e) $10^2 \div [10 \div (-2)]^2$

$10^2 \div [-5]^2$

$100 \div 25 = 4$

f) $[18 \div (-6)]^3 \times 2$

$[-3]^3 \times 2$

-27×2

$\boxed{= -54}$

2.4 Exponent Laws I

Connect

Patterns arise when we multiply and divide powers with the same base.

➤ To multiply $(-7)^3 \times (-7)^5$:

$$\begin{aligned}(-7)^3 \times (-7)^5 &= (-7)(-7)(-7) \times (-7)(-7)(-7)(-7)(-7)\\ &= (-7)(-7)(-7)(-7)(-7)(-7)(-7)(-7)\\ &= (-7)^8\end{aligned}$$

The base of the product is -7. The exponent is 8.
The sum of the exponents of the powers that were multiplied is $3 + 5 = 8$.
This relationship is true for the product of any two powers with the same base.
We use variables to represent the powers in the relationship:

Exponent Law for a Product of Powers

To multiply powers with the same base, add the exponents.

$a^m \times a^n = a^{m+n}$

The variable a is any integer, except 0.

The variables m and n are any whole numbers.

*The actual law goes in to more depth, but for our grade 9 class we will stick to the above law.

$$(2.5)^{21.8} \times (2.5)^{-16.2} = (2.5)^{5.6}$$

➤ To divide $8^7 \div 8^4$:

$$8^7 \div 8^4 = \frac{8^7}{8^4}$$

$$= \frac{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8}$$

$$= \frac{\cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times 8 \times 8 \times 8}{\cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1 \times \cancel{8}^1}$$

$$= \frac{8 \times 8 \times 8}{1}$$

$$= 8 \times 8 \times 8$$

$$= 8^3$$

So, $8^7 \div 8^4 = 8^3$

The base of the quotient is 8. The exponent is 3.
The difference of the exponents of the powers that were divided is $7 - 4 = 3$.
This relationship is true for the quotient of any two powers with the same base.

Exponent Law for a Quotient of Powers

To divide powers with the same base, subtract the exponents.

$a^m \div a^n = a^{m-n}$ $\quad m \geq n$

a is any integer, except 0; m and n are any whole numbers.

Example 2 Evaluating Expressions Using Exponent Laws

Evaluate.

a) $(-2)^4 \times (-2)^7$

b) $3^2 \times 3^4 \div 3^3$

Simplify first using the exponent laws.

a) The bases are the same. Add exponents.

$$\begin{aligned}(-2)^4 \times (-2)^7 &= (-2)^{(4+7)}\\ &= (-2)^{11}\\ &= -2048\end{aligned}$$

b) All the bases are the same so add the exponents of the two powers that are multiplied. Then, subtract the exponent of the power that is divided.

$$\begin{aligned}3^2 \times 3^4 \div 3^3 &= 3^{(2+4)} \div 3^3\\ &= 3^6 \div 3^3\\ &= 3^{(6-3)}\\ &= 3^3\\ &= 27\end{aligned}$$

Example 3 Using Exponent Laws and the Order of Operations

Evaluate.

a) $6^2 + 6^3 \times 6^2$

b) $(-10)^4[(-10)^6 \div (-10)^4] - 10^7$

A Solution

a) Multiply first. Add the exponents.

$$\begin{aligned}6^2 + 6^3 \times 6^2 &= 6^2 + 6^{(3+2)}\\ &= 6^2 + 6^5 && \text{Evaluate each power.}\\ &= 36 + 7776 && \text{Then add.}\\ &= 7812\end{aligned}$$

No shortcut

b) Evaluate the expression in the square brackets first. Divide by subtracting the exponents.

$$\begin{aligned}(-10)^4[(-10)^6 \div (-10)^4] - 10^7 &= (-10)^4[(-10)^{(6-4)}] - 10^7\\ &= (-10)^4(-10)^2 - 10^7 && \text{Multiply: add the exponents}\\ &= (-10)^{(4+2)} - 10^7\\ &= (-10)^6 - 10^7 && \text{Evaluate each power.}\\ &= 1\,000\,000 - 10\,000\,000 && \text{Then subtract.}\\ &= -9\,000\,000\end{aligned}$$

Practice Problems

Pages 76 and 77.

Questions:

$\underbrace{4, 5, 6,}_{\text{For Wed.}}$ 7, 8, 13aceg, 15.