4.1 Writing Equations to Describe Patterns

Connect

A landscape designer uses wooden boards as edging for the plots in a herb garden.

1 plot 2 plots 3 plots 4 plots

The number of boards, b, is *related* to the number of plots, p.

➤ Determine a pattern in the number of boards.

Number of Plots, p	Number of Boards, b
1	4
2	7
3	10
4	13

(+1, +1, +1 between the plot values; +3, +3, +3 between the board values)

As the number of plots increases by 1, the number of boards increases by 3. Repeated addition of 3 is the same as multiplication by 3. This suggests that the number of boards may be 3 times the number of plots. So, the equation $b = 3p$ may represent this relationship.

This is 1 less than the number 4 in the table. So, we add 1 to $3p$ to describe the number of boards correctly.

The terms $3p + 1$ form an *expression* that represents the number of boards for any number of plots p. An equation is: $b = 3p + 1$

Number of Plots, p	Number of Boards, b
1	$3(1) + 1 = 4$
2	$3(2) + 1 = 7$
3	$3(3) + 1 = 10$
4	$3(4) + 1 = 13$

Example 1 Writing an Equation to Represent a Written Pattern

An airplane is cruising at a height of 10 000 m. It descends to land. This table shows the height of the plane every minute after it began its descent. The height of the plane changes at a constant rate.

Time (t minutes)	Height (h metres)
0	10 000
1	9 700
2	9 400
3	9 100
4	8 800

a) Write an expression for the height in terms of the time since the plane began its descent.
b) Write an equation that relates the height of the plane to the time since it began its descent.
c) What is the height of the plane after 15 min?
d) How long after beginning its descent does the plane land?

a) When the time increases by 1 min, the height decreases by 300 m.
Add a third column to the table and write the height in terms of time.

	Time (t minutes)	Height (h metres)	Height in Terms of Time	
	0	10 000	$10\,000 - 0 = 10\,000$	
+1	1	9 700	$10\,000 - 300(1) = 9700$	−300
+1	2	9 400	$10\,000 - 300(2) = 9400$	−300
+1	3	9 100	$10\,000 - 300(3) = 9100$	−300
+1	4	8 800	$10\,000 - 300(4) = 8800$	−300
+1	:		:	−300
+1	t		$10\,000 - 300(t)$	−300

b) For an equation that relates height to time, equate the expression in part a to the height, h.
An equation is: $h = 10\,000 - 300t$

c) To determine the height of the plane after 15 min, substitute $t = 15$ in the equation:

$$\begin{aligned} h &= 10\,000 - 300t \\ &= 10\,000 - 300(15) \\ &= 10\,000 - 4500 \\ &= 5500 \end{aligned}$$

After 15 min, the plane is at a height of 5500 m.

d) When the plane lands, its height is 0.
Substitute $h = 0$ in the equation $h = 10\,000 - 300t$, then solve for t.

$$\begin{aligned} h &= 10\,000 - 300t \\ 0 &= 10\,000 - 300t \\ 300t + 0 &= 10\,000 - 300t + 300t \\ 300t &= 10\,000 \\ \frac{300t}{300} &= \frac{10\,000}{300} \\ t &= 33.\overline{3} \end{aligned}$$

The plane lands about 33 min after beginning its descent.

Writing Linear Equations from a Table of Values

x	y
0	4
1	1
2	-2
3	-5
4	-8
5	-11

Linear equations: The largest exponent on a variable is 1.

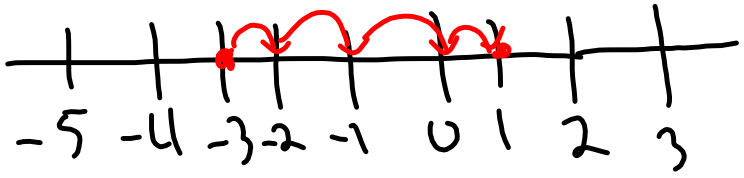
- Start with the form ***y=mx+b***
- Make sure "x" numbers differ by 1.
- The difference from one "y" value to another is equal to "m".
- Use a value of "x" and "y" to figure out "b".
  - > Or work backwards until x = 0 (the value of y when x = 0 is "b" in the mathematical relation).

$y = mx + b$

$y = -3x + b$

$1 = -3(1) + b$

$1 = -3 + b$ so $b = 4$



$y = -3x + 4$