UNIT 1

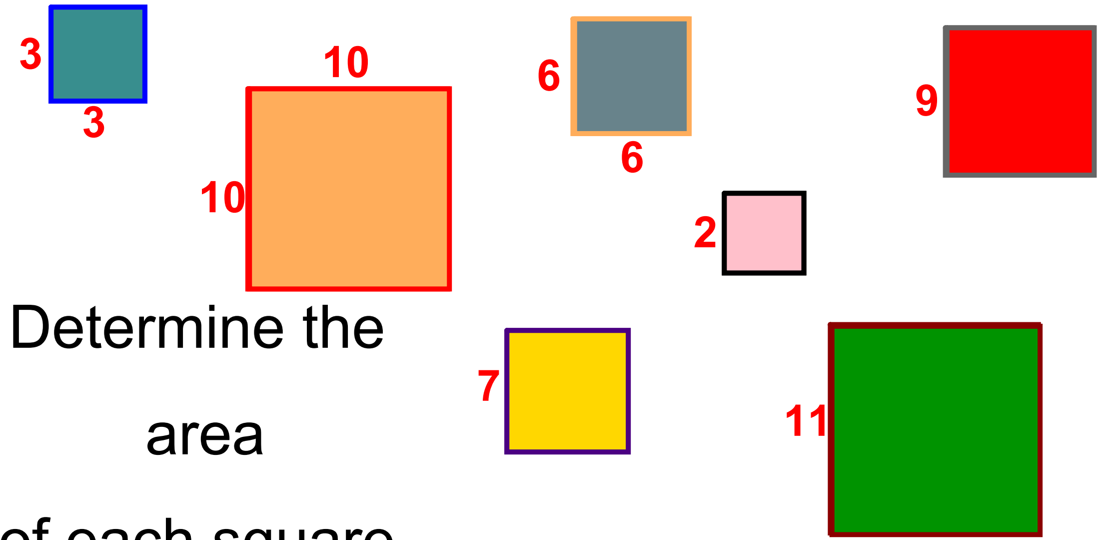
Square Roots and Surface Area

What You'll Learn

- Determine the square roots of fractions and decimals that are perfect squares.
- Approximate the square roots of fractions and decimals that are non-perfect squares.
- Determine the surface areas of composite 3-D objects to solve problems.

Why It's Important

Real-world measures are often expressed as fractions or decimals.
We use the square roots of these measures when we work with formulas such as the Pythagorean Theorem.
An understanding of surface area allows us to solve practical problems such as calculating:
the amount of paper needed to wrap a gift;
the number of cans of paint needed to paint a room;
and the amount of siding needed to cover a building



Determine the

area

of each square.

(Click to reveal area.)

1.1 Square Roots of Perfect Squares

A new parking lot is a square with an area of 900 m^2. What is the side length of the square?

Area of a Product	Side length as a Square Root
9	_____
16	_____
81	_____
49	_____
169	_____
_____	10
_____	15

You can calculate (or approximate) the square root of any real number. Crazy stuff happens when you try to square root numbers that are not real.

Try this: $\sqrt{4}$ =

What number multiplied by itself twice is four?

Example 1 Determining a Perfect Square Given its Square Root

Calculate the number whose square root is:

a) $\frac{3}{8}$ b) 1.8

A Solution

a) Visualize $\frac{3}{8}$ as the side length of a square.

The area of the square is: $\left(\frac{3}{8}\right)^2 = \frac{3}{8} \times \frac{3}{8}$

$= \frac{9}{64}$

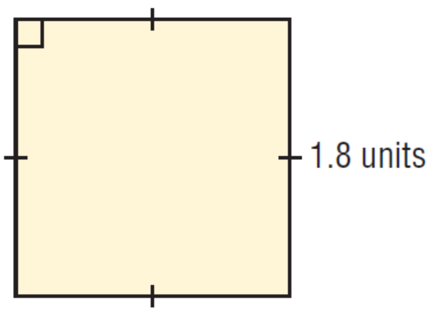
So, $\frac{3}{8}$ is a square root of $\frac{9}{64}$.

b) Visualize 1.8 as the side length of a square.

The area of the square is: $1.8^2 = 1.8 \times 1.8$

$= 3.24$

So, 1.8 is a square root of 3.24.



Important!

A fraction in simplest form is a **perfect square** if it can be written as a product of two equal fractions.

When a decimal can be written as a fraction that is a perfect square, then the decimal is also a perfect square. The square root is a terminating or repeating decimal.

Example 2 Identifying Fractions that Are Perfect Squares

Is each fraction a perfect square? Explain your reasoning.

a) $\frac{8}{18}$ b) $\frac{16}{5}$ c) $\frac{2}{9}$

A Solution

a) $\frac{8}{18}$

Simplify the fraction first. Divide the numerator and denominator by 2.

$\frac{8}{18} = \frac{4}{9}$

Since $4 = 2 \times 2$ and $9 = 3 \times 3$, we can write:

$\frac{4}{9} = \frac{2}{3} \times \frac{2}{3}$

Since $\frac{4}{9}$ can be written as a product of two equal fractions, it is a perfect square.

So, $\frac{8}{18}$ is also a perfect square.

b) $\frac{16}{5}$

The fraction is in simplest form.
So, look for a fraction that when multiplied by itself gives $\frac{16}{5}$.

The numerator can be written as $16 = 4 \times 4$, but the denominator cannot be written as a product of equal factors.

So, $\frac{16}{5}$ is not a perfect square.

c) $\frac{2}{9}$

The fraction is in simplest form.
So, look for a fraction that when multiplied by itself gives $\frac{2}{9}$.

The denominator can be written as $9 = 3 \times 3$, but the numerator cannot be written as a product of equal factors.

So, $\frac{2}{9}$ is not a perfect square.

Example 3 Identifying Decimals that Are Perfect Squares

Is each decimal a perfect square? Explain your reasoning.

a) 6.25 **b)** 0.627

Method 1

a) Write 6.25 as a fraction.

$6.25 = \frac{625}{100}$

Simplify the fraction. Divide the numerator and denominator by 25.

$6.25 = \frac{25}{4}$

$\frac{25}{4}$ can be written as $\frac{5}{2} \times \frac{5}{2}$.

So, $\frac{25}{4}$, or 6.25 is a perfect square.

Method 2

Use a calculator.

Use the square root function.

a) $\sqrt{6.25} = 2.5$

The square root is a terminating decimal so 6.25 is a perfect square.

b) Write 0.627 as a fraction.

$0.627 = \frac{627}{1000}$

This fraction is in simplest form. Neither 627 nor 1000 can be written as a product of equal factors, so 0.627 is not a perfect square.

b) $\sqrt{0.627} \doteq 0.791\ 833\ 316$

The square root appears to be a decimal that neither terminates nor repeats, so 0.627 is not a perfect square. To be sure, write the decimal as a fraction, then determine if the fraction is a perfect square, as shown in *Method 1*.

Practice Questions

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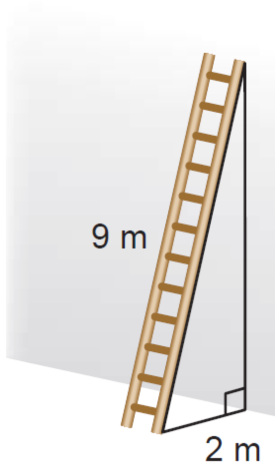
1.2 Square Roots of Non-Perfect Squares

A ladder is leaning against a wall.
For safety, the distance from the base of a ladder to the wall must be about $\frac{1}{4}$ of the height up the wall.
How could you check if the ladder is safe?

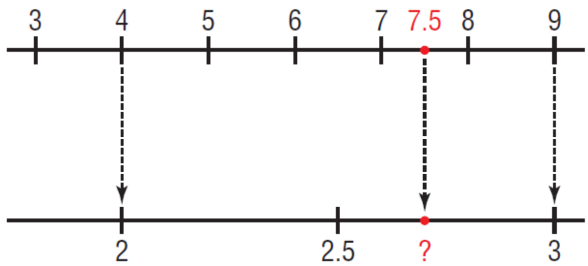


Connect

Many fractions and decimals are not perfect squares.
That is, they cannot be written as a product of two equal fractions.
A fraction or decimal that is not a perfect square is called a **non-perfect square**.

Here are two strategies for estimating a square root of a decimal that is a non-perfect square.

- Using benchmarks,
  To estimate $\sqrt{7.5}$, visualize a number line and the closest perfect square on each side of 7.5.
  $\sqrt{4} = 2$ and $\sqrt{9} = 3$
  7.5 is closer to 9 than to 4, so $\sqrt{7.5}$ is closer to 3 than to 2.
  From the diagram, an approximate value for $\sqrt{7.5}$ is 2.7.
  We write $\sqrt{7.5} \doteq 2.7$



- Using a calculator
  $\sqrt{7.5} \doteq 2.738\ 612\ 788$
  This decimal does not appear to terminate or repeat.
  There may be many more numbers after the decimal point that cannot be displayed on the calculator.
  To check, determine: $2.738\ 612\ 788^2 = 7.500\ 000\ 003$
  Since this number is not equal to 7.5, the square root is an approximation.

Example 1 Estimating a Square Root of a Fraction

Determine an approximate value of each square root.

a) $\sqrt{\frac{8}{5}}$ b) $\sqrt{\frac{3}{10}}$ c) $\sqrt{\frac{3}{7}}$ d) $\sqrt{\frac{19}{6}}$

▸ *A Solution*

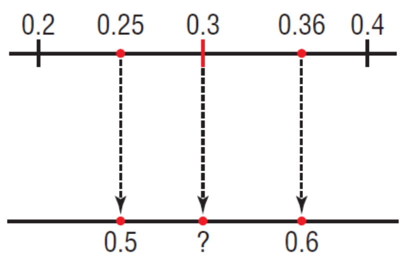
a) Use benchmarks. Think about the perfect squares closest to the numerator and denominator. In the fraction $\frac{8}{5}$, 8 is close to the perfect square 9, and 5 is close to the perfect square 4.

So, $\sqrt{\frac{8}{5}} \doteq \sqrt{\frac{9}{4}}$

$\sqrt{\frac{9}{4}} = \frac{3}{2}$

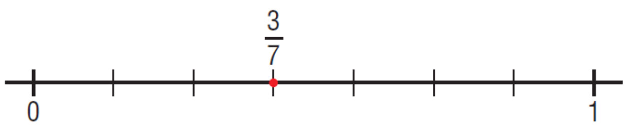
So, $\sqrt{\frac{8}{5}} \doteq \frac{3}{2}$

b) Write the fraction as a decimal, then think about benchmarks.
Write $\frac{3}{10}$ as a decimal: 0.3
Think of the closest perfect squares on either side of 0.3.
$\sqrt{0.25} = 0.5$ and $\sqrt{0.36} = 0.6$



0.3 is approximately halfway between 0.25 and 0.36, so choose 0.55 as a possible estimate for a square root.
To check, evaluate:
$0.55^2 = 0.3025$
0.3025 is close to 0.3, so 0.55 is a reasonable estimate.
So, $\sqrt{\frac{3}{10}} \doteq 0.55$

c) Choose a fraction close to $\frac{3}{7}$ that is easier to work with.



$\frac{3}{7}$ is a little less than $\frac{1}{2}$.

$\frac{1}{2} = 0.5$

$\sqrt{0.5} \doteq \sqrt{0.49}$

And, $\sqrt{0.49} = 0.7$

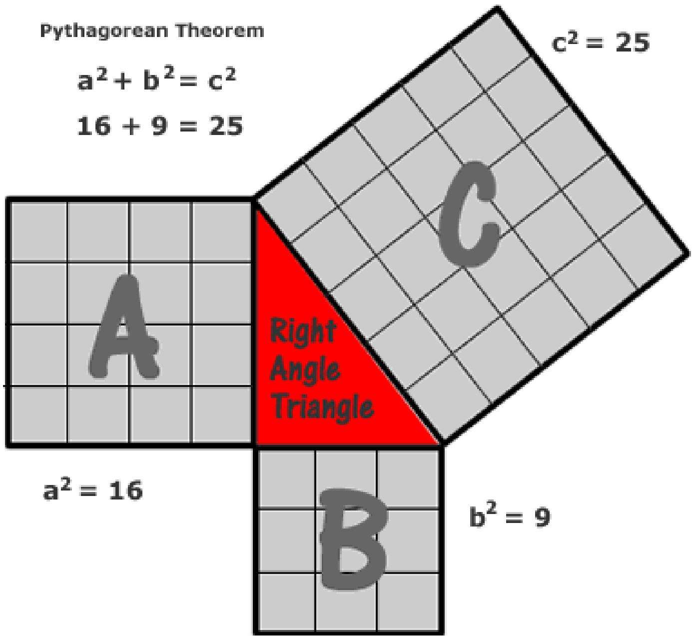
So, $\sqrt{\frac{3}{7}} \doteq 0.7$

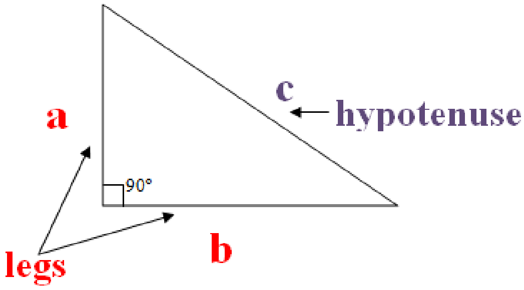
d) Use the square root function on a calculator.

$\sqrt{\frac{19}{6}} \doteq 1.779\ 513\ 042$

To the nearest hundredth, $\sqrt{\frac{19}{6}} \doteq 1.78$

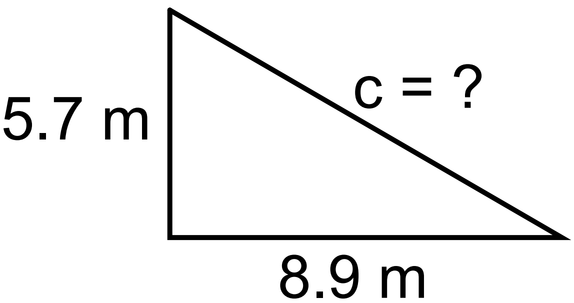
Applying Square Roots: Pythagorean Theorem

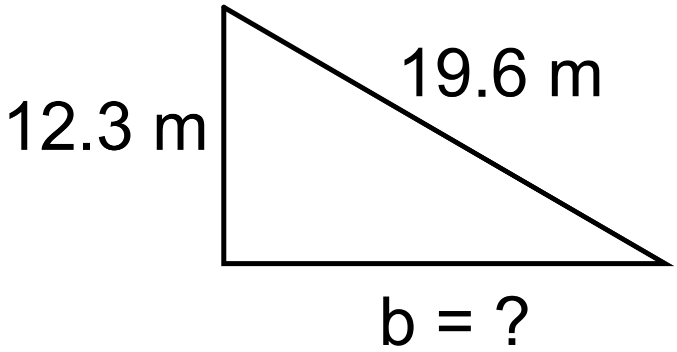


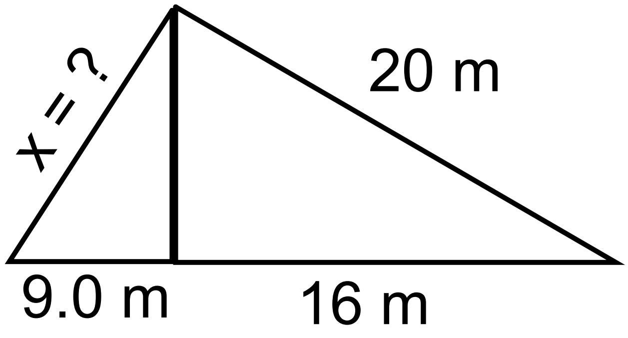


$a^2 + b^2 = c^2$

Examples




12.3 m
19.6 m
b = ?



Pythagorean Assignment

Example 3 Applying the Pythagorean Theorem

The sloping face of this ramp is to be covered in carpet.

a) Estimate the length of the ramp to the nearest tenth of a metre.

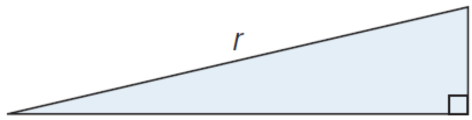
b) Use a calculator to check the answer.

c) Calculate the area of carpet needed.

A Solution

a) The ramp is a right triangular prism with a base that is a right triangle.
The base of the prism is its side view.
To calculate the length of the ramp, r, use the Pythagorean Theorem.

$$\begin{aligned} r^2 &= 6.5^2 + 1.5^2 \\ &= 42.25 + 2.25 \\ &= 44.5 \\ r &= \sqrt{44.5} \end{aligned}$$



44.5 is between the perfect squares 36 and 49, and closer to 49.
So, $\sqrt{44.5}$ is between 6 and 7, and closer to 7.
Estimate $\sqrt{44.5}$ as 6.7.
To check, evaluate: $6.7^2 = 44.89$
This is very close to 44.5, so $r \doteq 6.7$
The ramp is about 6.7 m long.

Since the dimensions of the ramp were given to the nearest tenth, the answer is also written in this form.

b) Use a calculator to check: $\sqrt{44.5} \doteq 6.670\,832\,032$
This number is 6.7 to the nearest tenth, so the answer is correct.

c) The sloping face of the ramp is a rectangle with dimensions 6.7 m by 2.2 m.
The area of the rectangle is about: $6.7 \times 2.2 = 14.74$
Round the answer up to the nearest square metre to ensure there is enough carpet.
So, about 15 m^2 of carpet are needed.

Practice Problems

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