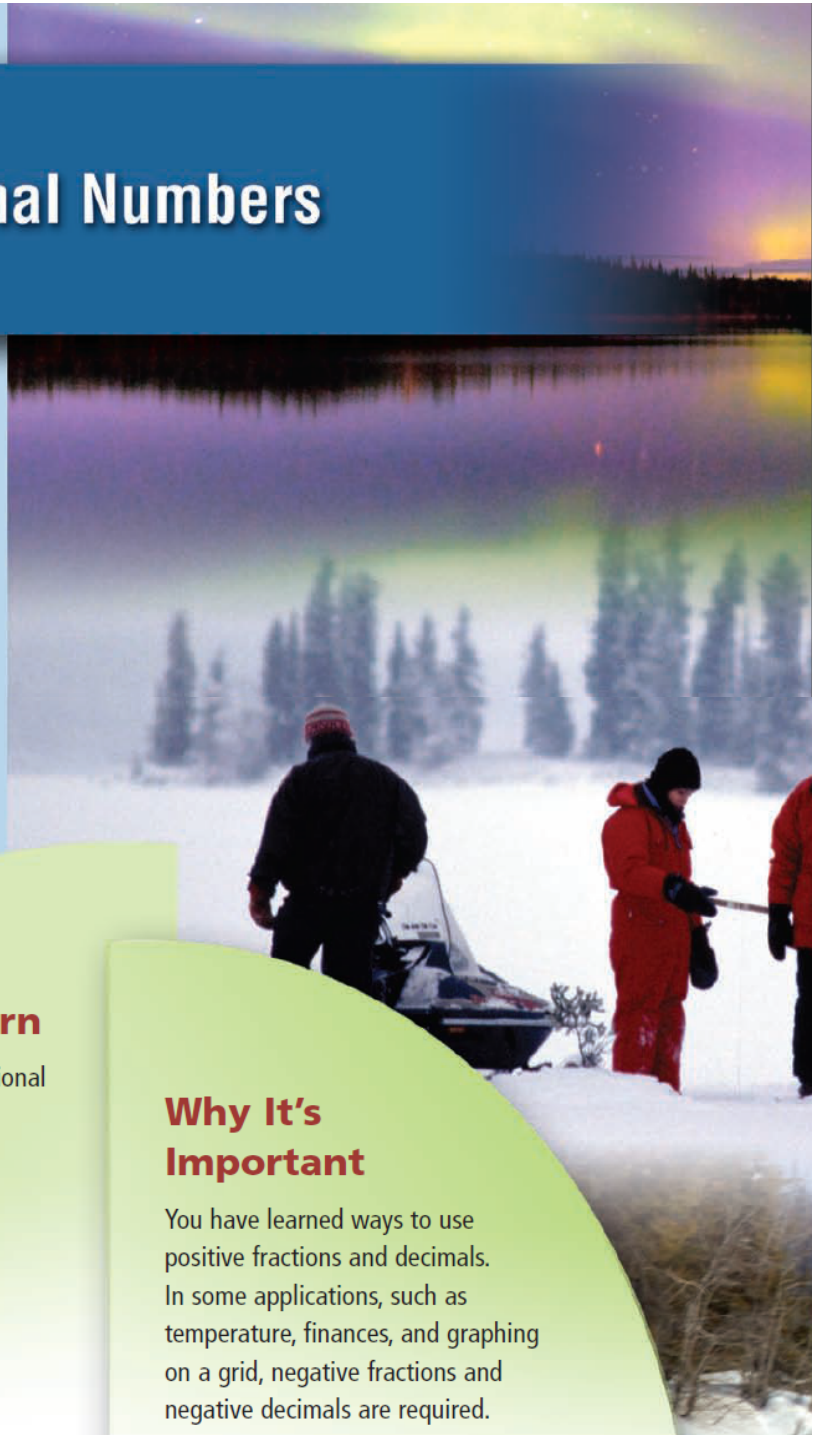**UNIT**

3

Rational Numbers

Suppose you are ice fishing on Blachford Lake, NWT.
The temperature at midnight is −12°C. At 6 A.M. the next day, the temperature is −11°C. What must the temperature have been at some time during the night?
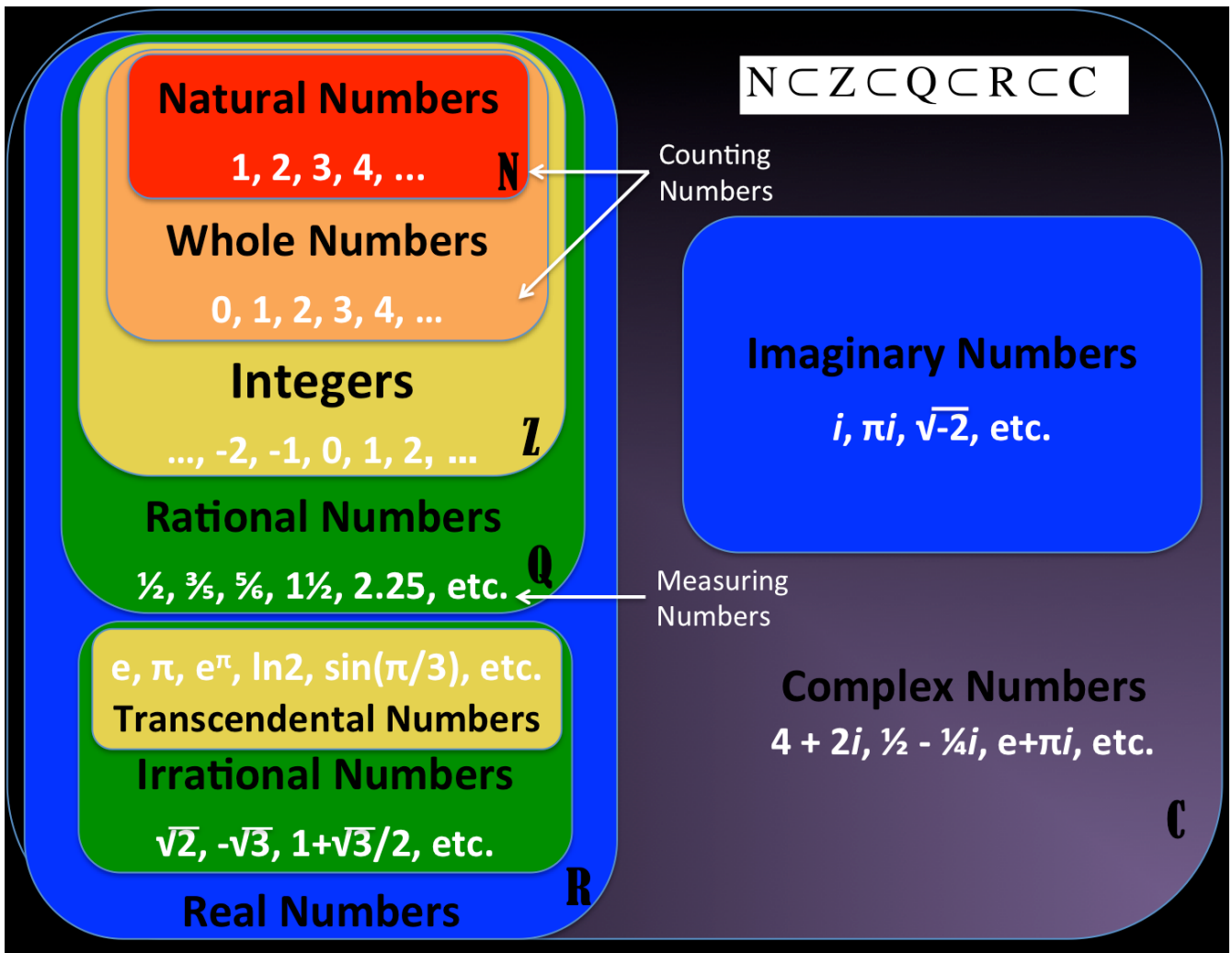
What You'll Learn

- Compare and order rational numbers.
- Solve problems by adding, subtracting, multiplying, and dividing rational numbers.
- Explain and apply the order of operations with rational numbers, with and without technology.

Why It's Important

You have learned ways to use positive fractions and decimals. In some applications, such as temperature, finances, and graphing on a grid, negative fractions and negative decimals are required.

Number Set in Mathematics

$$N \subset Z \subset Q \subset R \subset C$$

Natural Numbers

1, 2, 3, 4, ...   **N**

Whole Numbers

0, 1, 2, 3, 4, ...

Integers

..., -2, -1, 0, 1, 2, ...   **Z**

Rational Numbers

½, ⅗, ⅚, 1½, 2.25, etc.   **Q**

e, π, eᵖ, ln2, sin(π/3), etc.

Transcendental Numbers

Irrational Numbers

√2, -√3, 1+√3/2, etc.

Real Numbers   **R**

Counting
Numbers

Imaginary Numbers

i, πi, √-2, etc.

Measuring
Numbers

Complex Numbers

4 + 2*i*, ½ - ¼*i*, e+π*i*, etc.

C

3.1   What Is a Rational Number?

▶ **Definition of a Rational Number**

A rational number is any number that can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.
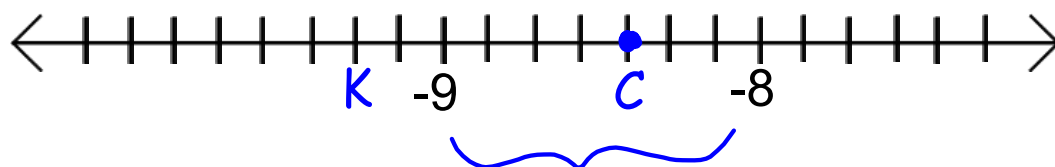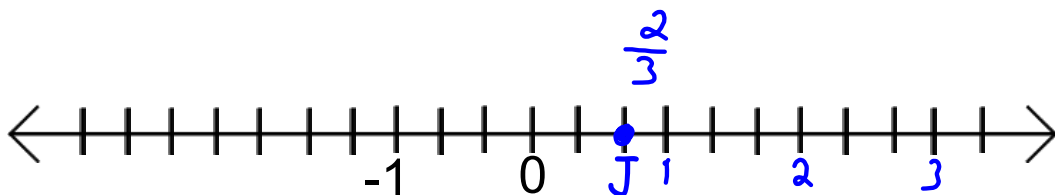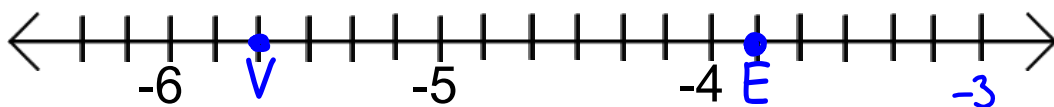
$$Q = \left\{ \frac{m}{n} \,\middle|\, m, n \in I, n \neq 0 \right\}$$

Reading that Math:

The rational numbers, **Q**, are defined as, **=**, the set of all numbers, **{}**, that can be written as a fraction, $\frac{m}{n}$, where **m** and **n** are an element of, ∈, the set of Integers, **I**, where **n** cannot equal, ≠, zero. **Math is wordy! Which is why symbols are used. Then the math can be read by anyone as the symbols are used internationally.**
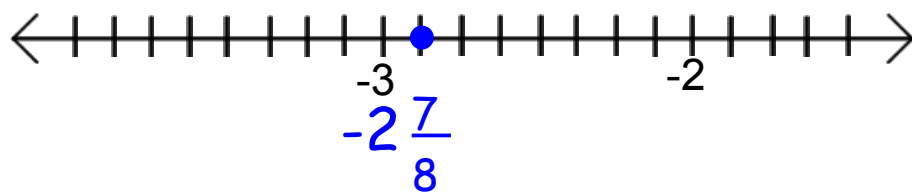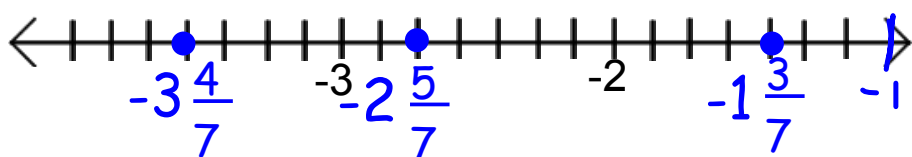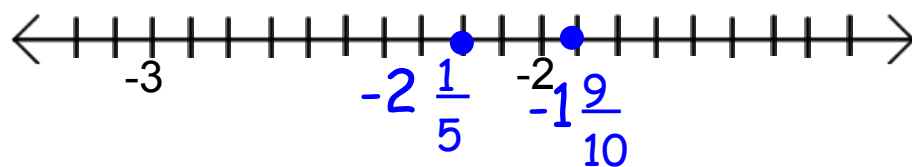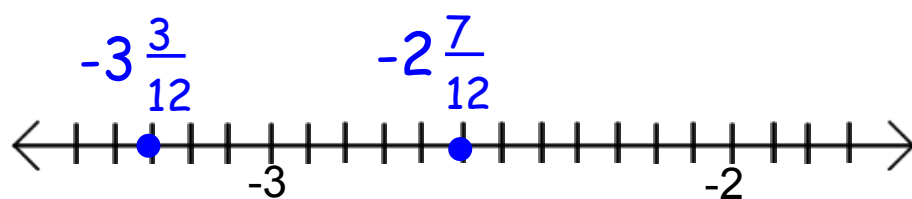
Number Lines

$V = \boxed{-5\frac{4}{6}}$ $E = -3\frac{5}{6}$

$\frac{2}{3}$

7 divisions

$C = -8\frac{3}{7}$ $K = -9\frac{2}{7}$

$-3\dfrac{3}{12}$ $-2\dfrac{7}{12}$

$-2\dfrac{1}{5}$ $-1\dfrac{9}{10}$

$-3\dfrac{4}{7}$ $-2\dfrac{5}{7}$ $-1\dfrac{3}{7}$

$-2\dfrac{7}{8}$

You try these:

8. Write the rational number represented by each letter on the number line, as a decimal.

a)

$-7\frac{9}{10}$ $-7\frac{2}{10}$ or $-7\frac{1}{5}$

-8   A   B   -7

b)

$-4\frac{2}{5}$ $-3\frac{1}{5}$

-5   C   -4   D -3

c)

$-\frac{7}{10}$ $-\frac{2}{10}$ or $-\frac{1}{5}$

-1   J   K   0

d)

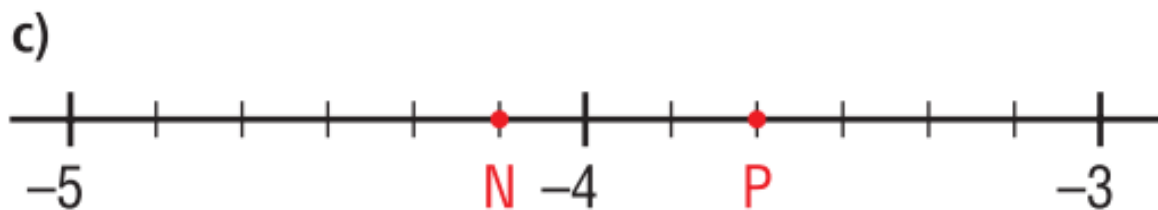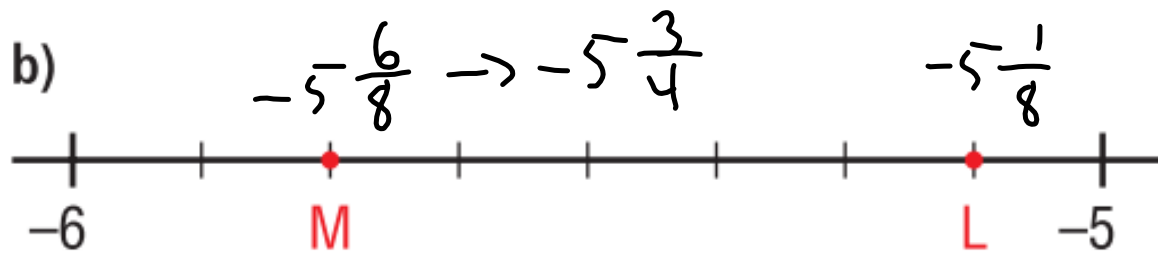-15.37 -15.32

-15.4   G   H   -15.3
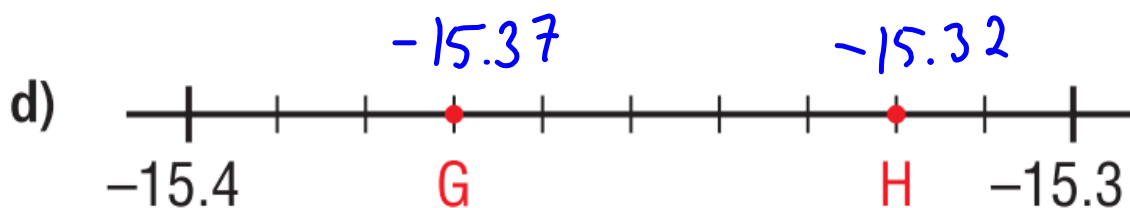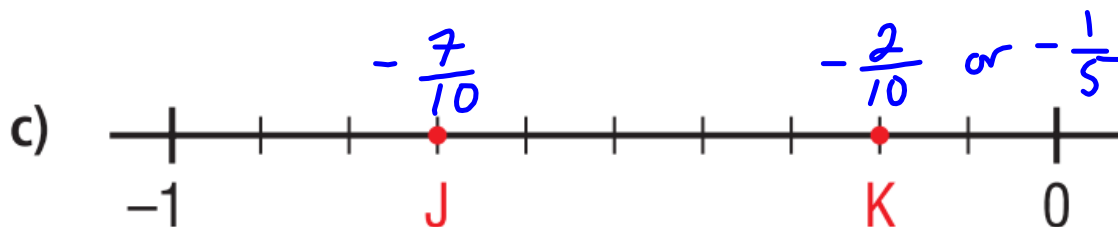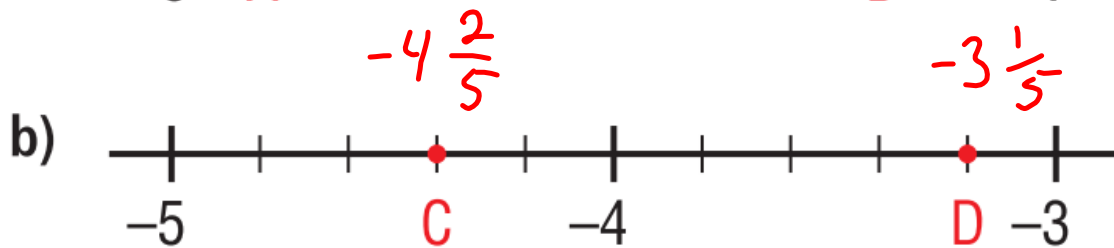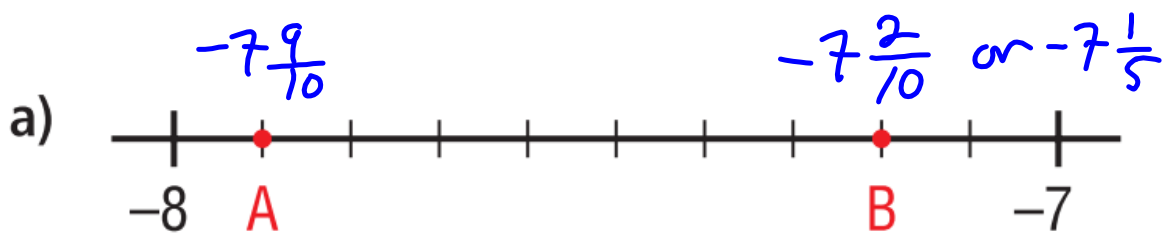
and these:

10. Write the rational number represented by each letter on the number line, as a fraction.

a)

$-11\frac{1}{4}$　　　$-10\frac{3}{4}$

-12　　　　　E　-11　　F　　　　　-10

b)

$-5\frac{6}{8} \rightarrow -5\frac{3}{4}$　　　　　$-5\frac{1}{8}$

-6　　　　M　　　　　　　　L　　-5

c)

-5　　　　　　N -4　　P　　　　-3

d)

$-\frac{3}{4}$　　　Q $-\frac{1}{2}$　　　　$-\frac{1}{4}$ R　　　0
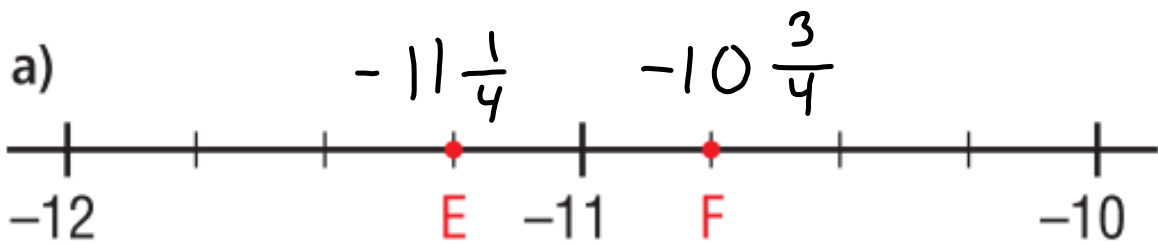
Any number that can be written as a ratio of integers are rational, so don't forget about these:

A fraction can be written as a terminating or repeating decimal:
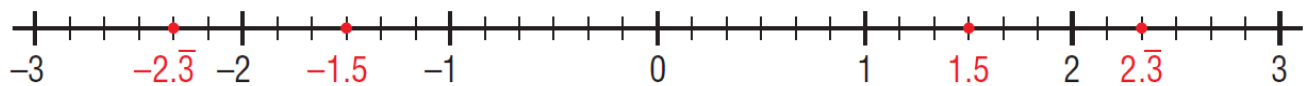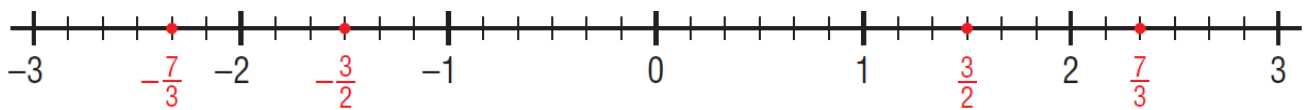
Any mixed number can be written as an improper fraction:

$$3\frac{1}{4} = \frac{13}{4} \quad \text{and} \quad -3\frac{1}{4} = -\frac{13}{4}$$
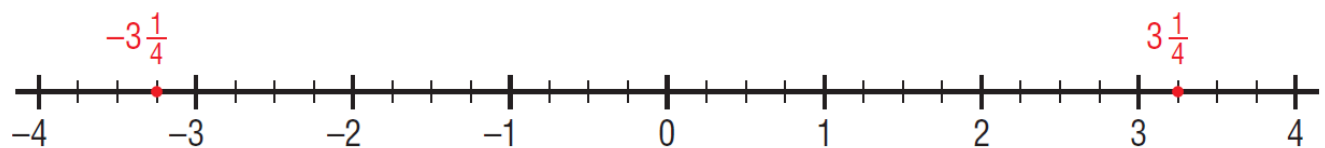
So, mixed numbers are rational numbers.

Any integer can be written as a fraction with denominator 1; for example, $-12 = \frac{-12}{1}$, so integers are rational numbers.

Write $\frac{4}{5}$ as a decimal.

0.8

Warm Up

What is a rational number?

You must be specific about three things!

State two rational numbers between -5.4 and -5.5

Express A as a mixed fraction.

A

5   6

Write two equivalent fractions!

$\frac{-8}{9}$

Find two rational numbers between...

(Decimals may be used on this side.)   (NO Decimals please!!.)

$\dfrac{-3}{8}$ $\dfrac{-4}{8}$ $\dfrac{5}{8} \begin{smallmatrix} \times 3 \\ \times 3 \end{smallmatrix}$ $\dfrac{6}{8} \begin{smallmatrix} \times 3 \\ \times 3 \end{smallmatrix}$

-0.375 -0.5 $\dfrac{15}{24}$ $\dfrac{18}{24}$

1st: -0.4

2nd: -0.45

$\boxed{\dfrac{17}{24} , \dfrac{16}{24}}$

Show your work!

Which rational number is larger??

(Decimals may be used on this side.)   (NO Decimals please!!.)

$\dfrac{-12}{15}$ $\dfrac{-13}{16}$ $\dfrac{2}{3}\times\dfrac{4}{4}$ $\dfrac{3}{4}\times\dfrac{3}{3}$

$\boxed{-0.8}$ (circled) -0.8125 make denominators the same.

$-0.8 > 0.8125$ $\dfrac{8}{12}$ $\boxed{<}$ $\dfrac{9}{12}$

Show your work!

Example 1   **Writing a Rational Number between Two Given Numbers**
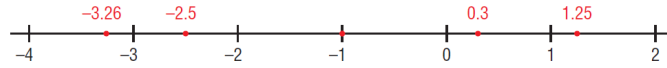
Write 3 rational numbers between each pair of numbers.

a) 1.25 and -3.26 b) -0.25 and -0.26 c) $-\frac{1}{2}$ and $\frac{1}{4}$ d) $-\frac{1}{2}$ and $-\frac{1}{4}$

▶ **A Solution**

There are many rational numbers between any two given numbers.
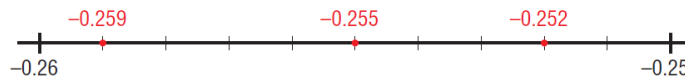Sketch or visualize a number line in each case.

a) 1.25 and -3.26

Label a number line with integers from -4 to 2.

From the number line, 3 possible rational numbers are:
$-2.5, -1,$ and 0.3

b) -0.25 and -0.26

Label a number line with these rational numbers.
Divide the line into 10 equal parts.

From the number line, 3 possible rational numbers are:
$-0.252, -0.255,$ and -0.259

d) $-\frac{1}{2}$ and $-\frac{1}{4}$

Label a number line from -1 to 0. Divide the line into quarters.

Write equivalent fractions for $-\frac{1}{2}$ and $-\frac{1}{4}$ with denominators of 8
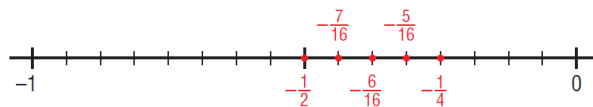to identify fractions between the two numbers.

$$-\frac{1}{2} = -\frac{2}{4} = -\frac{4}{8} \qquad\qquad -\frac{1}{4} = -\frac{2}{8}$$

Between $-\frac{4}{8}$ and $-\frac{2}{8}$, there is only one fraction, $-\frac{3}{8}$, with denominator 8.
So, write equivalent fractions with denominator 16:

$$-\frac{1}{2} = -\frac{2}{4} = -\frac{4}{8} = -\frac{8}{16} \qquad\qquad -\frac{1}{4} = -\frac{2}{8} = -\frac{4}{16}$$

Divide the number line into sixteenths.

From the number line, 3 possible rational numbers are: $-\frac{5}{16}, -\frac{6}{16},$ and $-\frac{7}{16}$

*Might be easier for this one to convert to decimals, get the
numbers, and change back to fractions.

$-\frac{1}{2}$ and $-\frac{1}{4}$ = -0.5 and -0.25

So, -0.3, -0.4, and -0.45 are between them.

As fractions: $-\dfrac{3}{10} -\dfrac{2}{5},$ and $-\dfrac{9}{20}$

| **Example 2** | **Ordering Rational Numbers in Decimal or Fraction Form** |

a) Use a number line. Order these numbers from least to greatest.

$0.35, 2.5, -0.6, 1.7, -3.2, -0.\overline{6}$

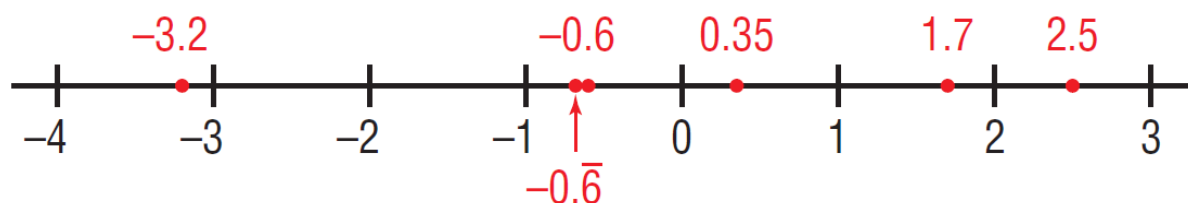**b)** Order these numbers from greatest to least. Record the numbers on a number line.

$-\dfrac{3}{8}, \dfrac{5}{9}, -\dfrac{10}{4}, -1\dfrac{1}{4}, \dfrac{7}{10}, \dfrac{8}{3}$

▶ Solutions

a) $0.35, 2.5, -0.6, 1.7, -3.2, -0.\overline{6}$

Mark each number on a number line.

$-0.\overline{6} = -0.666\ 666...;$ so, $-0.\overline{6} < -0.6$

For least to greatest, read the numbers from left to right: $-3.2, -0.\overline{6}, -0.6, 0.35, 1.7, 2.5$

b) $-\dfrac{3}{8}, \dfrac{5}{9}, -\dfrac{10}{4}, -1\dfrac{1}{4}, \dfrac{7}{10}, \dfrac{8}{3}$

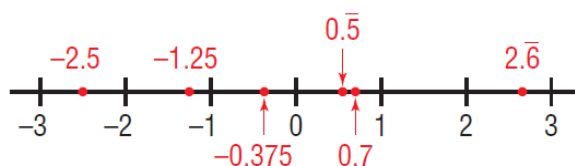Write each number as a decimal.

Use a calculator when necessary.

$$-\frac{3}{8} = -0.375 \qquad \frac{5}{9} = 0.\overline{5}$$

$$-\frac{10}{4} = -2.5 \qquad -1\frac{1}{4} = -1.25$$

$$\frac{7}{10} = 0.7 \qquad \frac{8}{3} = 2.\overline{6}$$

Mark each decimal on a number line.
Use the order of the decimals to
order the fractions.
From greatest to least, the
numbers are:

$\dfrac{8}{3}, \dfrac{7}{10}, \dfrac{5}{9}, -\dfrac{3}{8}, -1\dfrac{1}{4}, -\dfrac{10}{4}$

Example 3	Ordering Rational Numbers in Fraction and Decimal Form

Order these rational numbers from least to greatest.

$1.13, -\dfrac{10}{3}, -3.4, 2.\overline{7}, \dfrac{3}{7}, -2\dfrac{2}{5}$

Record the numbers on a number line.

► **A Solution**

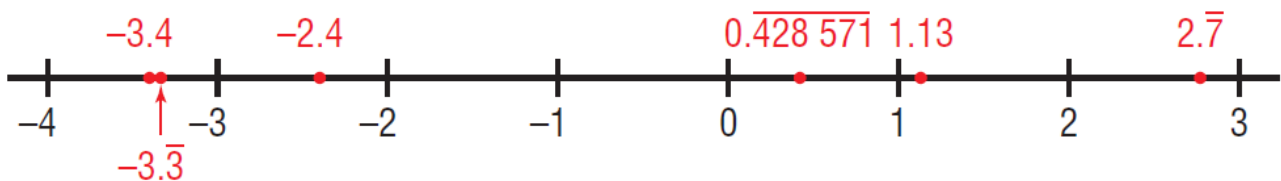$1.13, -\dfrac{10}{3}, -3.4, 2.\overline{7}, \dfrac{3}{7}, -2\dfrac{2}{5}$

Write the fractions and mixed number as decimals.

$-\dfrac{10}{3} = -3.\overline{3}$

$\dfrac{3}{7} = 0.\overline{428\ 571}$

$-2\dfrac{2}{5} = -2.4$

Mark each decimal on a number line.

For least to greatest, read the decimals from left to right.

The order is:

$-3.4, -\dfrac{10}{3}, -2\dfrac{2}{5}, \dfrac{3}{7}, 1.13, 2.\overline{7}$

Textbook Practice:

Practice for Monday → Pg 101 #5 - 7, 9, 11, 12.

Pg. 102 #14aceg, 21.

Pg. 103 # 27.