
a) Given the launch velocity of a projectile is $v$, and target coordinates are ( $x, y$ ) relative to the launch position; derive the equation below which is necessary for finding the angle of attack:

$$
\frac{g x^{2}}{2 v^{2}} \tan ^{2} \theta+x \tan \theta+\frac{g x^{2}}{2 v^{2}}-y=0
$$

b) Given the equation above, calculate the angles that would hit the target:
i. Located at $(x, y)=(59,18)$ with an initial speed of $35 \mathrm{~m} / \mathrm{s}$.
ii. Located at $(x, y)=(44,-21)$ with an initial speed of $42 \mathrm{~m} / \mathrm{s}$.
c) Note that the quadratic gives two possible answers. (PhET projectile simulation might help visualize the situation: http://phet.colorado.edu/en/simulation/legacy/projectile-motion)
i. Conceptually, why is it possible for a target to be hit by using one of two different launch angles in the case $y>0$ ?
ii. Same question but for $y<0$ ?
d) It is possible that there exists a target location ( $x, y$ ) that cannot be hit by a projectile with a velocity, $v$. In that case, how would the math equations communicate such a problem? Choose numbers for $(x, y)$ and $v$ that will result in no possible angle and show this by solving the equation from (a) with your values.
e) There exists a target location $x$, where $y \neq 0$, that can only be hit by one and only one angle for a given velocity, v.
i. Mathematically, what would have to happen in solving the equation from (a) for only one angle to be possible?
ii. Based on your response from (i), what is the formula for $\tan (\theta)$ that allows for the calculation of the only possible angle.
iii. Derive a formula for such a horizontal target location, $x$, knowing $v, g$, and $y$.
f) Use your results from (e) to calculate the horizontal target location, $x$, and the only angle that will hit the target when $v=25 \mathrm{~m} / \mathrm{s}$ and $\mathrm{y}=15 \mathrm{~m}$.

