

*definition of equilibrium: the state of an object when the vector sum of all the forces acting on it is zero.*

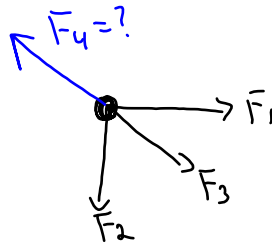
If an object is at *rest* and is in *equilibrium*, then we say that it is in a state of "*static equilibrium*."

**Equilibrant: is the one vector, when added to 2 or more other vectors produces a state of equilibrium. It is equal to the resultant but opposite in direction.**

Three forces act simultaneously on point P. The first force is 10 N east. The second force is 15 N south. The third force is 28 N, E46°S. Find the equilibrant. (46N, W50°N)

$$F_1 = 10 \text{ [E]} \quad F_3 = 28 \text{ [E}46^\circ\text{S]}$$

$$F_2 = 15 \text{ [S]} \quad F_4 = ?$$



East-West

$$F_{net} = \sum \text{ Forces} = 0 \text{ N.}$$

$$0 = F_{1E} + F_{2E} + F_{3E} + F_{4E}$$

$$0 = 10 + 0 + 28 \cos 46 + F_{4E}$$

$$\underline{-29.5 \text{ N}} = F_{4E}$$

North-South

$$F_{net} = F_{1N} + F_{2N} + F_{3N} + F_{4N}$$

$$0 = 0 + (-15) + (-28 \sin 46) + F_{4N}$$

$$\underline{35.1 \text{ N}} = F_{4N}$$

$$F_4 = \sqrt{(F_{4E})^2 + (F_{4N})^2}$$

$$F_4 = 45.8 \text{ N}$$

$$\theta = \tan^{-1} \left| \frac{F_{4N}}{F_{4E}} \right|$$

$$\theta = 50^\circ$$

$$F_4 = 46 \text{ [W}50^\circ\text{N]}$$

$\vec{F}_1 = 782 \text{ N}[E12^\circ N]$ ,  $\vec{F}_2 = 629 \text{ N}[W24^\circ S]$   
What is  $\vec{F}_3$ ? ( $F_3$  is the equilibrant)

$$\vec{F}_3 = 212 [W 26^\circ N]$$

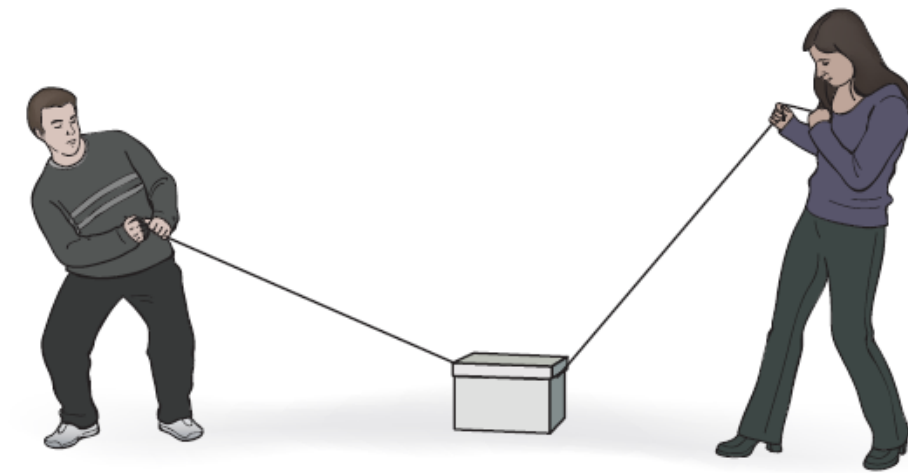
# Pushing or Pulling at an Angle

Grade: 12

Subject: Physics 122

Date: 2014

Consider this diagram for the following questions:

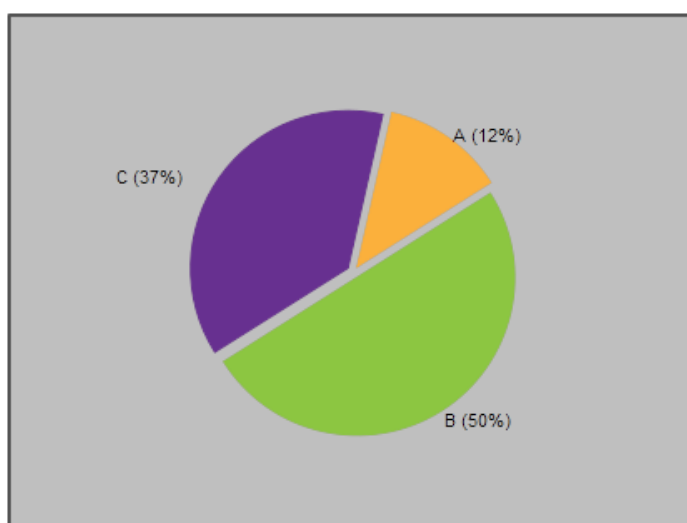


1 How does the normal force compare to the case where no one is pulling on the box?

A Normal force would be the same in either case.

**B** Normal force is reduced based on the diagram.

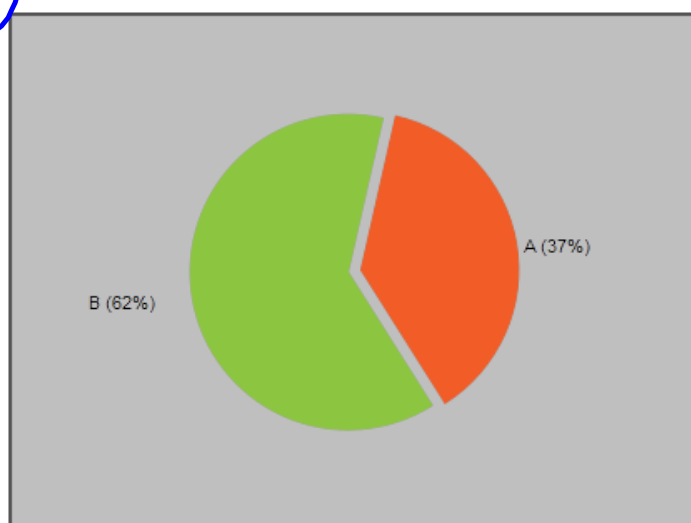
C Normal force is increased based on the diagram.



2 For this box not to have any horizontal motion, what condition must be met?

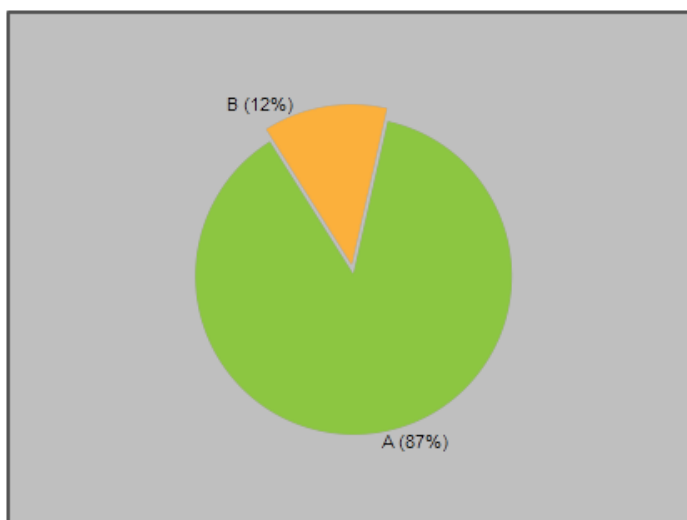
A Man applies more force than woman.

B Man applies less force than woman.



3 How does the vertical component of the applied force change when the angle is increased?

- A Increased
- B Decreased
- C Remains the same



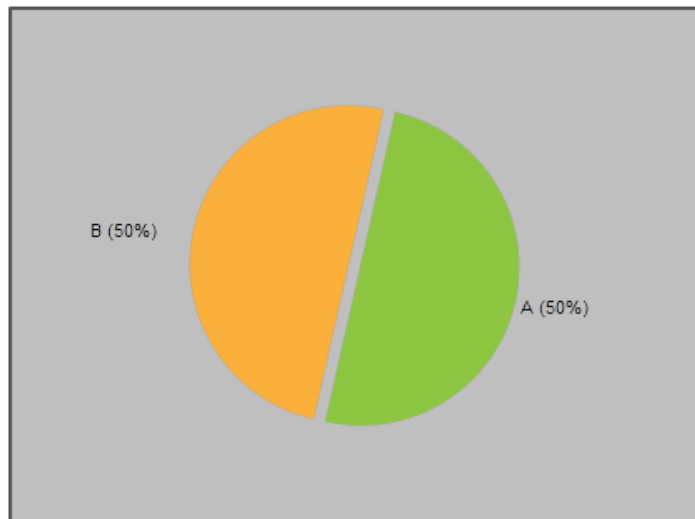


4 If the woman doubles her applied force then the horizontal component will \_\_\_\_\_.

A double

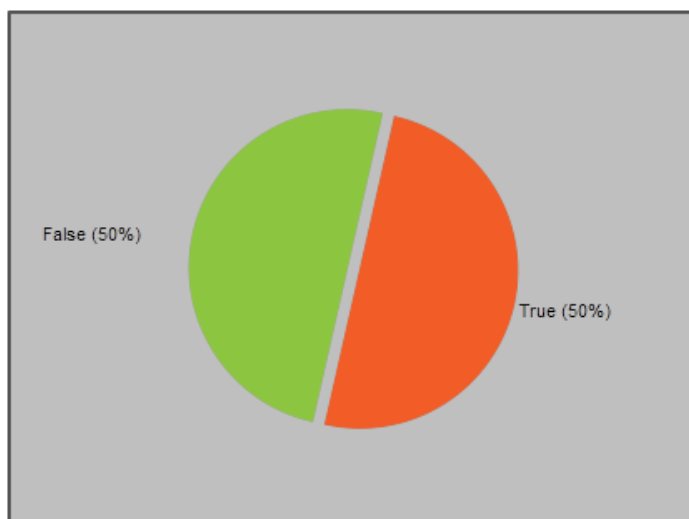
B be halved

C remain the same



5 If the man doubles the angle of the applied force (without changing  $F_a$ ) it will double the vertical component of  $F_a$ .

True  
False

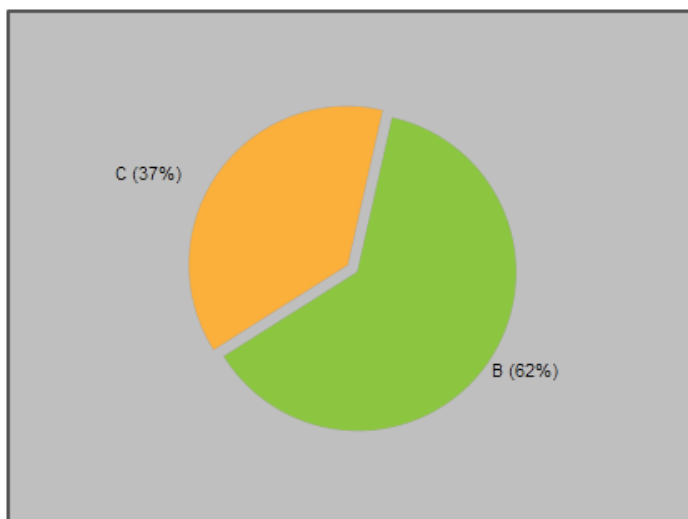


6 If both people increase their angles then the force of friction will \_\_\_\_\_.

A increase

**B decrease**

C remain the same

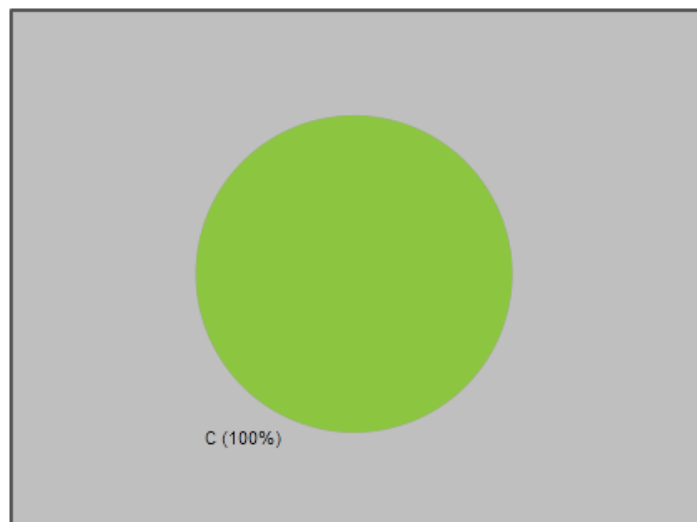


7 If both people reduce their pulling angles the force of gravity on the box will \_\_\_\_.

A increase

B decrease

C remain the same

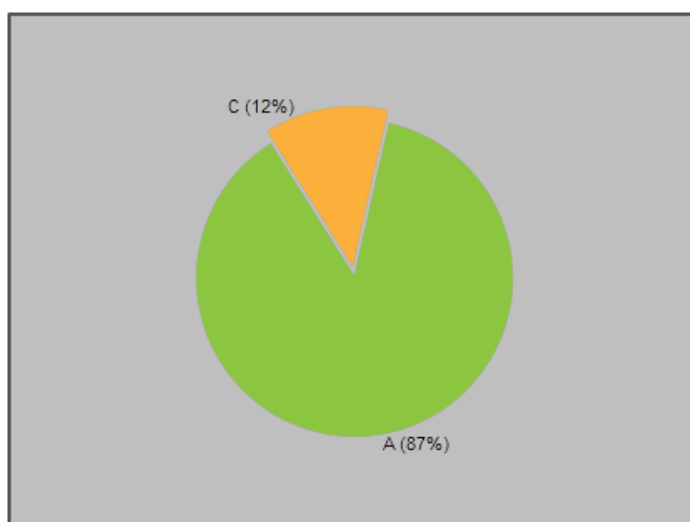


8 Suppose the box in the diagram is replaced with one that is more massive (that is the only change). The normal force will \_\_\_\_.

A increase

B decrease

C remain the same

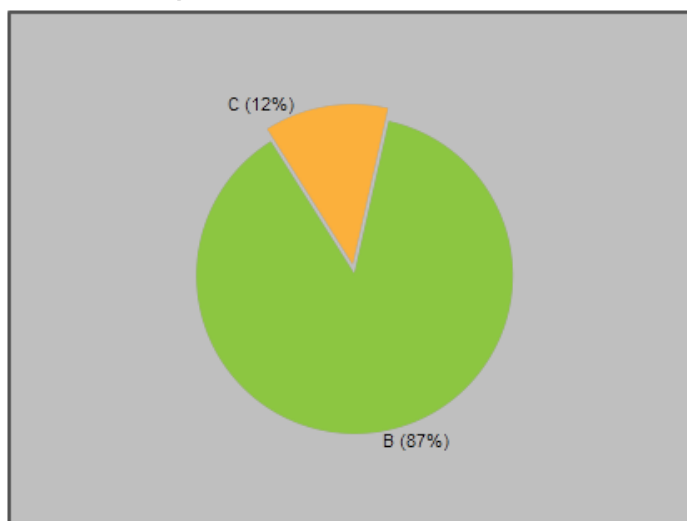


9 Suppose in this situation the box accelerates to the left. How could the woman compensate without changing her applied force?

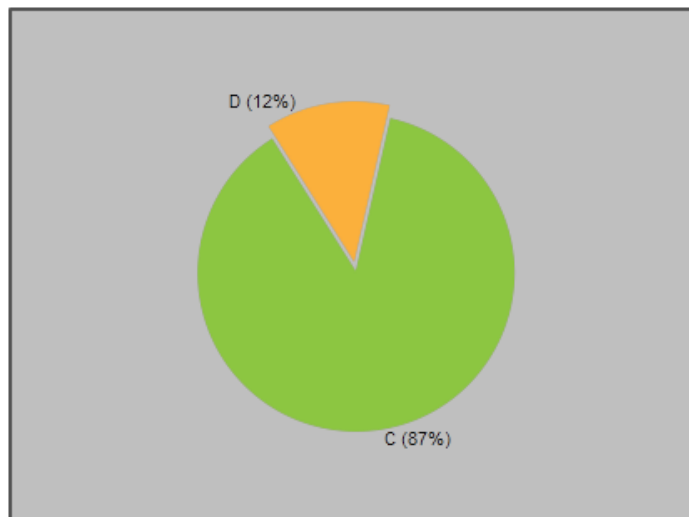
A increase the angle of her applied force

B decrease the angle of her applied force

C it is not possible to compensate

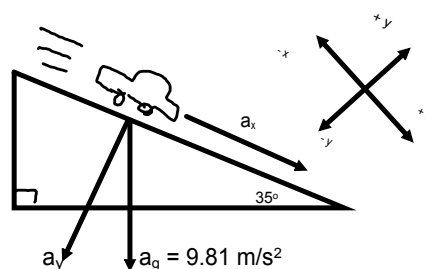
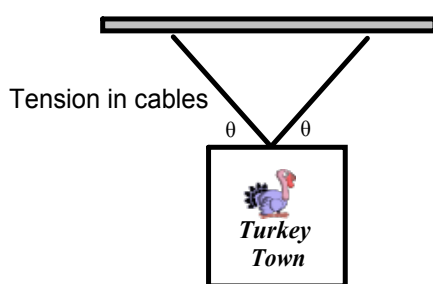
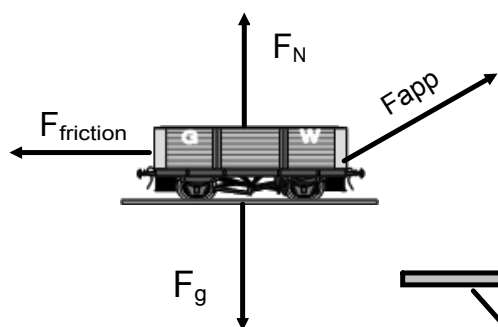


- 10 Suppose the box is in static equilibrium when a third person attaches a rope to the front side of the box and pulls (so this applied force would be out of your screen - in the 3rd dimension). What will happen to the box? (neglect friction)
- A The box will remain in static equilibrium.
  - B The box will accelerate away from the third person.
  - C The box will accelerate toward the third person.
  - D The box will move at a constant velocity towards the third person.



## Three Types of Force Problems

- 1 - Pushing or pulling an object along a horizontal surface.
- 2 - Tension and hanging signs.
- 3 - Objects on an incline.



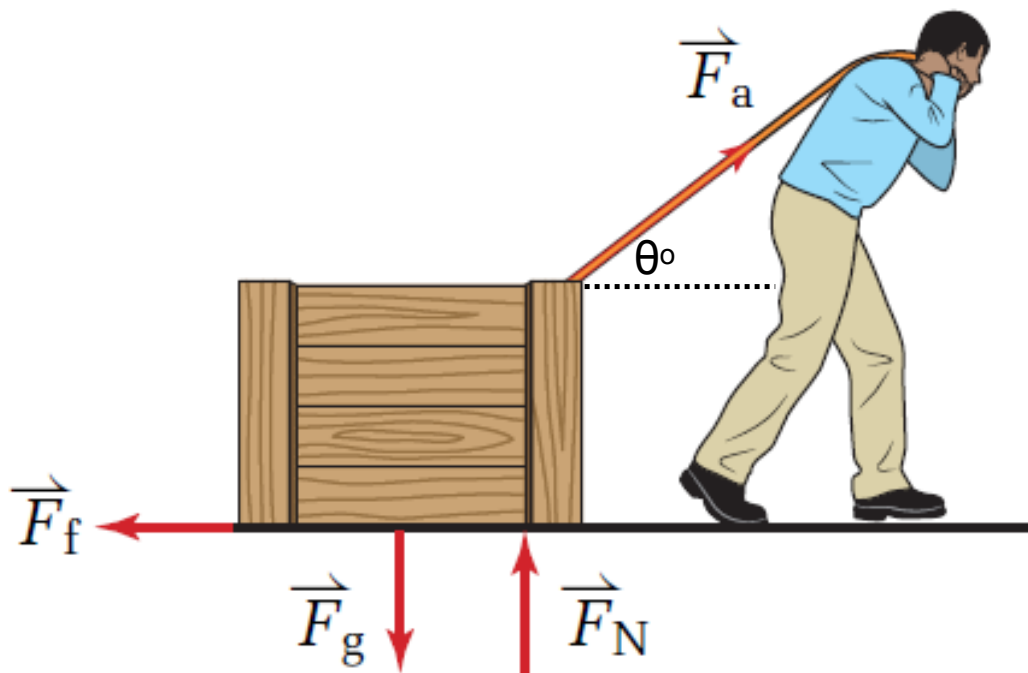


## Force Problems - Type I

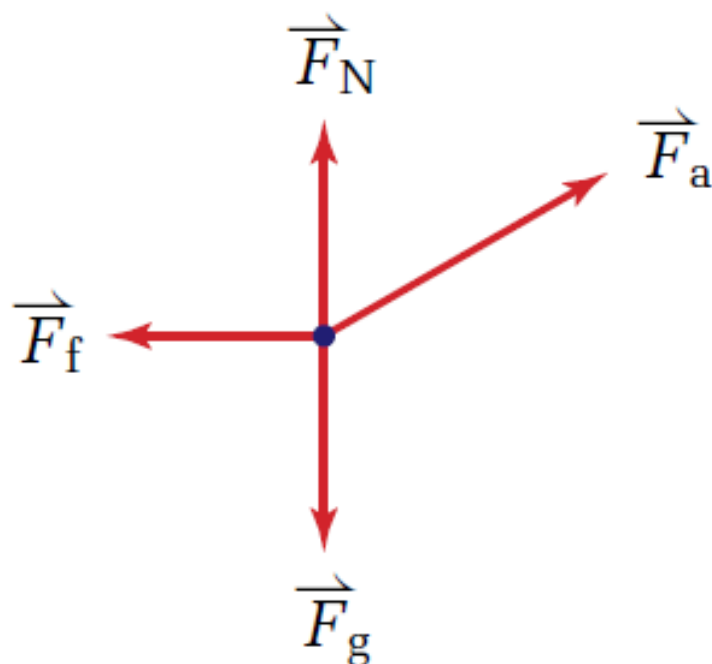
### Pushing or Pulling at an Angle

Read MHR: Page 168

**A**



**B**



A 55 kg snow blower is pushed along the ground at an angle of  $35^\circ$  down from the horizontal with an applied force of 175 N. The coefficient of kinetic friction is 0.19.

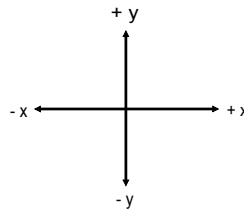
a) Calculate  $F_{ax}$  and  $F_{ay}$ .

b) Calculate  $F_N$ .

c) Calculate  $F_f$ .

d) Calculate  $F_{netx}$ .

e) Calculate  $a_x$ .



$$\begin{aligned} \text{a) } F_{ax} &= F_a \cos \theta \\ &= 175 \cos 35^\circ = \underline{143 \text{ N}} \end{aligned}$$

$$\begin{aligned} F_{ay} &= -F_a \sin \theta \\ &= -175 \sin 35^\circ = \underline{-100 \text{ N}} \end{aligned}$$

*down / pushed*

$$\text{b) } F_N = ? \quad F_{nety} = \sum \text{ Forces in } y\text{-dir.}$$

$$\begin{aligned} F_{nety} &= F_g + F_N + F_{ay} \\ \text{not moving vertically} \rightarrow 0 &= -(55)(9.81) + F_N + (-100) \end{aligned}$$

$$0 = -539.6 + F_N - 100$$

$$\boxed{640 \text{ N} = F_N}$$

$$\begin{aligned} \text{c) } F_f = ? \quad |F_f| &= \mu |F_N| \\ |F_f| &= (0.19)(640) \\ \boxed{|F_f|} &= \underline{122 \text{ N}} \end{aligned}$$

$$\begin{aligned} \text{d) } F_{netx} = ? \quad F_{netx} &= \sum \text{ Forces} \\ F_{netx} &= F_a + F_f \\ &= 143 + (-122) \end{aligned}$$

*opposite motion*

$$\boxed{F_{netx} = 21 \text{ N}}$$

$$\text{e) } a_x = ? \quad F_{netx} = m a_x$$

$$21 = 55 a_x$$

$$\boxed{0.38 \text{ m/s}^2 = a_x}$$

A 35 kg wagon is pulled along the ground at an angle of  $25^\circ$  to the horizontal with an applied force of 97 N.

- a) Calculate  $F_{ax}$  and  $F_{ay}$ .
- b) Calculate  $F_N$ .
- c) Calculate the force of friction if  $\mu = 0.22$ .
- d) Calculate the  $F_{netx}$ .
- e) Calculate  $a_x$ .

## Force Problems - Type I

MHR - Chapter 5 - Page 174

17. A student pushes a 25 kg lawn mower with a force of 150 N. The handle makes an angle of  $35^\circ$  to the horizontal.
- (a) Find the vertical and horizontal components of the applied force.
- (b) Calculate the normal force supporting the lawn mower while it is being pushed.
- (c) Calculate the net force propelling the mower if a frictional force of 85 N exists.
- (d) Calculate the horizontal acceleration of the lawn mower. (Remember: Only part of the  $F_{\text{applied}}$  is parallel to the direction of horizontal acceleration.)

- a) 86 N, down  
 $1.2 \times 10^2$  N, right  
 b)  $3.3 \times 10^2$  N, up  
 c) 38 N, right  
 d)  $1.5 \text{ m/s}^2$ , right

MHR - Chapter 5 - Page 208

24. A toboggan with a mass of 15 kg is being pulled with an applied force of 45 N at an angle of  $40^\circ$  to the horizontal. What is the acceleration if the force of friction opposing the motion is 28 N?
25. A grocery cart is being pushed with a force of 450 N at an angle of  $30.0^\circ$  to the horizontal. If the mass of the cart and the groceries is 42 kg,
- (a) Calculate the force of friction if the coefficient of friction is 0.60.
- (b) Determine the acceleration of the cart.

$0.43 \text{ m/s}^2$ , right

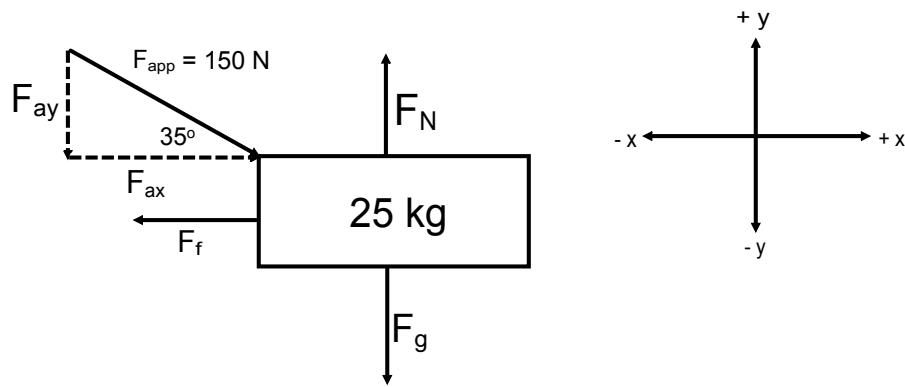
- a)  $3.8 \times 10^2$  N, left  
 b)  $0.23 \text{ m/s}^2$ , right

MHR - Chapter 5 - Page 209

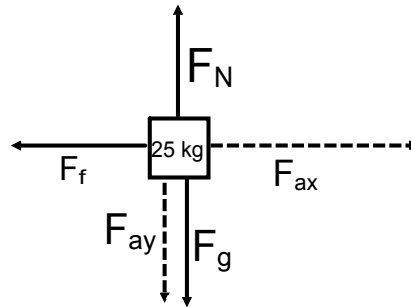
36. A 45.0 kg box is pulled with a force of 205 N by a rope held at an angle of  $46.5^\circ$  to the horizontal. The velocity of the box increases from 1.00 m/s to 1.50 m/s in 2.50 s. Calculate
- (a) the net force acting horizontally on the box.
- (b) the frictional force acting on the box.
- (c) the horizontal component of the applied force.
- (d) the coefficient of kinetic friction between the box and the floor.

- a) 9.0 N, right  
 b) 132 N, left  
 c) 141 N, right  
 d) 0.451

#17



Free Body Diagram



$$(a) F_{ax} = +150\cos(35) \\ = 123\text{ N}$$

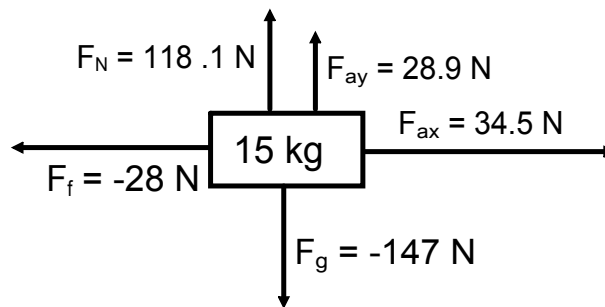
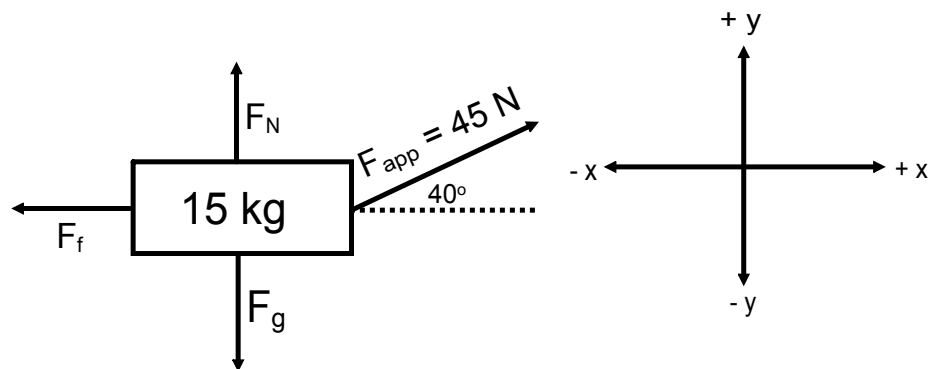
$$F_{ay} = -150\sin(35) \\ = -86\text{ N}$$

$$(b) F_{nety} = F_{ay} + F_g + F_N \\ 0 = -86\text{ N} - mg + F_N \\ 0 = -86 - 25(9.81) + F_N \\ 0 = -86 - 245.25 + F_N \\ 0 = -331.25 + F_N \\ +330\text{ N} = F_N$$

$$(c) F_{netx} = \text{Sum of horizontal forces} \\ = F_{ax} + F_f \\ = 123 + -85 \\ = +38\text{ N}$$

$$(d) F_{net} = ma \\ a = \frac{F_{net}}{m} \\ a = \frac{+38\text{ N}}{25\text{ kg}} \\ a = +1.5\text{ m/s}^2$$

# 24



$$\begin{aligned}
 F_{\text{net}x} &= F_{\text{ax}} + F_f \\
 &= 34.5 + (-28) \\
 &= +6.5 \text{ N}
 \end{aligned}$$

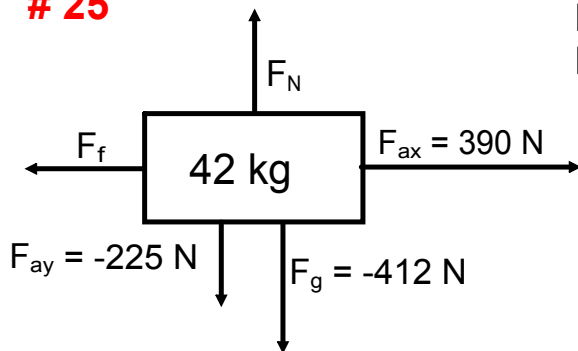
$$F_{\text{net}} = ma$$

$$a = \frac{F_{\text{net}}}{m}$$

$$a = +6.5 \text{ N}/15 \text{ kg}$$

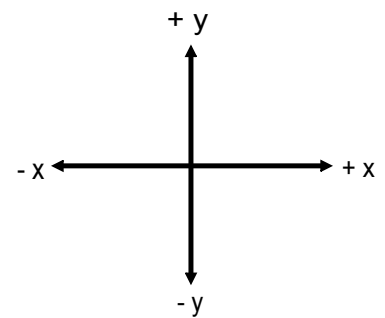
$$a = +0.43 \text{ m/s}^2$$

# 25



$$F_{ax} = 450\cos(30) \text{ N}$$

$$F_{ay} = 450\sin(30) \text{ N}$$



$$(a) \quad F_f = \mu F_N, \mu = 0.60$$

$$F_{nety} = F_{ay} + F_g + F_N$$

$$0 = -412 \text{ N} + -225 \text{ N} + F_N$$

$$F_N = + 637 \text{ N}$$

$$F_f = 0.60(637 \text{ N})$$

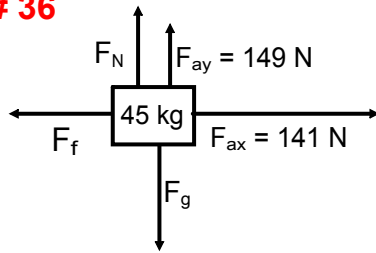
$$F_f = 380 \text{ N [left]}$$

$$(b) \quad a = \frac{F_{net}}{m} = \frac{F_{ax} + F_f}{m}$$

$$a = (389.7 \text{ N} + -382 \text{ N}) \div 42 \text{ kg}$$

$$a = +0.19 \text{ m/s}^2$$

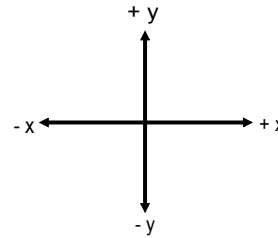
# 36



$$F_{\text{app}} = 205 \text{ N @ } 46.5^\circ$$

$$F_{\text{ax}} = 205 \cos(46.5)$$

$$F_{\text{ay}} = 205 \sin(46.5)$$



(a)  $F_{\text{net}} = ma$  *find acceleration*

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{1.50 \text{ m/s} - 1.00 \text{ m/s}}{2.50 \text{ s}}$$

$$a = 0.20 \text{ m/s}^2$$

$$F_{\text{net}} = (45 \text{ kg})(0.20 \text{ m/s}^2) = 9.0 \text{ N}$$

(c)  $F_{\text{ax}} = 141 \text{ N}$

$$F_{\text{net}} = F_{\text{ax}} + F_f$$

$$9.0 = 141 + F_f$$

$$\boxed{-132 \text{ N} = F_f}$$

(d)  $F_{\text{kf}} = \mu F_N$

$$F_{\text{nety}} = F_{\text{ay}} + F_g + F_N$$

$$0 = -441 \text{ N} + 149 \text{ N} + F_N$$

$$F_N = +292 \text{ N}$$

$$\mu = \frac{F_f}{F_N}$$

$$\mu = \frac{+132 \text{ N}}{292 \text{ N}}$$

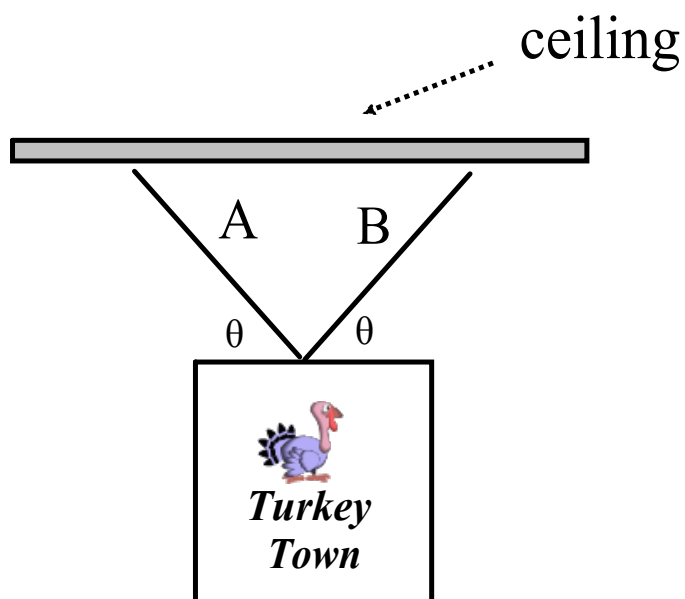
$$\mu = 0.451$$

can use "+132" as the formula implies the magnitude of  $F_f$ .

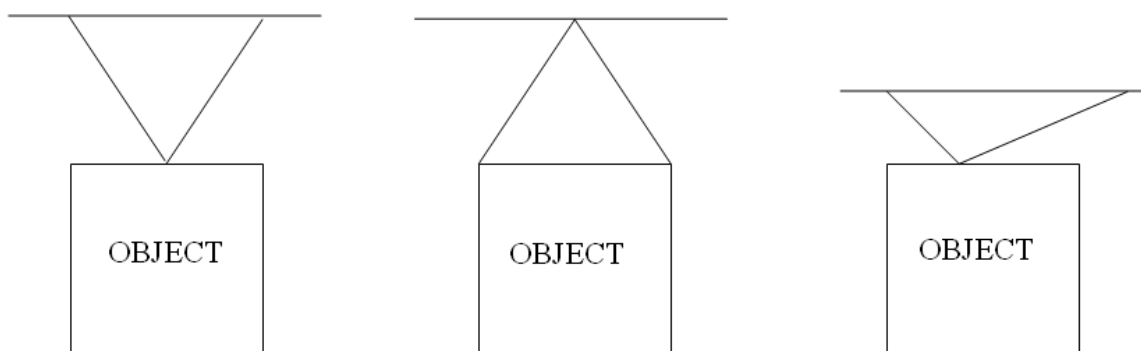


Type II - Signs/Pictures/Hanging Objects

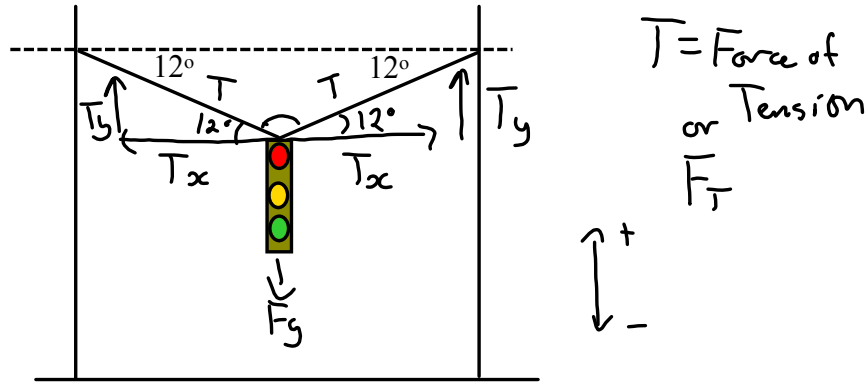
If an object is hung by a rope (wire, chain, etc.), we can resolve the force of tension along the rope.



*An object can be hung in a variety of ways.*



## Example



A traffic light hangs in the center of the road from cables as shown in the figure.

(a) If the mass of the traffic light is 65 kg, what is the vertical force that each cable exerts on the light to prevent it from falling? (b) What is the tension in each cable?

(a) The y-component of the tension in each cable must add together to support the light's weight; the light is in static equilibrium. Since the angles are the same the tension in each cable and their components are the same.

$$F_{\text{net},y} = \sum \text{Forces}$$

$$0 = F_g + 2T_y$$

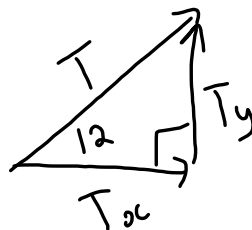
← 2 Force of tensions in y-d.r.

$$0 = -(65)(9.8) + 2T_y$$

$$638 = 2T_y$$

$$\boxed{319 \text{ N} = T_y}$$

(b) Use trig to solve for the tension in each cable.



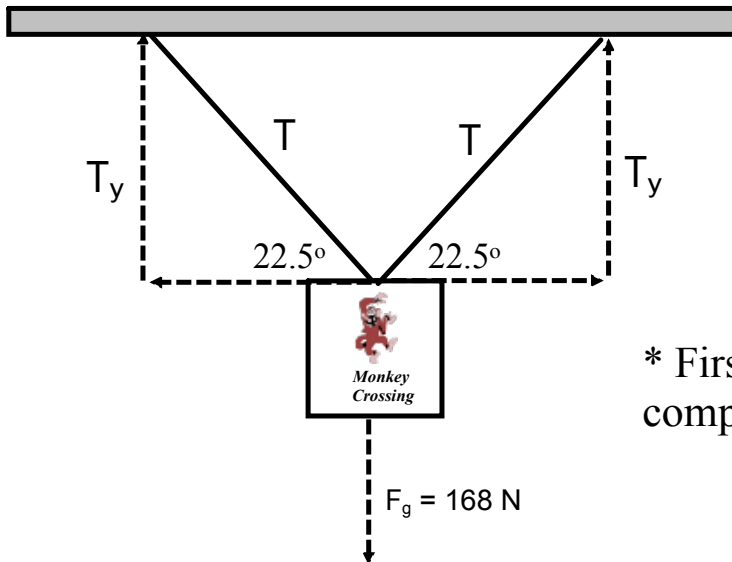
$$T = ?$$

$$\sin 12 = \frac{T_y}{T}$$

$$\sin 12 = \frac{319}{T}$$

$$0.2079 = \frac{319}{T}$$

$$T = \frac{319}{0.2079} = \boxed{1534 \text{ N}}$$



A sign that weighs 168 N is supported by two ropes, A and B, that make  $22.5^\circ$  angles with the horizontal.

Determine the tension along the ropes.

\* First label the diagram to view the components of each rope's tension.

Determine y-component of tension:

$$F_{\text{net}y} = 0 \text{ N}$$

$$F_{\text{net}y} = 2T_y + F_g$$

$$0 \text{ N} = 2T_y - 168 \text{ N}$$

$$T_y = 84 \text{ N}$$

Determine tension in each rope (remember they are the same if the angles are the same):

$$T = (T_y) / (\sin \theta^\circ)$$

$$T = 84 \text{ N} / \sin 22.5^\circ$$

$$T = 220 \text{ N}$$

$$\sin \theta = \frac{T_y}{T}$$

$$T_y = T \sin \theta$$

## Attachments

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ramp-forces-and-motion\_en.jar