Solutions

SOLUTIONS => Chapter 5 Review

1.

a) $8\sqrt{5}$
$= \sqrt{(8)^2(5)}$
$= \sqrt{(64)(5)}$
$= \sqrt{320}$

b) $-2\sqrt[5]{3}$
$= \sqrt[5]{(-2)^5(3)}$
$= \sqrt[5]{(-32)(3)}$
$= \sqrt[5]{-96}$

c) $3y^3\sqrt{7}$
$= \sqrt{(3)^2(y^3)^2(7)}$
$= \sqrt{(9)(y^6)(7)}$
$= \sqrt{63y^6}$

d) $-3z(\sqrt[3]{4z})$
$= \sqrt[3]{(-3)^3(z)^3(4z)}$
$= \sqrt[3]{(-27)(z^3)(4z)}$
$= \sqrt[3]{-108z^4}$

2.

a) $\sqrt{72}$
$= \sqrt{(2)(2)(2)(3)(3)}$
$= (2)(3)\sqrt{2}$
$= 6\sqrt{2}$

b) $3\sqrt{40}$
$= 3\sqrt{(2)(2)(2)(5)}$
$= 3(2)\sqrt{(2)(5)}$
$= 6\sqrt{10}$

c) $\sqrt{27m^2}$
$= \sqrt{(3)(3)(3)(m)(m)}$
$= 3m\sqrt{3}$

d) $\sqrt[3]{80x^5y^6}$
$= \sqrt[3]{(2)(2)(2)(2)(5)(x)(x)(x)(x)(x)(y)(y)(y)(y)(y)(y)}$
$= 2xy^2\sqrt[3]{(2)(5)(x)(x)}$
$= 2xy^2\sqrt[3]{10x^2}$

3

a) $-\sqrt{13} + 2\sqrt{13}$
$= 1\sqrt{13}$

b) $4\sqrt{7} - 2\sqrt{112}$
$= 4\sqrt{7} - 2\sqrt{(2)(2)(2)(2)(7)}$
$= 4\sqrt{7} - 2(2)(2)\sqrt{7}$
$= 4\sqrt{7} - 8\sqrt{7}$
$= -4\sqrt{7}$

c) $-\sqrt[3]{3} + \sqrt[3]{24}$
$= -\sqrt[3]{3} + \sqrt[3]{(2)(2)(2)(3)}$
$= -\sqrt[3]{3} + 2\sqrt[3]{3}$
$= 1\sqrt[3]{3}$

Solutions

4a) $4\sqrt{45x^3} - \sqrt{27x} + 17\sqrt{3x} - 9\sqrt{125x^3}$

$= 4\sqrt{(3)(3)(5)(x)(x)(x)} - \sqrt{(3)(3)(3)(x)} + 17\sqrt{3x} - 9\sqrt{(5)(5)(5)(x)(x)(x)}$

$= 4(3)(x)\sqrt{5x} - 3\sqrt{3x} + 17\sqrt{3x} - 9(5)(x)\sqrt{5x}$

$= 12x\sqrt{5x} - 3\sqrt{3x} + 17\sqrt{3x} - 45x\sqrt{5x}$

$= -33x\sqrt{5x} + 14\sqrt{3x}, \quad x \geq 0$

b) $\frac{2}{5}\sqrt{44a} + \sqrt{144a^3} - \frac{\sqrt{11a}}{2}$

$= \frac{2}{5}\sqrt{(2)(2)(11)a} + \sqrt{(2)(2)(2)(2)(3)(3)(a)(a)(a)} - \frac{1}{2}\sqrt{11a}$

$= \frac{2}{5}(2)\sqrt{11a} + (2)(2)(3)(a)\sqrt{a} - \frac{1}{2}\sqrt{11a}$

$= \frac{4}{5}\sqrt{11a} + 12a\sqrt{a} - \frac{1}{2}\sqrt{11a}$

$= \frac{4}{5}\sqrt{11a} - \frac{1}{2}\sqrt{11a} + 12a\sqrt{a}$

$= \frac{8}{10}\sqrt{11a} - \frac{5}{10}\sqrt{11a} + 12a\sqrt{a}$

$= \frac{3}{10}\sqrt{11a} + 12a\sqrt{a}, \quad a \geq 0$

5. $\boxed{8\sqrt{7}}$

↳ $2\sqrt{112}$

$= 2\sqrt{(2)(2)(2)(2)(7)}$

$= 2(2)(2)\sqrt{7}$

$= 8\sqrt{7}$ EQUIVALENT!

↳ $3\sqrt{42}$

NOT EQUIVALENT!

↳ $\sqrt{448}$

$= \sqrt{(2)(2)(2)(2)(2)(2)(7)}$

$= (2)(2)(2)\sqrt{7}$

$= 8\sqrt{7}$ EQUIVALENT!

↳ $4\sqrt{28}$

$= 4\sqrt{(2)(2)(7)}$

$= 4(2)\sqrt{7}$

$= 8\sqrt{7}$ EQUIVALENT!

Solutions

6. Order the following numbers from least to greatest: $3\sqrt{7}$, $\sqrt{65}$, $2\sqrt{17}$, 8.

$3\sqrt{7}$
$= \sqrt{(3)^2(7)}$
$= \sqrt{(9)(7)}$
$= \sqrt{63}$

$\sqrt{65}$

$2\sqrt{17}$
$= \sqrt{(2)^2(17)}$
$= \sqrt{(4)(17)}$
$= \sqrt{68}$

8
$= \sqrt{64}$

From least to greatest: $3\sqrt{7}, 8, \sqrt{65}, 2\sqrt{17}$

7. The speed, v, in kilometers per hour, of a car before a collision can be approximated from the length, d, in meters, of the skid mark left by the tire. On a dry day, one formula that approximates this speed is $v = \sqrt{169d}$, $d \geq 0$.

a) Rewrite the formula as a mixed radical.

$v = \sqrt{(13)(13)(d)}$
$v = 13\sqrt{d}$

b) What is the approximate speed of a car if the skid mark measures 13.4 m? Express your answer to the nearest kilometer per hour.

$v = 13\sqrt{13.4m}$
$v = 48$ km/h

Solutions

10.

a) $\sqrt{2}(\sqrt{6})$
$= \sqrt{12}$
$= \sqrt{(2)(2)(3)}$
$= 2\sqrt{3}$

b) $(-3f\sqrt{15})(2f^3\sqrt{5})$
$= -6f^4\sqrt{75}$
$= -6f^4\sqrt{(5)(5)(3)}$
$= -6f^4(5)\sqrt{3}$
$= -30f^4\sqrt{3}$

c) $(\sqrt[4]{8})(3\sqrt[4]{18})$
$= 3\sqrt[4]{144}$
$= 3\sqrt[4]{(2)(2)(2)(2)(3)(3)}$
$= 3(2)\sqrt[4]{9}$
$= 6\sqrt[4]{9}$

11.

a) $(2-\sqrt{5})(2+\sqrt{5})$
$= 4+2\sqrt{5}-2\sqrt{5}-5$
$= 4-5$
$= -1$

b) $(5\sqrt{3}-\sqrt{8})^2$
$=(5\sqrt{3}-\sqrt{8})(5\sqrt{3}-\sqrt{8})$
$=25(3) - 5\sqrt{24} - 5\sqrt{24} + 8$
$= 75-10\sqrt{24}+8$
$= 83-10\sqrt{24}$
$= 83-10\sqrt{(2)(2)(2)(3)}$
$= 83-10(2)\sqrt{6}$
$= 83-20\sqrt{6}$

c) $(a+3\sqrt{a})(a+7\sqrt{4a})$
$= a^2+7a\sqrt{4a}+3a\sqrt{a}+21\sqrt{4a^2}$
$= a^2+7a(2)\sqrt{a}+3a\sqrt{a}+21(2a)$
$= a^2+14a\sqrt{a}+3a\sqrt{a}+42a$
$= a^2+17a\sqrt{a}+42a,\ a\geq 0$

Solutions

13.

a) $\frac{\sqrt{6}}{\sqrt{12}}$

$= \frac{\sqrt{6}}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}}$

$= \frac{\sqrt{72}}{12}$

$= \frac{\sqrt{(2)(2)(2)(3)(3)}}{12}$

$= \frac{(2)(3)\sqrt{2}}{12}$

$= \frac{6\sqrt{2}}{12}$

$= \frac{\sqrt{2}}{2}$

b) $\frac{-1}{\sqrt[3]{25}}$

$= \frac{-1}{\sqrt[3]{25}} \cdot \frac{(\sqrt[3]{25})^2}{(\sqrt[3]{25})^2}$

$= -\frac{(\sqrt[3]{25})^2}{25}$

c) $-4\sqrt{\frac{2a^2}{9}}$, $a \geq 0$

$= -4\frac{\sqrt{2a^2}}{\sqrt{9}}$

$= -\frac{4a\sqrt{2}}{3}$

14.

a) $\frac{-2}{4-\sqrt{3}}$

$= \frac{-2}{4-\sqrt{3}} \cdot \frac{(4+\sqrt{3})}{(4+\sqrt{3})}$

$= \frac{-8-2\sqrt{3}}{16-3}$

$= \frac{-8-2\sqrt{3}}{13}$

b) $\frac{\sqrt{7}}{2\sqrt{5}-\sqrt{7}}$

$= \frac{\sqrt{7}}{2\sqrt{5}-\sqrt{7}} \cdot \frac{(2\sqrt{5}+\sqrt{7})}{(2\sqrt{5}+\sqrt{7})}$

$= \frac{2\sqrt{35}+7}{(4)(5)-7}$

$= \frac{2\sqrt{35}+7}{20-7}$

$= \frac{2\sqrt{35}+7}{13}$

Solutions

c) $\frac{18}{6+\sqrt{27m}}$

$= \frac{18}{6+\sqrt{27m}} \cdot \frac{(6-\sqrt{27m})}{(6-\sqrt{27m})}$

$= \frac{108-18\sqrt{27m}}{36-27m}$

$= \frac{108-18\sqrt{(3)(3)(3)m}}{36-27m}$

$= \frac{108-18(3)\sqrt{3m}}{36-27m}$

$= \frac{108-54\sqrt{3m}}{36-27m}$

$= \frac{12-6\sqrt{3m}}{4-3m}$

$m \geq 0$ and $m \neq \frac{4}{3}$

d) $\frac{a+\sqrt{b}}{a-\sqrt{b}}$

$= \frac{a+\sqrt{b}}{a-\sqrt{b}} \cdot \frac{(a+\sqrt{b})}{(a+\sqrt{b})}$

$= \frac{a^2+2a\sqrt{b}+b}{a^2-b}$

$b \geq 0$ and $b \neq a^2$

16.

a) $\left(\frac{-5\sqrt{3}}{\sqrt{6}}\right)\left(\frac{-\sqrt{7}}{3\sqrt{21}}\right)$

$= \frac{5\sqrt{21}}{3\sqrt{126}}$

$= \frac{5\sqrt{21}}{3\sqrt{(2)(3)(3)(7)}}$

$= \frac{5\sqrt{21}}{3(3)\sqrt{14}}$

$= \frac{5\sqrt{21}}{9\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}}$

$= \frac{5\sqrt{294}}{9(14)}$

$= \frac{5\sqrt{(2)(3)(7)(7)}}{126}$

$= \frac{35\sqrt{6}}{126} = \frac{5\sqrt{6}}{18}$

b) $\left(\frac{2a\sqrt{a^3}}{9}\right)\left(\frac{12}{-\sqrt{8a}}\right)$

$= \frac{24a\sqrt{a^3}}{-9\sqrt{8a}}$

$= \frac{24a^2\sqrt{a}}{-9\sqrt{8a}} \cdot \frac{\sqrt{8a}}{\sqrt{8a}}$

$= \frac{24a^2\sqrt{8a^2}}{-9(8a)}$

$= \frac{24a^2\sqrt{(2)(2)(2)(a)(a)}}{-9(8a)}$

$= \frac{24a^2(2)(a)\sqrt{2}}{-72a}$

$= \frac{48a^3\sqrt{2}}{-72a}$

$= -\frac{2a^2\sqrt{2}}{3}$

Solutions

18.

a) $-\sqrt{x} = -7$

1 $x \geq 0$

2 $(-\sqrt{x})^2 = (-7)^2$

$x = 49$

b) $\sqrt{4-x} = -2$

1 $4 - x \geq 0$

$\frac{-x}{-1} \geq \frac{-4}{-1}$

$x \leq 4$

2 $\sqrt{4-x} = -2$

NO SOLUTION!

c) $5 - \sqrt{2x} = -1$

1 $\frac{2x}{2} \geq \frac{0}{2}$

$x \geq 0$

2 $5 - \sqrt{2x} = -1$

$5 + 1 = \sqrt{2x}$

$6 = \sqrt{2x}$

$(6)^2 = (\sqrt{2x})^2$

$\frac{36}{2} = \frac{2x}{2}$

$18 = x$

d) $1 + \sqrt{\frac{7x}{3}} = 8$

1 $\frac{7x}{3} \geq 0$

$\frac{7x}{7} \geq \frac{0}{7}$

$x \geq 0$

2 $1 + \sqrt{\frac{7x}{3}} = 8$

$\sqrt{\frac{7x}{3}} = 8 - 1$

$\sqrt{\frac{7x}{3}} = 7$

$\frac{\sqrt{7x}}{\sqrt{3}} = 7$

$\sqrt{7x} = 7\sqrt{3}$

$(\sqrt{7x})^2 = (7\sqrt{3})^2$

$7x = (49)(3)$

$\frac{7x}{7} = \frac{147}{7}$

$x = 21$

Solutions

(2) 19a) $\sqrt{5x-3} = \sqrt{7x-12}$

$(\sqrt{5x-3})^2 = (\sqrt{7x-12})^2$

$5x-3 = 7x-12$

$-3+12 = 7x-5x$

$\frac{9}{2} = \frac{2x}{2}$

$\frac{9}{2} = x$

Restrictions:

(1) $5x-3 \geq 0$ $\quad$ $7x-12 \geq 0$

$\frac{5x}{5} \geq \frac{3}{5}$ $\quad$ $\frac{7x}{7} \geq \frac{12}{7}$

$x \geq \frac{3}{5}$ $\quad$ $\boxed{x \geq \frac{12}{7}}$

(2) b) $\sqrt{y-3} = y-3$

$(\sqrt{y-3})^2 = (y-3)^2$

$y-3 = y^2-6y+9$

$0 = y^2-6y-y+9+3$

$0 = y^2-7y+12$

$0 = (y-4)(y-3)$

$0 = y-4$ or $0 = y-3$

$4 = y$ or $3 = y$

(1) $y-3 \geq 0$

$y \geq 3$

(2) c) $\sqrt{7n+25} - n = 1$

$\sqrt{7n+25} = 1+n$

$(\sqrt{7n+25})^2 = (1+n)^2$

$7n+25 = 1+2n+n^2$

$0 = n^2+2n-7n+1-25$

$0 = n^2-5n-24$

$0 = (n+3)(n-8)$

$0 = n+3$ or $0 = n-8$

$-3 = n$ or $\boxed{8 = n}$

$\Downarrow$

Extraneous Root!

(1) $7n+25 \geq 0$

$\frac{7n}{7} \geq \frac{-25}{7}$

$n \geq -\frac{25}{7}$

Solutions

21. On a calm day, the distance, d, in kilometers, that the coast guard crew on the Coast Guard cutter Vakta can see to the horizon depends on their height, h, in meters, above the water. The formula $d = \sqrt{\frac{3h}{2}}, h \geq 0$ models this relationship. What is the height of the crew above the water if the distance to the horizon is 7.1 km?

$$d = \sqrt{\frac{3h}{2}}$$

$$7.1 = \sqrt{\frac{3h}{2}}$$

$$(7.1)^2 = \left(\sqrt{\frac{3h}{2}}\right)^2$$

$$50.41 = \frac{3h}{2}$$

$$(50.41)(2) = 3h$$

$$\frac{100.82}{3} = \frac{3h}{3}$$

$$33.6\text{m} = h$$