With what minimum speed does a baseball have to come off the bat to clear the Green Monster at Fenway Park? The wall is 115.5 m from home plate and 11.3 m high. Assume the batter makes contact 1.0 m off the ground at an angle of $65^{\circ}$.
$>$ This problem is asking us to solve for the speed of the ball, $v$. What makes it challenging is that no velocities in any direction or time is given.
$>$ We will start with our first principle equations and develop a formula that only has the variable $v$.
$>$ First, list known information and equations in each direction.

| $\boldsymbol{x}$-direction | $\boldsymbol{y}$-direction |
| :--- | :--- |
| $d_{o x}=0 \mathrm{~m}$ | $d_{o y}=1.0 \mathrm{~m}$ |
| $d_{f x}=115.5 \mathrm{~m}$ | $d_{f y}=11.3 \mathrm{~m}$ |
| eq1: $v_{x}=v \cos 65$ (horizontal component) | eq4: $v_{o y}=v \sin 65$ (vertical component) |
| eq2: $v_{x}=\frac{d_{f x}-d_{o x}}{t}=\frac{115.5-0}{t}$ (velocity formula) | eq5: $11.3=1.0+v_{o y} t+\frac{1}{2}(-9.81) t^{2}$ (vertical position) |
| Eliminating $v_{x}$ gives us: | We use the $d_{f y}$ equation because it contains variables |
| eq3: $v \cos 65=\frac{115.5}{t}$ | given in the question. |

$>$ Now to do some equation manipulation. Remember the purpose is to create one equation with the only know variable the velocity, $v$.

- There is nothing more we can do with the equations in the $x$-direction. We eliminated $v_{x}$ and we are left with an equation with $v$ and $t$. Let us move on to the $y$-direction.
- We do not have, nor want to calculate, $v_{o y}$ so we will eliminate it by substituting eq4 into eq5:
eq6:

$$
11.3=1.0+(v \sin 65) t-4.9 t^{2}
$$

- This helps solve the problem because now we have two equations and two unknowns (eq3 \& eq6):
eq3:

$$
v \cos 65=\frac{115.5}{t}
$$

eq6:

$$
11.3=1.0+(v \sin 65) t-4.9 t^{2}
$$

- One more substitution and we will have our final equation. Since we are trying to find velocity, $v$, we will eliminate the time variable, $t$, in eq6.
- First we need to rearrange eq3 to solve for time, $t$ :
eq3:

$$
t=\frac{115.5}{v \cos 65}
$$

- Now substitute eq3 into eq6 to eliminate time:

$$
11.3=1.0+(v \sin 65)\left(\frac{115.5}{v \cos 65}\right)-4.9\left(\frac{115.5}{v \cos 65}\right)^{2}
$$

- The $v$-variable in the middle term divides out, a trig identity simplifies $\sin$ and $\cos$ and using exponent laws on the quadratic term gives:

$$
11.3=1.0+115.5 \tan 65-4.9\left(\frac{115.5^{2}}{v^{2}(\cos 65)^{2}}\right)
$$

- This now becomes easier to solve, just be careful with your math and you should get $v=39.3 \mathrm{~m} / \mathrm{s}$

