Determining the Acceleration of the Top of a Stretched Slinky and Allowed to Free Fall

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Abstract

A very interesting phenomenon occurs when a stretched slinky is released and free falls to the ground. The bottom of the slinky levitates because the force of gravity is balanced by an upward force of tension. The slinky does not fall until the top catches up, at which point the entire slinky falls to the floor. According to Newton's Laws of Gravity any object in free fall accelerates towards the center of the Earth with an acceleration of $g = -9.81\ \text{m/s}^2$. This experiment hypothesizes that the top of the slinky will accelerate at g no matter its stretch length. Two videos were taken, one that allows the slinky to stretch to equilibrium under its own weight and the other has a 1.0 kg mass connected to the bottom of the slinky which gives it a greater fall distance. The videos were analyzed with the Tracker Video Analysis program which allows for detailed position-time and vertical velocity-time analyses. In an effort to have statistically relevant results the top of the slinky in each video was tracked four separate times which allowed for the use of a standard error of the mean analysis of the results.

Introduction and Hypothesis

A slinky suspended in the air by holding one end and letting gravity stretch the other end offers a look at a complex physics system (Figure 1). Upon its release the bottom on the slinky moves very little, levitates, until the top of the slinky reaches the equilibrium position, at which point the entire slinky falls to the floor. Even going so far as attaching a mass at the bottom of the slinky does not change that effect, it only acts to increase the stretched length of the slinky and how fast it appears to fall.

This experiment consists of the video analysis of two stretched slinkys. One is stretched by its own mass, and the other had a 1.0 kg mass attached at the bottom. It is hypothesized that the change in position with time of the top of the slinky, in both cases, will result in an acceleration downwards at the normal acceleration of gravity, $g = -9.81\ \text{m/s}^2$, in accordance with Newton's 2nd Law of Motion (and Laws of Gravity) that a constant force produces a constant acceleration. In this case the constant force is gravity and the constant acceleration is $-9.81\ \text{m/s}^2$.

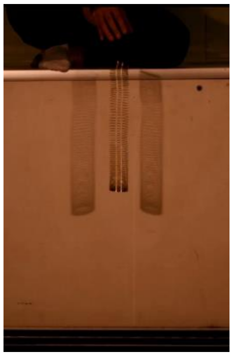


Figure 1. A hanging slinky stretched by gravity.

Materials and Programs

- 1 standard slinky
- 1 meter stick
- A 1.0 kg mass (that can attach to the slinky)
- Edgertronic slow motion camera
- Additional lighting (two Lowel Tota light sources)
- Tracker Video Analysis Software (×4)

Procedure

The materials were collected and the camera and lights were set up in the lab. The ideal place to film with the Edgertronic camera is outside in the sunlight, however, due to weather and time constraints the video was recorded indoors. The slinky was stretched and released with all of its motion resulting from the force of gravity. That motion was recorded with the Edgertronic slow motion camera. The camera was set to record at 700 fps with a 1/700 second shutter speed. Those settings allowed for clear, near high definition, video recording. The slinky was stretched again, but this time a 1.0 kg mass was attached at the bottom and everything was position so that the mass was a few inches above the floor. The longer, stretched slinky, was released and recorded with the same settings as the slinky with no attached mass. The videos were both saved to a computer and the Tracker Video Analysis Software was used to analyse the motion of the slinkys. As precise as possible, the Edgertronic camera was positioned at 90° to the slinky's direction of motion. An angle other than 90° would result in a parallax effect and only measure a component of the slinky's motion and not its full magnitude.

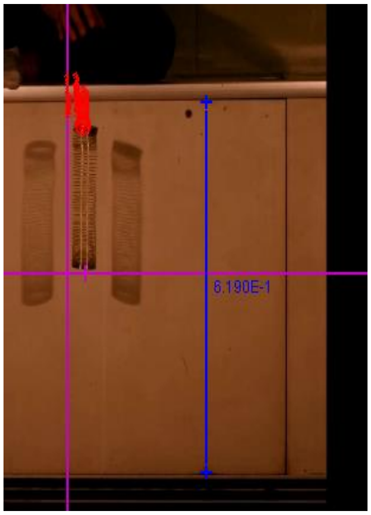


Figure 2: A screen shot from the Tracker Video Analysis program showing the coordinate system (crossed lines on the left) and the calibration stick (vertical line on the right). The markings above the slinky are the tracks made by the program.

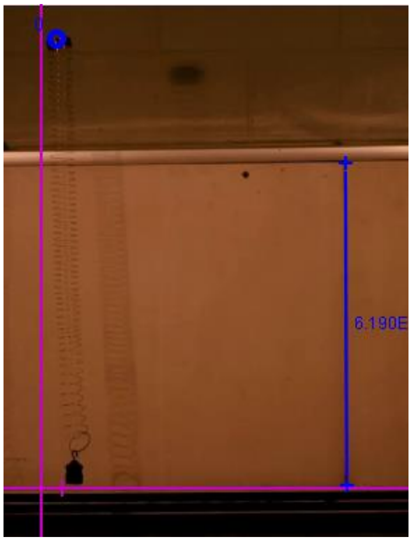


Figure 3: Similar to Figure 2, but with the 1.0 kg mass attached to the bottom of the slinky. Note the longer stretch length at that the mass is a few inches above the floor.

The Tracker program is an Open Source Physics computer program that will track an object in a video and allow for a detailed analysis of the object's motion (for more information on the Tracker program follow the link to the website: http://physlets.org/tracker/). Once the video has been uploaded in Tracker, the program must be calibrated before analysis can occur. The calibration process involved setting up a coordinate system, configuring the video playback settings (to tell the program that the video was recorded at 700 fps), and a calibration stick. Refer to Figures 2 and 3 for screen shots of the tracker program detailing the coordinate system and calibration stick for each video. The origin of the coordinate system is set near the bottom left of the slink for each analysis (doing this allows for an analysis of how far the top of the slinky is from the bottom of the slinky or, as in the second video, the bottom of the attached mass). For accurate results the calibration stick must measure a real distance as accurately as possible. Here, the height of the heater was measured with a meter stick to be 0.619 m and that value was used for the calibration stick. The entire tracking process was repeated four times for each video to allow for a meaningful statistical analysis.

The Data

All of the measurements for each video came from the position-time analysis of the top of the slinky as it fell. Recall that the position-time analysis was repeated three additional times for each video in an effort to provide an error analysis. Position was tracked directly with its value determined by the calibration stick and location of the coordinate system. The time is determined by knowing the video recording rate (frames per second). The program calculates the time between video frames as 1/fps, or 1/700 seconds. All of the slinky's motion is in the vertical, y, direction so only the vertical position-time graphs are relevant. Figure 4 shows the position-time data (for the first of four analyses) for the slinky that was freely suspended and released (no attached mass) while Figure 5 is the data (also for the first of four analyses) for the slinky with a 1.0 kg mass attached at the bottom. For the analyses, the x-axis (where $y = 0$) is set to be close to the bottom of the slinky (Figure 2) when no mass is attached, and near the bottom of the hanging mass (Figure 3) once it has been attached. For reference the slinky has a default length of 0.15 m and a mass of 0.75 kg.

The goal of this experiment was to determine the acceleration of the top of each slinky. Each position-time analysis appears to relate to a constant vertical velocity, however, the vertical velocity-time, v_y, data are relevant and should be discussed. The vertical velocity data for the first (of four) analysis of the mass-free and the mass-attached slinkys are presented in Figures 6 and 7, respectively (each graph was given the same vertical scale to easily show the effects on vertical velocity of attaching the mass to the slinky).

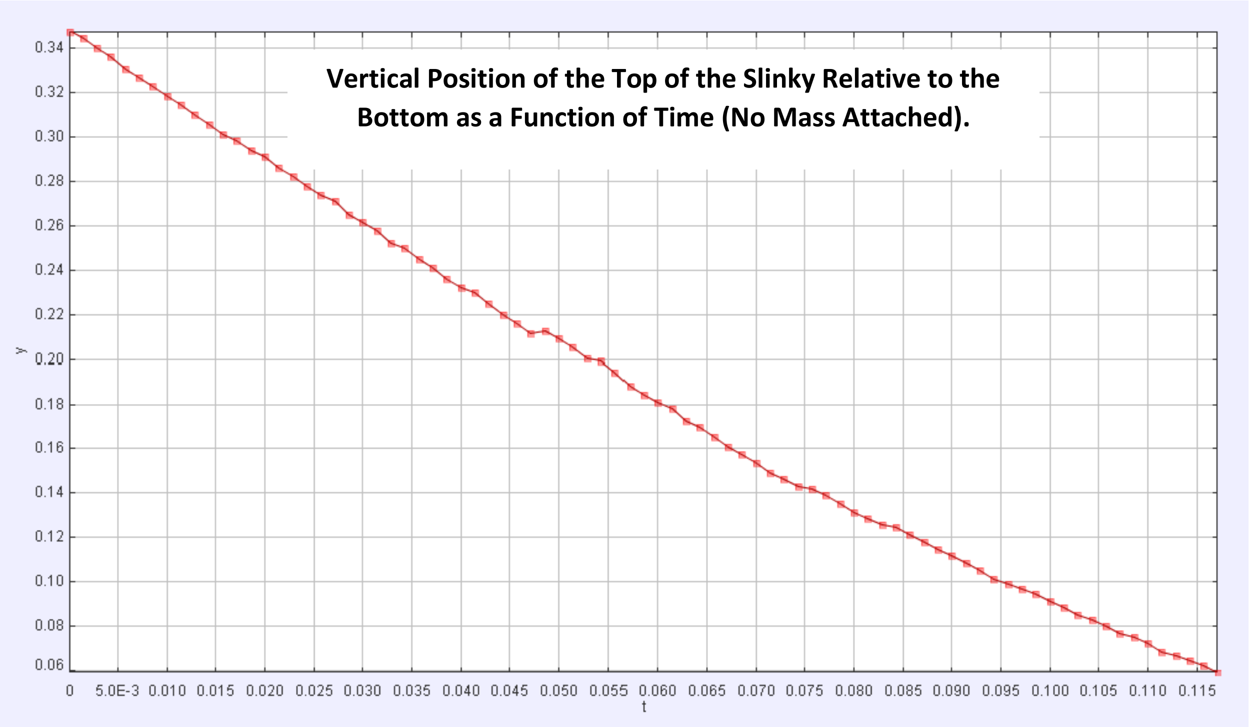


Figure 4: The position-time analysis of top of the falling slinky that does not have a mass attached. The vertical axis, y, measures height in meters above the bottom of the slinky. The horizontal axis measures time in seconds.

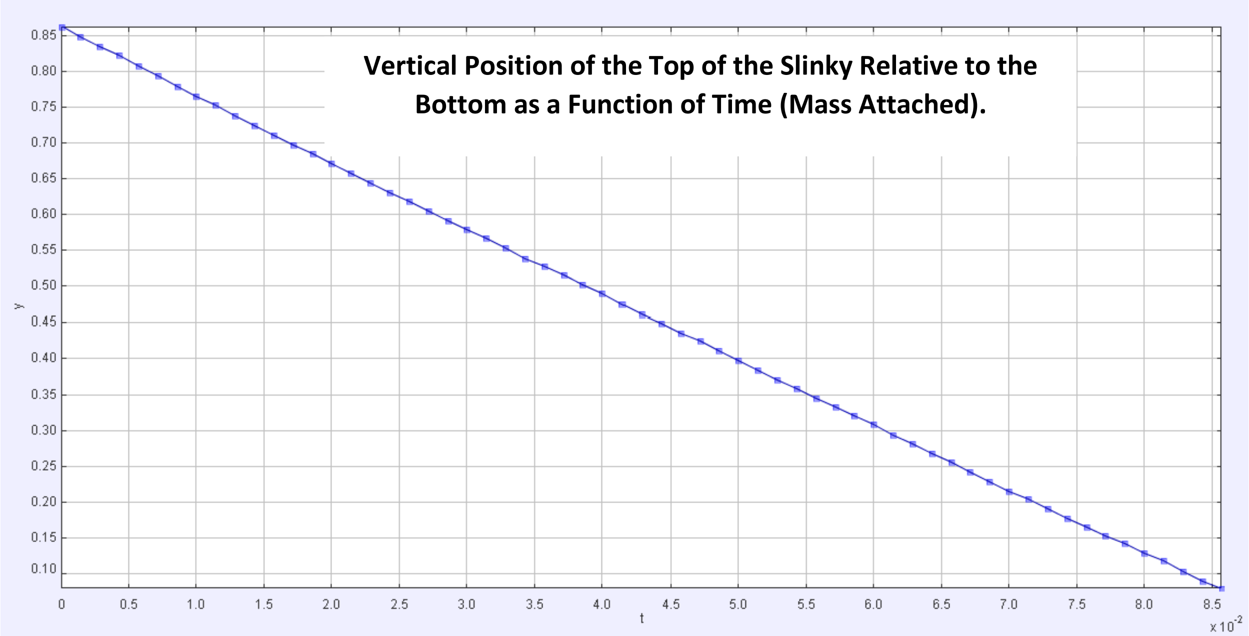


Figure 5: The position-time analysis of top of the falling slinky that has a mass attached. The vertical axis, y, measures height in meters above the bottom of the slinky. The horizontal axis measures time in seconds (note each time value is scaled $\times 10^{-2}$). The other three analyses produced similar graphical results.

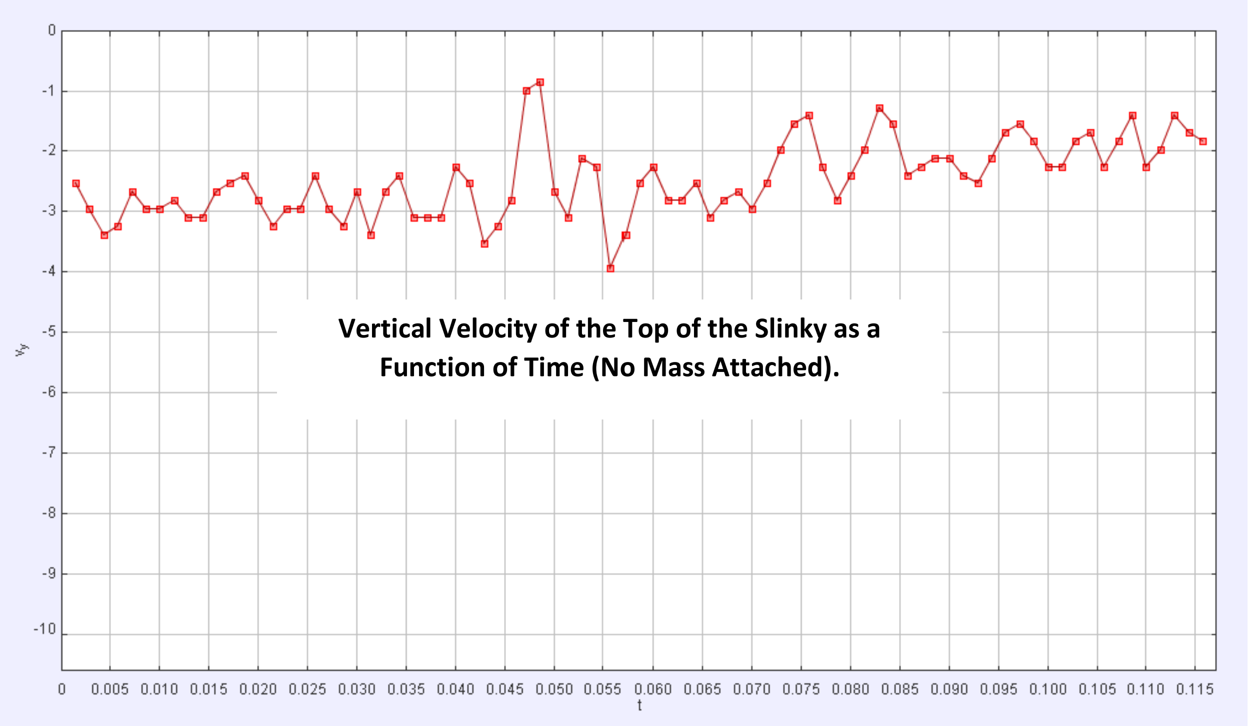


Figure 6: This graph shows the vertical velocity (m/s), v_y, as a function of time (s) for the mass-free slinky.

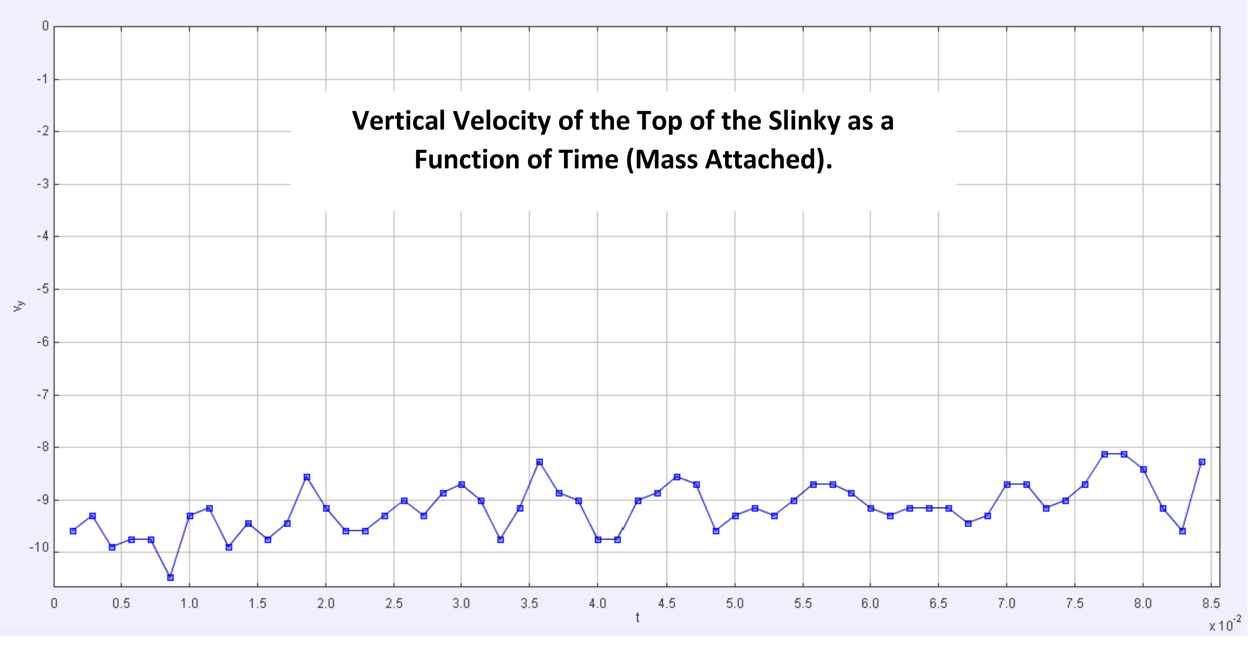


Figure 7: This graph shows the vertical velocity (m/s), v_y, as a function of time (s) for the slinky with an attached mass. The other three analyses produced similar graphs. Note again that the time axis is scaled $\times 10^{-2}$. The additional three analyses produced similar results.

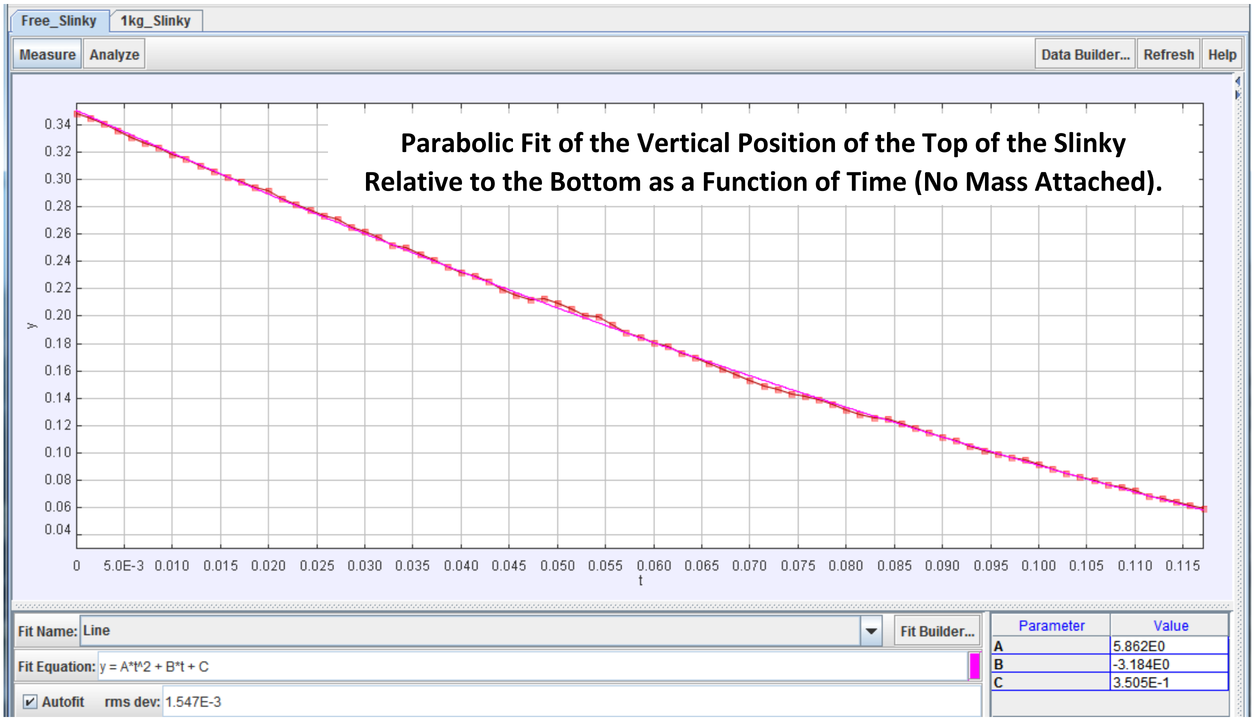
Analysis

The bulk of the following analysis section of this report is for the first data analysis of each video. It is meant as a guide through how and why each analysis was completed. A summary of all four analyses of each video can be found at the end of this section in Tables 1 – 4. Due to there being three significant digits used for the calibration stick (from measuring the heater height with a meter stick), each number reported here will contain three significant digits as well.

At first glance the position-time data from Figure 4 appear to immediately contradict the hypothesis that the top of the slinky accelerates at –9.81 m/s^2. An object with a constant acceleration will be represented by a non-linear, parabolic, math function. The data in Figure 4 appear linear, which implies that the slinky fell with a constant velocity as opposed to a constant acceleration. The vertical velocity data in Figure 6, however, clearly shows what is happening with respect to the motion of the top of the slinky.

The data in Figure 6, although more scattered, does show a trend that the magnitude of the vertical velocity (the speed) increases as the slinky moved downward towards the bottom. Interestingly, although falling downward, there appears to be a positive change in velocity. This is a very surprising and an unforeseen result suggesting that a suspended and dropped slinky is a more complex problem than previously thought.

Now that it is clear the object is accelerating, that acceleration needs to be determined from the relevant data. Figure 8 shows a parabolic function fitting the position-time data. The resulting fit generates the equation $y = 5.86t^2 – 3.18t + 0.351$. Mathematically, twice the coefficient of the quadratic term yields the acceleration of the object (the math reason for this will become evident upon taking a high school calculus course). That values gives an acceleration of +11.7 m/s^2 and note the positive value communicates an



Fit Name: Line

Fit Equation: y = A*t^2 + B*t + C

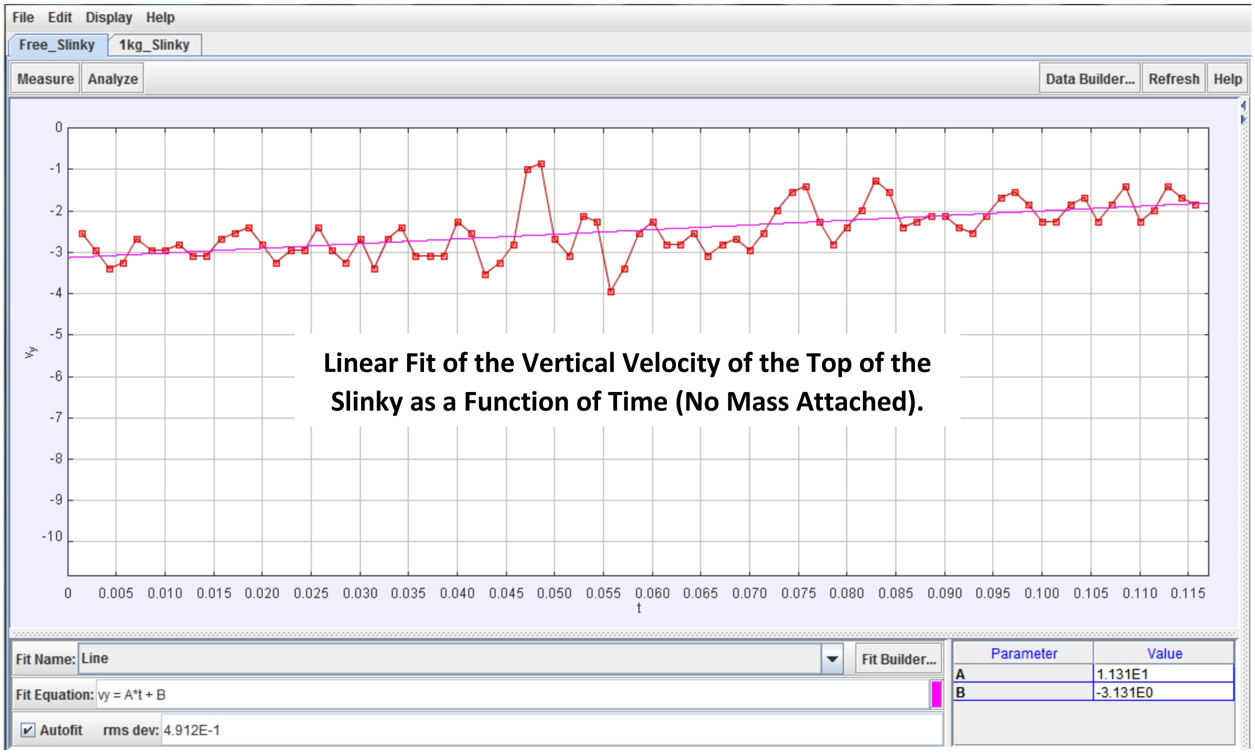
☑ Autofit rms dev: 1.547E-3

Parameter	Value
A	5.862E0
B	-3.184E0
C	3.505E-1

Figure 8: A parabolic function fitting using the Tracker program. The fit yields the equation $y = 5.86t^2 - 3.18t + 0.351$. The important number is the coefficient of the quadratic term as it relates to the acceleration of the top of the slinky.

upwards direction. It is important to remember that it is not the motion of the entire slinky that was measured, but rather just the top. This results suggests that the slinky is dynamic in its overall motion.

Further support that the top of the slinky is accelerating is the analysis of the vertical velocity-time data that was shown in Figure 6. Although an argument could be made that the v_y-t data could be fit with a nonlinear function (another parabola, for example), for the sake of simplicity, and because of the strong position-time fit to a parabola, a line-of-best-fit is applied here. The linear fit is shown in Figure 9. The resulting fit equation is $v_y = 11.3t - 3.13$. This is the physics formula for instantaneous velocity for an object moving with a constant velocity $v_f = at + v_o$. From this the average acceleration is +11.3 m/s^2 and is in strong agreement with the position-time analysis that yielded an average acceleration of +11.7 m/s^2.



Fit Name: Line

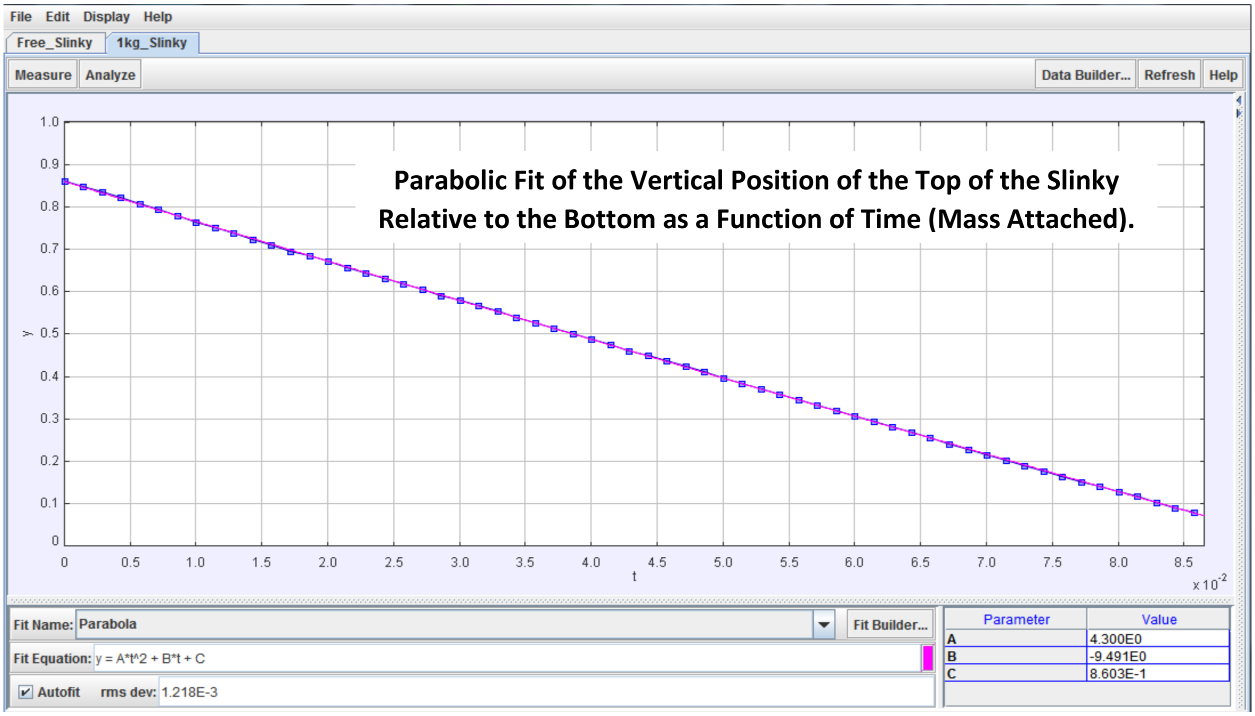
Fit Equation: vy = A*t + B

☑ Autofit rms dev: 4.912E-1

Parameter	Value
A	1.131E1
B	-3.131E0

Figure 9: A linear fit to the vertical velocity - time data resulted in an average acceleration of +11.3 m/s^2. That is in excellent agreement with the position-time analysis from Figure 8.

At this point the hypothesis of a downward acceleration of –9.81 m/s^2 for the top of the slink is already shown to be false, however, the analysis of the slinky with the mass attached shows additional, interesting results. The position-time data appears to follow a linear mathematical relationship, however, a parabolic fit was applied to be consistent with the analysis performed for the mass-free slinky. It is believed that the appearance of linearity is from tracking only a small part of the parabolic data and that should a much larger slinky be used a more visual parabolic function would emerge. The parabolic fit of the position-time data is shown in Figure 10. The fit equation is $y = 4.30t^2 - 9.49t + 0.860$ and yields an acceleration of 8.60 m/s^2.



Fit Name: Parabola

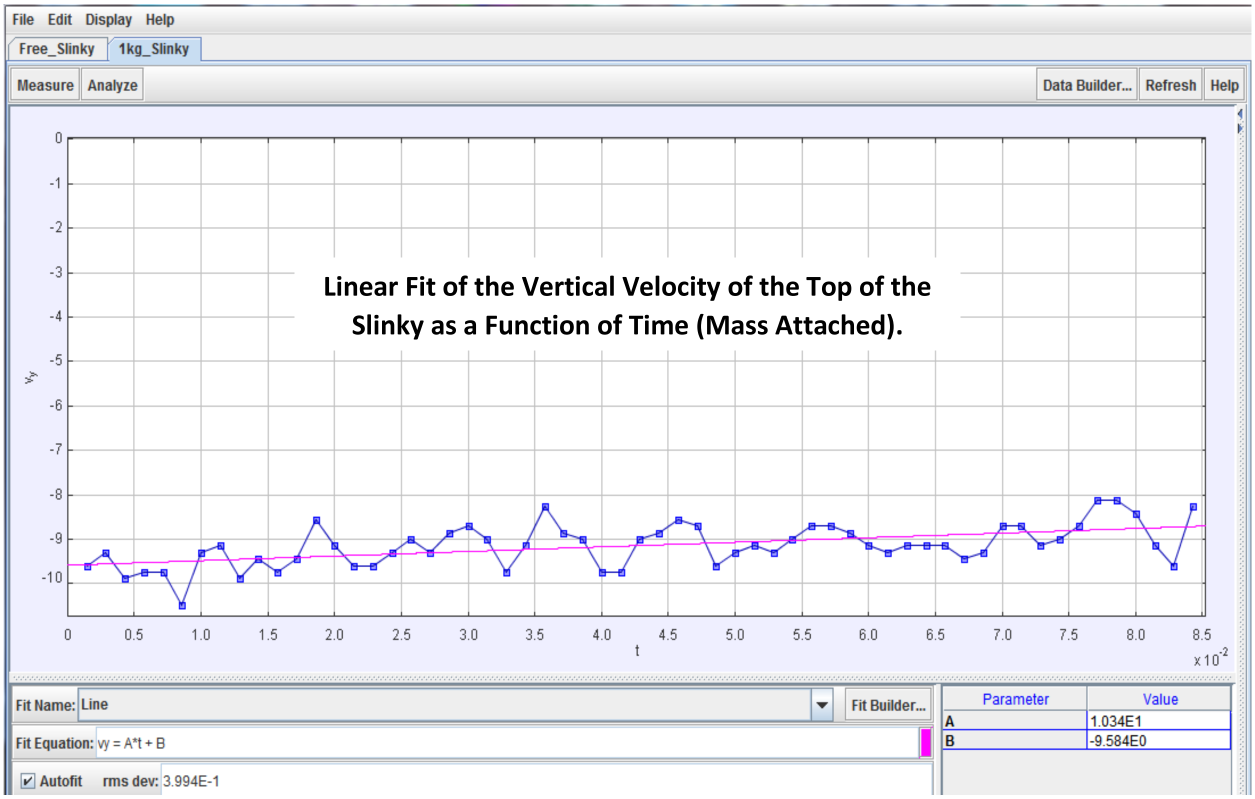
Fit Equation: y = A*t^2 + B*t + C

☑ Autofit rms dev: 1.218E-3

Parameter	Value
A	4.300E0
B	-9.491E0
C	8.603E-1

Figure 10: The parabolic fit of the vertical velocity-time data when the mass was attached to the slinky. The resulting equation is $y = 4.30t^2 - 9.49t + 0.860$. This gives an average acceleration of +8.60 m/s^2.

The vertical velocity-time graph for the slinky with the attached mass does not yield a strong agreement for acceleration as did the mass-free slinky. Figure 11 shows the v_y-t line-of-best-fit analysis that gives the formula $v_y = 10.3t - 9.58$. The average acceleration works out to be +10.3 m/s^2. This is not in good agreement with the parabolic fit of position-time unlike the case of the mass-free slinky. This could be the result of inaccurately assuming the acceleration is constant during the fall of the slinky. However, the average acceleration in both slinky drop videos are close to the same value. Recall that for the mass-free slinky the average acceleration was +11.3 m/s^2, this is interesting and suggests additional research should be undertaken, but this could still be coincidence. A summary of every video analysis can be found in Tables 1 – 4.



Linear Fit of the Vertical Velocity of the Top of the Slinky as a Function of Time (Mass Attached).

Fit Name: Line

Fit Equation: vy = A*t + B

☑ Autofit rms dev: 3.994E-1

Parameter	Value
A	1.034E1
B	-9.584E0

Figure 11: A linear fit to the vertical velocity - time data when the mass was attached resulted in an average acceleration of +10.34 m/s^2. That is in weak agreement with the position-time analysis from Figure 10, however, it is in a stronger agreement with the v_y-t analysis in Figure 9.

Repeating the analysis of each video four times allows for statistically significant results. The acceleration determined by each graphical method provides support that for the mass-free slinky it had an average acceleration of 10.3 m/s^2 and for the mass-attached slinky an average acceleration of 7.27 m/s^2.

Mass-Free Slinky Position – Time Analysis		
Analysis	**Acceleration (m/s^2)**	**RMS Dev (m/s^2)**
1	11.7	0.00155
2	10.3	0.00107
3	10.9	0.00141
4	9.17	0.00111
Mean	**10.5**	**0.00128**
SEM	**5.08 %**	

Table 1: A summary of the four independent analyses of position – time for the same video where there is no mass attached to the slinky. The RMS Dev values are low which supports the idea that the data fits a parabolic function. SEM is the "Standard Error of the Mean" that measures how the mean varies with each Tracker analysis that attempts to measure that same quantity. Here the SEM is 5.08 %.

Mass-Free Slinky Vertical Velocity – Time Analysis		
Analysis	**Acceleration (m/s^2)**	**RMS Dev (m/s^2)**
1	11.3	0.491
2	10.2	0.326
3	9.64	0.427
4	9.10	0.290
Mean	**10.0**	**0.383**
SEM	**4.70 %**	

Table 2: A Summary of the four independent analyses of vertical velocity – time for the same video (mass-free slinky). The RMS Dev values are low enough to support that the data fits a linear function. The standard error of the mean was 4.70 %.

Mass-Attached Slinky Position – Time Analysis		
Analysis	**Acceleration (m/s^2)**	**RMS Dev (m/s^2)**
1	8.600	0.00122
2	6.326	0.000729
3	6.266	0.000951
4	7.126	0.000778
Mean	**7.08**	**0.000920**
SEM	**7.68 %**	

Table 3: A summary of four independent analyses of position – time for the same video where the 1.0 kg mass was attached to the slinky. The RMS Dev values are low enough to support the idea that the data fits a parabolic function. The SEM value was 7.08 %, a notably higher error than this mass-free video.

Mass-Attached Slinky Vertical Velocity – Time Analysis		
Analysis	**Acceleration (m/s^2)**	**RMS Dev (m/s^2)**
1	10.3	0.399
2	5.99	0.306
3	5.87	0.426
4	7.66	0.373
Mean	**7.46**	**0.376**
SEM	**14.0 %**	

Table 4: A Summary of the four independent analyses of vertical velocity – time for the same video (mass-attached slinky). The RMS Dev values are low enough to support that the data fits a linear function. The standard error of the mean was 14.0 %, remarkably higher than all other analyses.

The results in Tables 1 – 4 provides further evidence contradicting the hypothesis that the top of each slinky has the same acceleration. In fact, it would appear that the addition of the mass changes the slinky system significantly. Each video has something in common, however, that the bottom of the slinky does not start to fall until the slinky has returned to its original length.

Initially, these findings do not appear to support the Newton's Law of Gravity in which objects, near the surface of the Earth (neglecting air resistance) accelerate towards the center of the Earth with a magnitude of 9.81 m/s^2. Exploring this issue further lead to researching for an explanation as to why the slinkys appeared to break the laws of gravity. Shimon Kolkowitz analyzed the levitating slinky in great detail in 2007 and calculated that the center of mass of the slinky accelerates downward at 9.81 m/s^2 and thus the system still obeys Newton’s laws and the observations of Galileo. Kolkowitz notes that the center of gravity continuously changes its position between the top and bottom of the slinky so a more detailed mathematical analysis and model would be necessary to successfully measure such a quantity in an experimental setting.

Error Analysis

There are many ways errors and uncertainties can manifest in the data analysis. An effort has been made to reduce these errors as much as possible. Random errors such as weather conditions and equipment quality existed but are very small. The Edgertronic camera is new and contains high quality hardware and video processing software.

Systemic errors were the most significant. The most difficult part of taking the video was obtaining adequate light in the lab setting and having the video as focused as possible. Once the video was taken the analysis was prone to human error in setting the calibration stick and tracking the top of the slinky in each video. To obtain exact results in each of the four video analyses, one would have to click on the exact same pixel each time and have the exact same calibration stick length, a very improbable task.

In an effort to determine the overall statistical quality of the experiment a standard error of the mean (SEM) was calculated. An SEM is a sort of mean-of-means statistic that measures how the mean of the acceleration varies with each video analysis. Recall that each acceleration is obtained from a mathematical fitting of the data, which is a calculation of a mathematical mean of the data. For example, the line-of-best-fit results in the mean slope value for that group of data. That linear fit is repeated three additional times and each of those produce a mean slope value and because a mean of the slope means is calculated a SEM can be used. The SEM values are included in Tables 1 – 4 and vary from 4.70% to 14%. The smaller the SEM the stronger the data accurately represents what really happened to the slinky. The large SEM value for the vertical velocity – time analysis suggests to either repeat the analysis of that video or that the v_y-t data is not an appropriate method to determine the slinky’s acceleration. More than likely it is a matter of needing a greater number of video tracking repetitions as the other SEMs were all low.

Although not an error per se, the root-mean-square deviation (RMS Dev) was also included in Tables 1 – 4. The RMS Dev relates to how well the data fits the mathematical function calculated by the Tracker program to model the data. The smaller the number the stronger the fit. This experiment yielded very small RMS Dev for all the position-time parabolas and also low values for the vertical velocity-time linear fittings. All in all, the statistical and error analysis support the notion that the acceleration values determined from this experiment can be deemed accurate.

Conclusion

This experiment yielded unexpected and fascinating results. A slinky held from one end and allowed to stretch and was then released. The videos, captured at 700 fps with the Edgertronic slow motion camera, clearly showed the bottom of the slinky levitating until the top of the slinky fell all the way down and only then did the slinky fall to the floor. Very early in the data analysis it was evident that the hypothesis was not going to be supported.

The hypothesis was that the top of a slinky that is suspended freely and then suspended with a 1.0 kg mass will accelerated downward at the acceleration due to gravity of 9.81 m/s^2. Early in the analysis it appeared that the position-time data followed a linear relationship (which debunks any notion of an acceleration), however, the vertical velocity-time data showed a clear change in velocity trend in every video analysis. Interesting still was that the change in velocity was positive indicating that the top of the slinkys were experiencing an upward average acceleration even while the slinky fell to the ground. For the mass-free slinky the average acceleration was around 10.0 – 10.5 m/s^2 with a standard error of the mean within 4.70 % to 5.08 %. The SEM are due to the human element involved in tracking the slinkys while random and systemic errors are small. The low SEM indicated accuracy in the analysis. The mass-attached slinky was more prone to errors. Its average velocity was 7.08 – 7.46 m/s^2 with a SEM within 7.68 % - 14.0 %; the high SEM is an indication that additional analyses of the video may be necessary as one of the acceleration values was much greater than the rest.

According to Newton's Laws of Gravity and the observation of Galileo Galilee, all objects accelerate downward at 9.81 m/s^2, which did not match this analysis very well. Since the experimental errors are small, that lead to the notion that some understanding of the physics is missing. The paper by Shimon Kolkowitz in 2007 cleared up the confusion by determining that the center of mass of the slinky does, in fact, accelerate downward at the acceleration due to gravity. Since the center of mass varies in location as the slinky falls, a more elaborate mathematical model and experimental design are necessary to reproduce Kolkowitz's work from 2007. One thing is for certain, a very simple, everyday phenomenon can result in a complex application of the laws of physics.

References

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