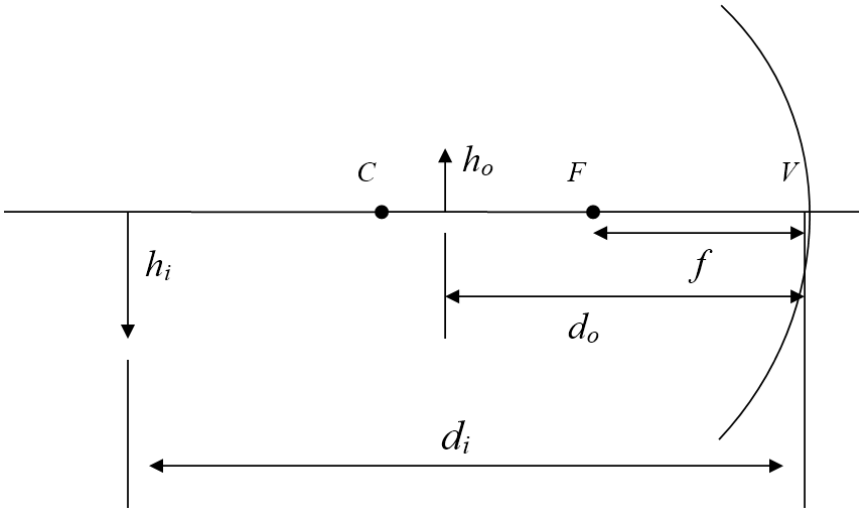
Image Characteristics: Equations



f = the focal length of the mirror.
d_o = the distance from the vertex to the object.
d_i = the distance from the vertex to the image.
h_o = the height of the object.
h_i = the height of the image.

Magnification

$$M = \frac{h_i}{h_o} \qquad \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M = -\frac{d_i}{d_o}$$

The Mirror Equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

Sign Conventions

Variables	Description
d_o, d_i	relative to vertex; + if real, - if virtual
R, f	relative to vertex; + if real, - if virtual
h_o, h_i	relative to principle axis; + if upright, - if inverted.

Example 1

An object is located 30.0 cm from a converging mirror with a radius of curvature of 10.0 cm.

(a) At what distance from the mirror will the image be formed?

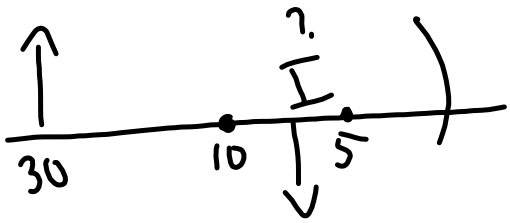
$d_o = 30\,cm$

$R = 10\,cm$

$d_i = ?$

$f = \frac{1}{2}R = 5\,cm$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$



Ans: $5 < d_i < 10$

$$\frac{1}{30} + \frac{1}{d_i} = \frac{1}{5}$$

$$\frac{1}{d_i} = \frac{1}{5} - \frac{1}{30}$$

$$\frac{1}{d_i} = \frac{1}{6}$$

$\boxed{d_i = 6\,cm}$

(b) If the object is 4.0 cm tall, how tall is the image?

$h_o = 4.0\,cm$

$h_i = ?$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$\frac{h_i}{4} = -\frac{6}{30}$$

$\boxed{h_i = -0.8\,cm}$

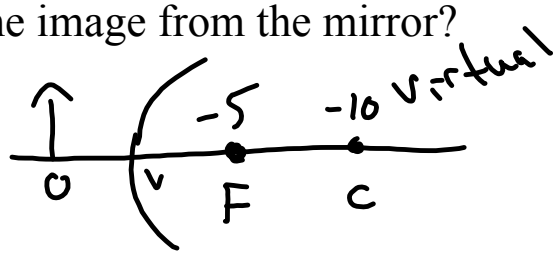
upside down

Read through text: Holt pg 457 - 458, 461.

A diverging mirror with a focal length of $f = -5.0$ cm produces an image of an object. If the object is located 15.0 cm from the mirror then, (a) What is the distance of the image from the mirror?

$f = -5\,cm$

$d_o = 15\,cm$



$-5 < d_i < 0$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{15} + \frac{1}{d_i} = \frac{1}{-5}$$

$$\frac{1}{d_i} = \frac{-1}{5} - \frac{1}{15}$$

$$\frac{1}{d_i} = \frac{-4}{15}$$

$$d_i = \frac{-15}{4}$$

$$\boxed{d_i = -3.75\,cm}$$

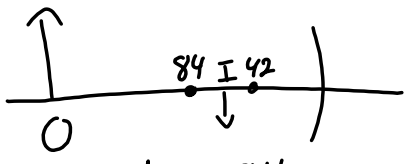
(b) What is the magnification?

$$M = -\frac{d_i}{d_o} = -\frac{(-3.75)\,cm}{15\,cm}$$

$$\boxed{M = 0.25}$$

Practice: Holt Problems Pg 466 #1-4.

(15) $f = 42\text{cm}$



$d_o > 84\text{cm}$

$M = -\frac{1}{3}$

↑ real, inverted image

$M = -\frac{1}{3}$

$f = 42\text{cm}$

$d_o = ?$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$M = -\frac{d_i}{d_o}$$

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{42} \quad (1)$$

(2) $-\frac{1}{3} = -\frac{d_i}{d_o}$ ← solve for d_i and sub in (1)

(2) $\frac{d_o}{3} = d_i$

(1) $\frac{1}{d_o} + \frac{1}{d_o/3} = \frac{1}{42}$

$$\frac{1}{d_o} + \frac{3}{d_o} = \frac{1}{42}$$

$$\frac{4}{d_o} = \frac{1}{42}$$

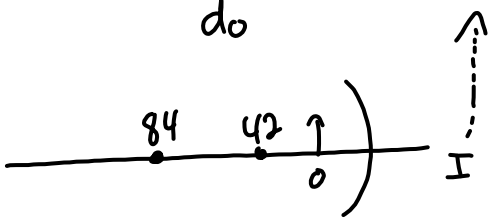
$$(4)(42) = d_o$$

$$\boxed{168\text{cm} = d_o}$$

(16) $f = 42\text{cm}$ $\quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

$M = +3$ $\quad M = \frac{-d_i}{d_o}$

↑ virtual, upright



$d_o < 42\text{cm}$

(1) $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{42}$

(2) $3 = \frac{-d_i}{d_o}$ ← $-3d_o = d_i$ Sub in (1)

$$\frac{1}{d_o} + \frac{1}{(-3d_o)} = \frac{1}{42}$$

$\frac{1}{2} + \frac{1}{4}$ $\quad \frac{3\times 1}{3d_o} - \frac{1}{3d_o} = \frac{1}{42}$

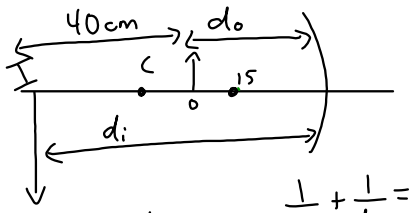
$$\frac{3}{3d_o} - \frac{1}{3d_o} = \frac{1}{42}$$

$$\frac{2}{3d_o} = \frac{1}{42}$$

$$84 = 3d_o$$

$$\frac{84}{3} = d_o$$

$$\boxed{28\text{cm} = d_o}$$



$40 + d_o = d_i$ $\qquad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$

① $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{15}$ $\qquad$ Sub ② → ①

② $40 + d_o = d_i$

$$\frac{1}{d_o} + \frac{1}{40+d_o} = \frac{1}{15}$$

$$\frac{1}{d_o} \times \frac{(40+d_o)}{(40+d_o)} + \frac{1}{(40+d_o)} \times \frac{d_o}{d_o} = \frac{1}{15}$$

$$\frac{40+d_o}{d_o(40+d_o)} + \frac{d_o}{d_o(40+d_o)} = \frac{1}{15}$$

$$\frac{2d_o + 40}{d_o(40+d_o)} = \frac{1}{15}$$

$$15(2d_o + 40) = 40d_o + d_o^2$$

$$30d_o + 600 = 40d_o + d_o^2$$

$$0 = d_o^2 + 10d_o - 600$$

$$0 = (d_o + 30)(d_o - 20)$$

$d_o = -30$ cm or $+20$ cm ✓

reject

$d_i = ?$

$$\frac{1}{20} + \frac{1}{d_i} = \frac{1}{15}$$

$40 + d_o = d_i$

$40 + 20 = d_i$

$60\text{cm} = d_i$

$$\frac{1}{d_i} = \frac{1}{15} - \frac{1}{20}$$

$$\frac{1}{d_i} = \frac{1}{60}$$

$d_i = 60$cm

$$M = \frac{-d_i}{d_o} = \frac{-60}{20}$$

$M = -3$