), N/(A \cdot m)$
magnetic flux	Φ	weber	Wb	$V \cdot s$ , $T \cdot m^2$ , $m^2 \cdot kg/(C \cdot s)$
radioactivity	$\Delta N/\Delta t$	becquerel	Bq	$s^{-1}$
radiation dose		gray	Gy	$J/kg \cdot m^2/s^2$
temperature (Celsius)	T	degree Celsius	°C	$T^{\circ}C = (T + 273.15) \text{ K}$
		atomic mass unit	u	$1u = 1.660 \ 566 \times 10^{-27} \ kg$
		electron volt	eV	$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$

# **Physical Constants and Data**

# **Fundamental Physical Constants**

Quantity	Symbol	Accepted value
speed of light in a vacuum	C	$2.998 \times 10^{8} \text{ m/s}$
gravitational constant	G	$6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
Coulomb's constant	k	$8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
charge on an electron	e	$1.602 \times 10^{-19} \text{ C}$
rest mass of an electron	$m_{ m e}$	$9.109 \times 10^{-31} \text{ kg}$
rest mass of a proton	$m_{\rm p}$	$1.673 \times 10^{-27} \text{ kg}$
rest mass of a neutron	$m_{\rm n}$	$1.675 \times 10^{-27} \text{ kg}$
Planck's constant	h	$6.626 \times 10^{-34} \text{ J} \cdot \text{s}$

# **Electric Circuit Symbols**



# **Other Physical Data**

Quantity	Symbol	Accepted value	
standard atmospheric pressure	P	$1.013 \times 10^5 \text{ Pa}$	
speed of sound in air		343 m/s (at 20°C)	
water: density (4°C)		$1.000 \times 10^3 \text{ kg/m}^3$	
latent heat of fusion		$3.34 \times 10^5 \text{ J/kg}$	
latent heat of vaporization		$2.26 \times 10^6 \text{ J/kg}$	
specific heat capacity (15°C)		4186 J/(kg°C)	
kilowatt hour	E	$3.6 \times 10^6 \text{ J}$	
acceleration due to Earth's gravity	g	9.81 m/s <sup>2</sup> (standard value; at sea level)	
mass of Earth	$m_{ m E}$	$5.98 \times 10^{24} \text{ kg}$	
mean radius of Earth	$r_{ m E}$	$6.38 \times 10^6 \text{ m}$	
mean radius of Earth's orbit	$R_{ m E}$	$1.49 \times 10^{11} \text{ m}$	
period of Earth's orbit	$T_{ m E}$	$365.25 \text{ days or } 3.16 \times 10^7 \text{ s}$	
mass of Moon	$m_{ m M}$	$7.36 \times 10^{22} \text{ kg}$	
mean radius of Moon	$r_{ m M}$	$1.74 \times 10^6 \text{ m}$	
mean radius of Moon's orbit	$R_{ m M}$	$3.84 \times 10^8 \text{ m}$	
period of Moon's orbit	$T_{ m M}$	27.3 days or $2.36 \times 10^6 \text{ s}$	
mass of Sun	$m_{ m s}$	$1.99 \times 10^{30} \text{ kg}$	
radius of Sun	$r_{ m s}$	$6.96 \times 10^8 \text{ m}$	

#### **Resistor Colour Codes**

Colour	Digit represented	Multiplier	Tolerance
black	0	× 1	
brown	1	$\times$ 1.0 $\times$ 10 <sup>1</sup>	
red	2	$\times$ 1.0 $\times$ 10 <sup>2</sup>	
orange	3	$\times$ 1.0 $\times$ 10 <sup>3</sup>	
yellow	4	$\times$ 1.0 $\times$ 10 <sup>4</sup>	
green	5	$ imes 1.0  imes 10^5$	
blue	6	$ imes 1.0  imes 10^6$	
violet	7	$\times$ 1.0 $\times$ 10 <sup>7</sup>	
gray	8	$\times$ 1.0 $\times$ 10 <sup>8</sup>	
☐ white	9	$\times$ 1.0 $\times$ 10 <sup>9</sup>	
gold		$\times$ 1.0 $\times$ 10 <sup>-1</sup>	5%
silver		$ imes 1.0  imes 10^{-2}$	10%
no colour			20%

