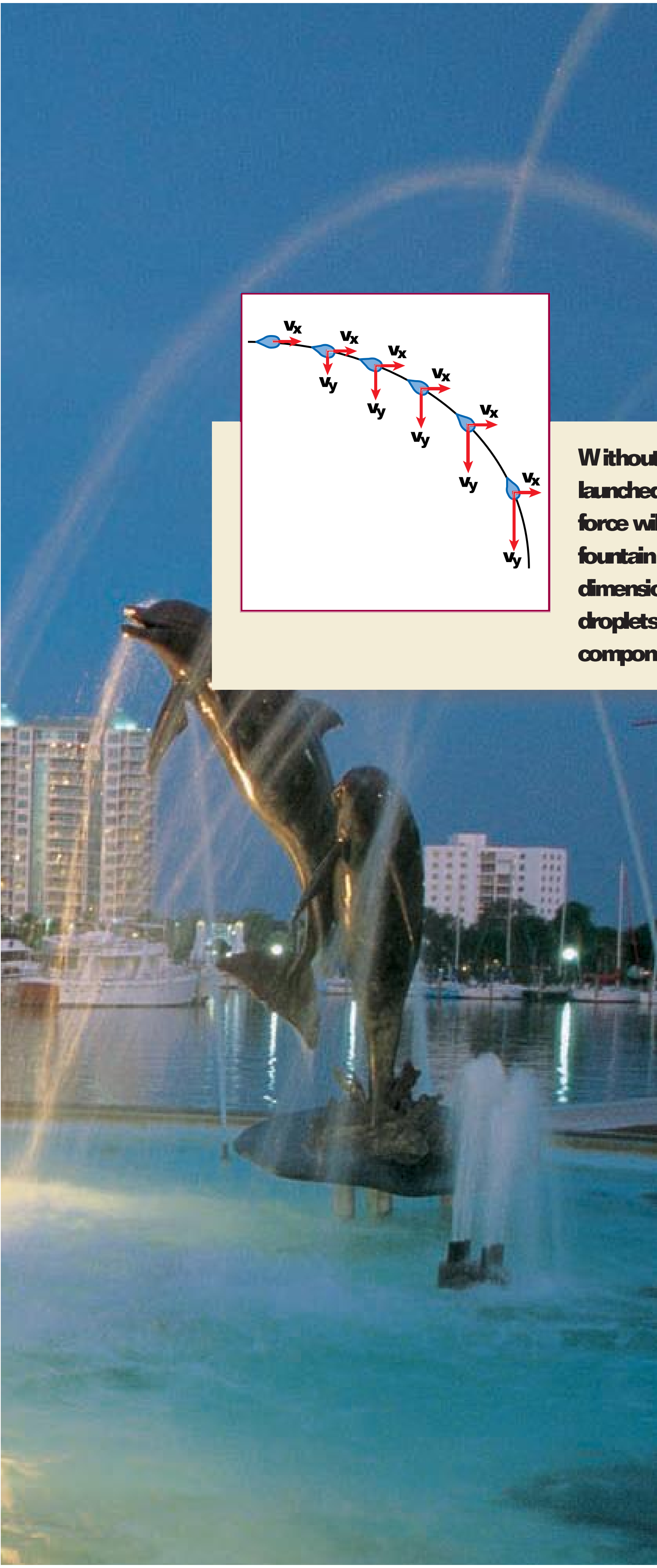

CHAPTER 3

Two-Dimensional Motion and Vectors

Without air resistance, any object that is thrown or launched into the air and that is subject to gravitational force will follow a parabolic path. The water droplets in this fountain are one example. The velocity of any object in two-dimensional motion such as one of these water droplets can be separated into horizontal and vertical components, as shown in the diagram.

WHAT TO EXPECT

In this chapter, you will use vectors to analyze two-dimensional motion and to solve problems in which objects are projected into the air.

Why It Matters

After you know how to analyze two-dimensional motion, you can predict where a falling object will land based on its initial velocity and position.

CHAPTER PREVIEW

SECTION 1

Introduction to Vectors

SECTION OBJECTIVES

- Distinguish between a scalar and a vector.
- Add and subtract vectors by using the graphical method.
- Multiply and divide vectors by scalars.

SCALARS AND VECTORS

In the chapter Motion in One Dimension, our discussion of motion was limited to two directions, forward and backward. Mathematically, we described these directions of motion with a positive or negative sign. That method works only for motion in a straight line. This chapter explains a method of describing the motion of objects that do not travel along a straight line.

Vectors indicate direction scalars do not

Each of the physical quantities encountered in this book can be categorized as either a scalar quantity or a vector quantity. A scalar is a quantity that has magnitude but no direction. Examples of scalar quantities are speed, volume, and the number of pages in this textbook. A vector is a physical quantity that has both direction and magnitude.

scalar

a physical quantity that has magnitude but no direction

vector

a physical quantity that has both magnitude and direction

As we look back to the chapter Motion in One Dimension, we can see that displacement is an example of a vector quantity. An airline pilot planning a trip must know exactly how far and which way to fly. Velocity is also a vector quantity. If we wish to describe the velocity of a bird, we must specify both its speed (say, 3.5 m s) and the direction in which the bird is flying (say, northeast). Another example of a vector quantity is acceleration.

Figure 1
The lengths of the vector arrows represent the magnitudes of these two soccer players velocities.

Vectors are represented by boldface symbols

In physics, quantities are often represented by symbols, such as *t* for time. To help you keep track of which symbols represent vector quantities and which are used to indicate scalar quantities, this book will use boldface type to indicate vector quantities. Scalar quantities will be in italics. For example, the speed of a bird is written as v = 3.5 m s. But a velocity, which includes a direction, is written as **v** = 3.5 m s to the northeast. When writing a vector on your paper, you can distinguish it from a scalar by drawing an arrow above the abbreviation for a quantity, such as $\vec{v}$ = 3.5 m s to the northeast.

One way to keep track of vectors and their directions is to use diagrams. In diagrams, vectors are shown as arrows that point in the direction of the vector. The length of a vector arrow in a diagram is proportional to the vector s magnitude. For example, in Figure 1 the arrows represent the velocities of the two soccer players running toward the soccer ball.

A resultant vector represents the sum of two or more vectors

When adding vectors, you must make certain that they have the same units and describe similar quantities. For example, it would be meaningless to add a velocity vector to a displacement vector because they describe different physical quantities. Similarly, it would be meaningless, as well as incorrect, to add two displacement vectors that are not expressed in the same units. For example, you cannot add meters and feet together.

Section 1 of the chapter Motion in One Dimension covered vector addition and subtraction in one dimension. Think back to the example of the gecko that ran up a tree from a 20 cm marker to an 80 cm marker. Then the gecko reversed direction and ran back to the 50 cm marker. Because the two parts of this displacement are opposite, they can be added together to give a total displacement of 30 cm. The answer found by adding two vectors in this way is called the **resultant.**

Did you know?

The word vector is also used by airline pilots and navigators. In this context, a vector is the particular path followed or to be followed, given as a compass heading.

resultant

a vector that represents the sum of two or more vectors

Vectors can be added graphically

Consider a student walking 1600 m to a friend s house and then 1600 m to school, as shown in Figure 2. The student s total displacement during his walk to school is in a direction from his house to the school, as shown by the dotted line. This direct path is the *v*ector sum of the student s displacement from his house to his friend s house and his displacement from the friend s house to school. How can this resultant displacement be found?

One way to find the magnitude and direction of the student s total displacement is to draw the situation to scale on paper. Use a reasonable scale, such as 50 m on land equals 1 cm on paper. First draw the vector representing the student s displacement from his house to his friend s house, giving the proper direction and scaled magnitude. Then draw the vector representing his walk to the school, starting with the tail at the head of the first vector. Again give its scaled magnitude and the right direction. The magnitude of the resultant vector can then be determined by using a ruler. Measure the length of the vector pointing from the tail of the first vector to the head of the second vector. The length of that vector can then be multiplied by 50 (or whatever scale you have chosen) to get the actual magnitude of the student s total displacement in meters.

The direction of the resultant vector may be determined by using a protractor to measure the angle between the resultant and the first vector or between the resultant and any chosen reference line.

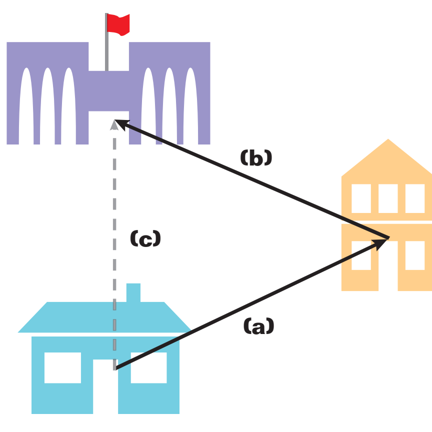

Figure 2
A student walks from his house to his friends house (a), then from his friends house to the school (b). The student s resultant displacement (c) can be found by using a ruler and a protractor.

PROPERTIES OF VECTORS

Now consider a case in which two or more vectors act at the same point. When this occurs, it is possible to find a resultant vector that has the same net effect as the combination of the individual vectors. Imagine looking down from the second level of an airport at a toy car moving at 0.80 m s across a walkway that moves at 1.5 m s. How can you determine what the car s resultant velocity will look like from your viewpoint?

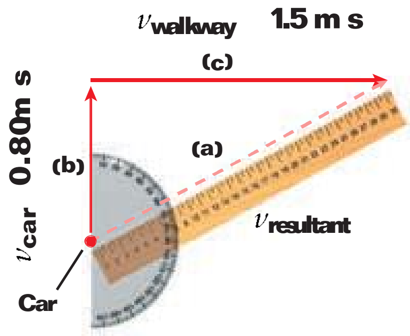

Figure 3
The resultant velocity (a) of a toy car moving at a velocity of 0.80 m s (b) across a moving walkway with a velocity of 1.5 m s (c) can be found using a ruler and a protractor.

Vectors can be moved parallel to themselves in a diagram

Note that the car s resultant velocity while moving from one side of the walkway to the other will be the combination of two independent motions. Thus, the moving car can be thought of as traveling first at 0.80 m s across the walkway and then at 1.5 m s down the walkway. In this way, we can draw a given vector anywhere in the diagram as long as the vector is parallel to its previous alignment (so that it still points in the same direction). Thus, you can draw one vector with its tail starting at the tip of the other as long as the size and direction of each vector do not change. This process is illustrated in Figure 3. Although both vectors act on the car at the same point, the horizontal vector has been moved up so that its tail begins at the tip of the vertical vector. The resultant vector can then be drawn from the tail of the first vector to the tip of the last vector. This method is known as the triangle (or polygon) method of addition.

Again, the magnitude of the resultant vector can be measured using a ruler, and the angle can be measured with a protractor. In the next section, we will develop a technique for adding vectors that is less time-consuming because it involves a calculator instead of a ruler and protractor.

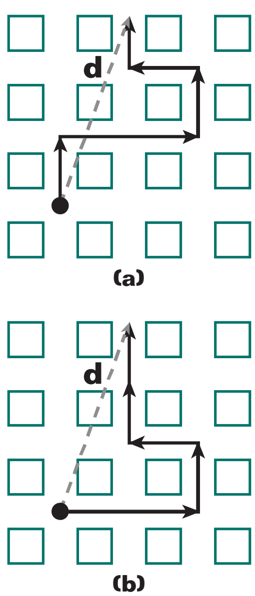

Figure 4
A marathon runner s displacement, d, will be the same regardless of whether the runner takes path (a) or (b) because the vectors can be added in any order.

Vectors can be added in any order

When two or more vectors are added, the sum is independent of the order of the addition. This idea is demonstrated by a runner practicing for a marathon along city streets, as represented in Figure 4. The runner executes the same four displacements in each case, but the order is different. Regardless of which path the runner takes, the runner will have the same total displacement, expressed as d. Similarly, the vector sum of two or more vectors is the same regardless of the order in which the vectors are added, provided that the magnitude and direction of each vector remain the same.

To subtract a vector, add its opposite

Vector subtraction makes use of the definition of the negative of a vector. The negative of a vector is defined as a vector with the same magnitude as the original vector but opposite in direction. For instance, the negative of the velocity of a car traveling 30 m s to the west is –30 m s to the west, or +30 m s to the east. Thus, adding a vector to its negative vector gives zero.

When subtracting vectors in two dimensions, first draw the negative of the vector to be subtracted. Then add that negative vector to the other vector by using the triangle method of addition.

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Multiplying or dividing vectors by scalars results in vectors

There are mathematical operations in which vectors can multiply other vectors, but they are not needed in this book. This book does, however, make use of vectors multiplied by scalars, with a vector as the result. For example, if a cab driver obeys a customer who tells him to go twice as fast, that cabs original velocity vector, v_{cab}, is multiplied by the scalar number 2. The result, written $2v_{cab}$, is a vector with a magnitude twice that of the original vector and pointing in the same direction.

On the other hand, if another cab driver is told to go twice as fast in the opposite direction, this is the same as multiplying by the scalar number 2. The result is a vector with a magnitude two times the initial velocity but pointing in the opposite direction, written as $2v_{cab}$.

SECTION REVIEW

1. Which of the following quantities are scalars, and which are vectors?
   a. the acceleration of a plane as it takes off
   b. the number of passengers on the plane
   c. the duration of the flight
   d. the displacement of the flight
   e. the amount of fuel required for the flight
2. A roller coaster moves 85 m horizontally, then travels 45 m at an angle of 30.0 above the horizontal. What is its displacement from its starting point? Use graphical techniques.
3. A novice pilot sets a planes controls, thinking the plane will fly at $2.50 \quad 10^2$ km h to the north. If the wind blows at 75 km h toward the southeast, what is the planes resultant velocity? Use graphical techniques.
4. While flying over the Grand Canyon, the pilot slows the plane down to one-half the velocity in item 3. If the winds velocity is still 75 km h toward the southeast, what will the planes new resultant velocity be? Use graphical techniques.
5. **Critical Thinking** The water used in many fountains is recycled. For instance, a single water particle in a fountain travels through an 85 m system and then returns to the same point. What is the displacement of this water particle during one cycle?

SECTION 2

Vector Operations

SECTION OBJECTIVES

- Identify appropriate coordinate systems for solving problems with vectors.
- Apply the Pythagorean theorem and tangent function to calculate the magnitude and direction of a resultant vector.
- Resolve vectors into components using the sine and cosine functions.
- Add vectors that are not perpendicular.

COORDINATE SYSTEMS IN TWO DIMENSIONS

In the chapter Motion in One Dimension, the motion of a gecko climbing a tree was described as motion along the y-axis. The direction of the displacement of the gecko was denoted by a positive or negative sign. The displacement of the gecko can now be described by an arrow pointing along the y-axis, as shown in Figure 5. A more versatile system for diagraming the motion of an object, however, employs vectors and the use of both the x- and y-axes simultaneously.

The addition of another axis not only helps describe motion in two dimensions but also simplifies analysis of motion in one dimension. For example, two methods can be used to describe the motion of a jet moving at 300 m s to the northeast. In one approach, the coordinate system can be turned so that the plane is depicted as moving along the y-axis, as in Figure 6(a). The jet s motion also can be depicted on a two-dimensional coordinate system whose axes point north and east, as shown in Figure 6(b).

One problem with the first method is that the axis must be turned again if the direction of the plane changes. Another problem is that the first method provides no way to deal with a second airplane that is not traveling in the same direction as the first airplane. Thus, axes are often designated using fixed directions. For example, in Figure 6(b), the positive y-axis points north and the positive x-axis points east.

Similarly, when you analyze the motion of objects thrown into the air, orienting the y-axis parallel to the vertical direction simplifies problem solving.

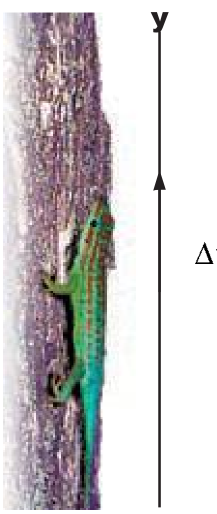


Figure 5
A gecko s displacement while climbing a tree can be represented by an arrow pointing along the y-axis.

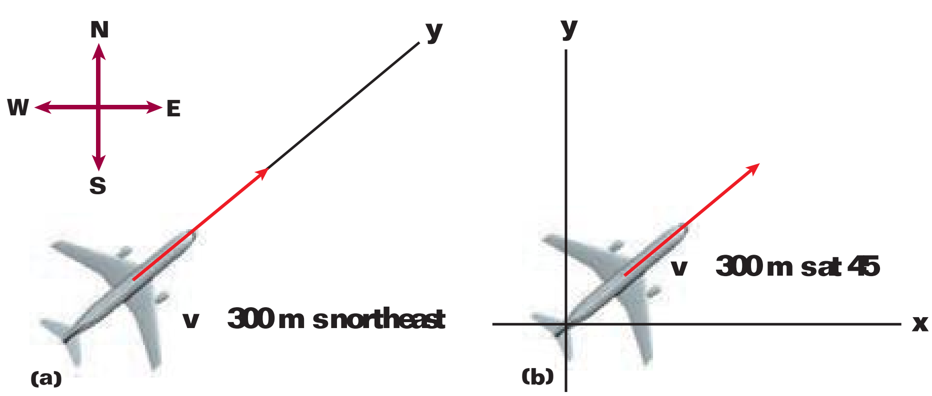


Figure 6
A plane traveling northeast at a velocity of 300 m s can be represented as either (a) moving along a y-axis chosen to point to the northeast or (b) moving at an angle of 45 to both the x- and y-axes, which line up with west-east and south-north, respectively.

TIP There are no firm rules for applying coordinate systems to situations involving vectors. As long as you are consistent, the final answer will be correct regardless of the system you choose. Perhaps your best choice for orienting axes is the approach that makes solving the problem easiest for you.

DETERMINING RESULTANT MAGNITUDE AND DIRECTION

In Section 1, the magnitude and direction of a resultant were found graphically. However, this approach is time consuming, and the accuracy of the answer depends on how carefully the diagram is drawn and measured. A simpler method uses the Pythagorean theorem and the tangent function.

Use the Pythagorean theorem to find the magnitude of the resultant

Imagine a tourist climbing a pyramid in Egypt. The tourist knows the height and width of the pyramid and would like to know the distance covered in a climb from the bottom to the top of the pyramid. Assume that the tourist climbs directly up the middle of one face.

As can be seen in Figure 7, the magnitude of the tourist s vertical displacement, Δy, is the height of the pyramid. The magnitude of the horizontal displacement, Δx, equals the distance from one edge of the pyramid to the middle, or half the pyramid s width. Notice that these two vectors are perpendicular and form a right triangle with the displacement, d.

As shown in Figure 8(a), the Pythagorean theorem states that for any right triangle, the square of the hypotenuse the side opposite the right angle equals the sum of the squares of the other two sides, or legs.

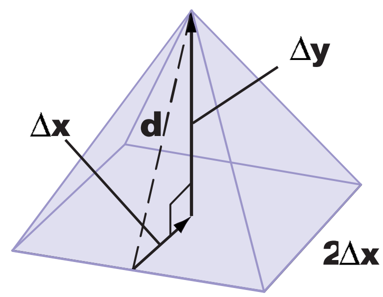


Figure 7
Because the base and height of a pyramid are perpendicular, we can find a tourist s total displacement, d, if we know the height, Δy, and width, $2\Delta x$, of the pyramid.

PYTHAGOREAN THEOREM FOR RIGHT TRIANGLES

$$c^2 = a^2 + b^2$$

$$(\text{length of hypotenuse})^2 = (\text{length of one leg})^2 + (\text{length of other leg})^2$$

In Figure 8(b), the Pythagorean theorem is applied to find the tourist s displacement. The square of the displacement is equal to the sum of the square of the horizontal displacement and the square of the vertical displacement. In this way, you can find out the magnitude of the displacement, d.

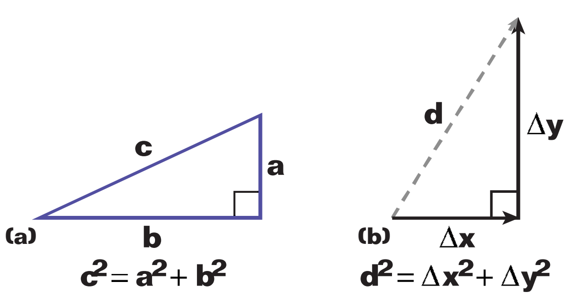
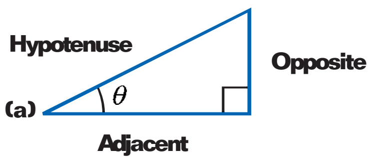
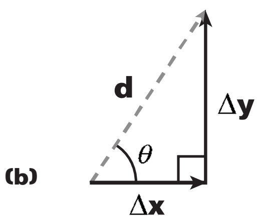


Figure 8
(a) The Pythagorean theorem can be applied to any right triangle. (b) It can also be applied to find the magnitude of a resultant displacement.

Use the tangent function to find the direction of the resultant



$\tan\theta = \frac{opp}{adj}$



$\tan\theta = \frac{\Delta y}{\Delta x}$

$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$

Figure 9
(a) The tangent function can be applied to any right triangle, and (b) it can also be used to find the direction of a resultant displacement.

In order to completely describe the tourist s displacement, you must also know the direction of the tourist s motion. Because Δx, Δy, and d form a right triangle, as shown in Figure 9(b), the inverse tangent function can be used to find the angle θ, which denotes the direction of the tourist s displacement.

For any right triangle, the tangent of an angle is defined as the ratio of the opposite and adjacent legs with respect to a specified acute angle of a right triangle, as shown in Figure 9(a).

As shown below, the magnitude of the opposite leg divided by the magnitude of the adjacent leg equals the tangent of the angle.

DEFINITION OF THE TANGENT FUNCTION FOR RIGHT TRIANGLES

$$\tan\theta = \frac{opp}{adj} \qquad \text{tangent of angle} = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

The inverse of the tangent function, which is shown below, gives the angle.

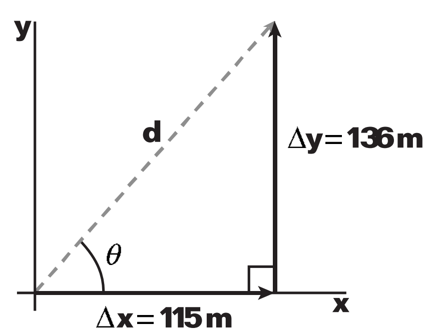
$$\theta = \tan^{-1}\left(\frac{opp}{adj}\right)$$

SAMPLE PROBLEM A

Finding Resultant Magnitude and Direction

PROBLEM

An archaeologist climbs the Great Pyramid in Giza, Egypt. The pyramid s height is 136 m and its width is 2.30×10^2 m. What is the magnitude and the direction of the displacement of the archaeologist after she has climbed from the bottom of the pyramid to the top?



SOLUTION

1. DEFINE

Given: $\Delta y = 136$ m $\qquad \Delta x = \frac{1}{2}(\text{width}) = 115$ m

Unknown: $d = ?$ $\qquad \theta = ?$

Diagram: Choose the archaeologist s starting position as the origin of the coordinate system.

2. PLAN

Choose an equation or situation:
The Pythagorean theorem can be used to find the magnitude of the archaeologist s displacement. The direction of the displacement can be found by using the tangent function.

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

Rearrange the equations to isolate the unknowns:

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$$

3. CALCULATE Substitute the values into the equations and solve:

$$d = \sqrt{(115\ m)^2 + (136\ m)^2}$$

$$\boxed{d = 178\ m}$$

$$\theta = \tan^{-1}\left(\frac{136\ m}{115\ m}\right)$$

$$\boxed{\theta = 49.8°}$$

TIP Be sure your calculator is set to calculate angles measured in degrees. Some calculators have a button labeled DRG that, when pressed, toggles between degrees, radians, and grads.

4. EVALUATE Because d is the hypotenuse, the archaeologist's displacement should be less than the sum of the height and half of the width. The angle is expected to be more than 45° because the height is greater than half of the width.

PRACTICE A

Finding Resultant Magnitude and Direction

1. A truck driver is attempting to deliver some furniture. First, he travels 8 km east, and then he turns around and travels 3 km west. Finally, he turns again and travels 12 km east to his destination.
   a. What distance has the driver traveled?
   b. What is the driver's total displacement?
2. While following the directions on a treasure map, a pirate walks 45.0 m north and then turns and walks 7.5 m east. What single straight-line displacement could the pirate have taken to reach the treasure?
3. Emily passes a soccer ball 6.0 m directly across the field to Kara. Kara then kicks the ball 14.5 m directly down the field to Luisa. What is the ball's total displacement as it travels between Emily and Luisa?
4. A hummingbird, 3.4 m above the ground, flies 1.2 m along a straight path. Upon spotting a flower below, the hummingbird drops directly downward 1.4 m to hover in front of the flower. What is the hummingbird's total displacement?

RESOLVING VECTORS INTO COMPONENTS

components of a vector

the projections of a vector along the axes of a coordinate system

In the pyramid example, the horizontal and vertical parts that add up to give the tourist s actual displacement are called **components.** The x component is parallel to the x-axis. The y component is parallel to the y-axis. Any vector can be completely described by a set of perpendicular components.

In this textbook, components of vectors are shown as outlined, open arrows. Components have arrowheads to indicate their direction. Components are scalars (numbers), but they are signed numbers, and the direction is important to determine their sign in a particular coordinate system.

You can often describe an object s motion more conveniently by breaking a single vector into two components, or *resolving* the vector. Resolving a vector allows you to analyze the motion in each direction.

This point may be illustrated by examining a scene on the set of a new action movie. For this scene, a biplane travels at 95 km h at an angle of 20 relative to the ground. Attempting to film the plane from below, a camera team travels in a truck that is directly beneath the plane at all times, as shown in Figure 10.

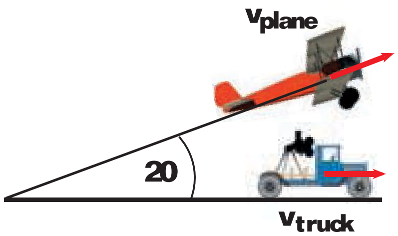


Figure 10
A truck carrying a film crew must be driven at the correct velocity to enable the crew to film the underside of a biplane. The plane flies at 95 km h at an angle of 20 relative to the ground.

To find the velocity that the truck must maintain to stay beneath the plane, we must know the horizontal component of the plane s velocity. Once more, the key to solving the problem is to recognize that a right triangle can be drawn using the plane s velocity and its x and y components. The situation can then be analyzed using trigonometry.

The sine and cosine functions are defined in terms of the lengths of the sides of such right triangles. The sine of an angle is the ratio of the leg opposite that angle to the hypotenuse.

DEFINITION OF THE SINE FUNCTION FOR RIGHT TRIANGLES

$$\sin q \quad \frac{opp}{hyp} \qquad \text{sine of an angle} \quad \frac{\text{opposite leg}}{\text{hypotenuse}}$$

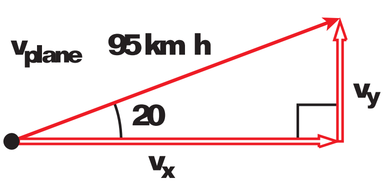


Figure 11
To stay beneath the biplane, the truck must be driven with a velocity equal to the x component (v_x) of the biplane s velocity.

In Figure 11, the leg opposite the 20 angle represents the y component, v_y, which describes the vertical speed of the airplane. The hypotenuse, v_{plane}, is the resultant vector that describes the airplane s total velocity.

The cosine of an angle is the ratio between the leg adjacent to that angle and the hypotenuse.

DEFINITION OF THE COSINE FUNCTION FOR RIGHT TRIANGLES

$$\cos q \quad \frac{adj}{hyp} \qquad \text{cosine of an angle} \quad \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

In Figure 11, the adjacent leg represents the x component, v_x, which describes the airplane s horizontal speed. This x component equals the speed that the truck must maintain to stay beneath the plane. Thus, the truck must maintain a speed of v_x (cos 20)(95 km h) 90 km h.

SAMPLE PROBLEM B

Resolving Vectors

PROBLEM

Find the components of the velocity of a helicopter traveling 95 km h at an angle of 35 to the ground.

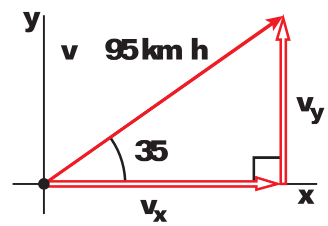
SOLUTION

1. DEFINE

Given: v 95 km h q 35

Unknown: v_x ? v_y ?

Diagram: The most convenient coordinate system is one with the x-axis directed along the ground and the y-axis directed vertically.



2. PLAN

Choose an equation or situation:
Because the axes are perpendicular, the sine and cosine functions can be used to find the components.

$$\sin q \quad \frac{v_y}{v}$$

$$\cos q \quad \frac{v_x}{v}$$

Rearrange the equations to isolate the unknowns:

$$v_y \quad v \sin q$$
$$v_x \quad v \cos q$$

TIP Don t assume that the cosine function can always be used for the x-component and the sine function can always be used for the y-component. The correct choice of function depends on where the given angle is located. Instead, always check to see which component is adjacent and which component is opposite to the given angle.

3. CALCULATE

Substitute the values into the equations and solve:

$$v_y \quad (95 \text{ km h})(\sin 35\ )$$

$$\boxed{v_y \quad 54 \text{ km h}}$$

$$v_x \quad (95 \text{ km h})(\cos 35\ )$$

$$\boxed{v_x \quad 78 \text{ km h}}$$

4. EVALUATE

Because the components of the velocity form a right triangle with the helicopter s actual velocity, the components must satisfy the Pythagorean theorem.

$$v^2 \quad v_x^2 \quad v_y^2$$
$$(95)^2 \quad (78)^2 \quad (54)^2$$
$$9025 \approx 9000$$

The slight difference is due to rounding.

PRACTICE B

Resolving Vectors

1. How fast must a truck travel to stay beneath an airplane that is moving 105 km h at an angle of 25 to the ground?
2. What is the magnitude of the vertical component of the velocity of the plane in item 1?
3. A truck drives up a hill with a 15 incline. If the truck has a constant speed of 22 m s, what are the horizontal and vertical components of the truck s velocity?
4. What are the horizontal and vertical components of a cat s displacement when the cat has climbed 5 m directly up a tree?

ADDING VECTORS THAT ARE NOT PERPENDICULAR

Until this point, the vector-addition problems concerned vectors that are perpendicular to one another. However, many objects move in one direction and then turn at an angle before continuing their motion.

Suppose that a plane initially travels 5 km at an angle of 35 to the ground, then climbs at only 10 relative to the ground for 22 km. How can you determine the magnitude and direction for the vector denoting the total displacement of the plane?

Because the original displacement vectors do not form a right triangle, you can not apply the tangent function or the Pythagorean theorem when adding the original two vectors.

Determining the magnitude and the direction of the resultant can be achieved by resolving each of the plane s displacement vectors into its x and y components. Then the components along each axis can be added together. As shown in Figure 12, these sums will be the two perpendicular components of the resultant, d. The resultant s magnitude can then be found by using the Pythagorean theorem, and its direction can be found by using the inverse tangent function.

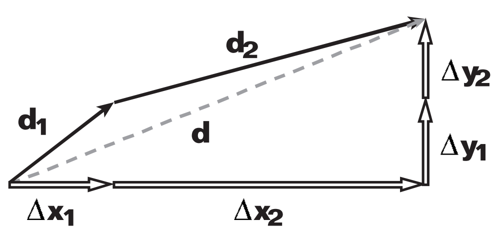


Figure 12
Add the components of the original displacement vectors to find two components that form a right triangle with the resultant vector.

SAMPLE PROBLEM C

STRATEGY Adding Vectors Algebraically

PROBLEM

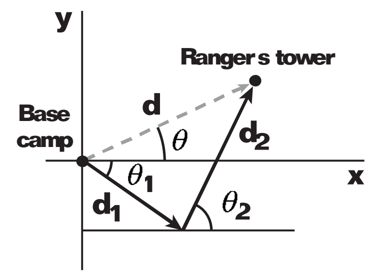
A hiker walks 27.0 km from her base camp at 35 south of east. The next day, she walks 41.0 km in a direction 65 north of east and discovers a forest ranger s tower. Find the magnitude and direction of her resultant displacement between the base camp and the tower.

SOLUTION

1. **Select a coordinate system. Then sketch and label each vector.**

Given: d_1 27.0 km q_1 35
d_2 41.0 km q_2 65

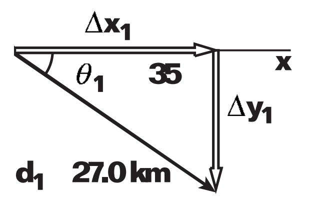
Unknown: d ? q ?



TIP: q_1 is negative, because clockwise angles from the positive x-axis are conventionally considered to be negative.

2. **Find the x and y components of all vectors.**
Make a separate sketch of the displacements for each day. Use the cosine and sine functions to find the displacement components.

$$\cos q \quad \frac{\Delta x}{d} \qquad \sin q \quad \frac{\Delta y}{d}$$



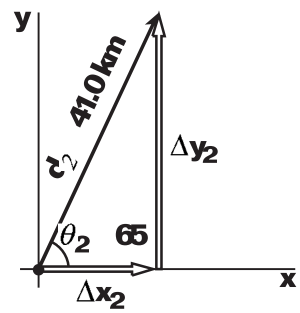
(a)

(a) For day 1: $\Delta x_1 \quad d_1 \cos q_1 \quad (27.0\text{ km}) \cos(35) \quad 22\text{ km}$

$\Delta y_1 \quad d_1 \sin q_1 \quad (27.0\text{ km}) \sin(35) \quad 15\text{ km}$

(b) For day 2: $\Delta x_2 \quad d_2 \cos q_2 \quad (41.0\text{ km}) (\cos 65) \quad 17\text{ km}$

$\Delta y_2 \quad d_2 \sin q_2 \quad (41.0\text{ km}) (\sin 65) \quad 37\text{ km}$



(b)

3. **Find the x and y components of the total displacement.**

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 \quad 22\text{ km} \quad 17\text{ km} \quad 39\text{ km}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 \quad 15\text{ km} \quad 37\text{ km} \quad 22\text{ km}$$

4. **Use the Pythagorean theorem to find the magnitude of the resultant vector.**

$$d^2 \quad (\Delta x_{tot})^2 \quad (\Delta y_{tot})^2$$

$$d \quad \sqrt{(\Delta x_{tot})^2 \quad (\Delta y_{tot})^2} \quad \sqrt{(39\text{ km})^2 \quad (22\text{ km})^2} \quad \boxed{45\text{ km}}$$

5. **Use a suitable trigonometric function to find the angle.**

$$q \quad \tan^{1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) \quad \tan^{1}\left(\frac{22\text{ km}}{39\text{ km}}\right) \quad \boxed{29 \text{ north of east}}$$

PRACTICE C

Adding Vectors Algebraically

1. A football player runs directly down the field for 35 m before turning to the right at an angle of 25° from his original direction and running an additional 15 m before getting tackled. What is the magnitude and direction of the runner s total displacement?

2. A plane travels 2.5 km at an angle of 35° to the ground and then changes direction and travels 5.2 km at an angle of 22° to the ground. What is the magnitude and direction of the plane s total displacement?

3. During a rodeo, a clown runs 8.0 m north, turns 55° north of east, and runs 3.5 m. Then, after waiting for the bull to come near, the clown turns due east and runs 5.0 m to exit the arena. What is the clown s total displacement?

4. An airplane flying parallel to the ground undergoes two consecutive displacements. The first is 75 km 30.0° west of north, and the second is 155 km 60.0° east of north. What is the total displacement of the airplane?

SECTION REVIEW

1. Identify a convenient coordinate system for analyzing each of the following situations:
   a. a dog walking along a sidewalk
   b. an acrobat walking along a high wire
   c. a submarine submerging at an angle of 30° to the horizontal

2. Find the magnitude and direction of the resultant velocity vector for the following perpendicular velocities:
   a. a fish swimming at 3.0 m s relative to the water across a river that moves at 5.0 m s
   b. a surfer traveling at 1.0 m s relative to the water across a wave that is traveling at 6.0 m s

3. Find the vector components along the directions noted in parentheses.
   a. a car displaced 45° north of east by 10.0 km (north and east)
   b. a duck accelerating away from a hunter at 2.0 m s^2 at an angle of 35° to the ground (horizontal and vertical)

4. **Critical Thinking** Why do nonperpendicular vectors need to be resolved into components before you can add the vectors together?